Learning and Inference in Markov Logic Networks

CS 486/686
University of Waterloo
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Outline

- Markov Logic Networks
  - Parameter learning
  - Lifted inference

Parameter Learning

- Where do Markov logic networks come from?

- Easy to specify first order formulas
- Hard to specify weights due to unclear interpretation

- Solution:
  - Learn weights from data
  - Preliminary work to learn first-order formulas from data

Parameter tying

- Observation: first-order formulas in Markov logic networks specify templates of features with identical weights

- Key: tie parameters corresponding to identical weights

- Parameter learning:
  - Same as in Markov networks
  - But many parameters are tied together

Parameter tying

- Parameter tying ➔ few parameters
  - Faster learning
  - Less training data needed
- Maximum likelihood: $\theta^* = \text{argmax}_\theta P(\text{data}|\theta)$
  - Complete data: convex opt., but no closed form
    - Gradient descent, conjugate gradient, Newton's method
  - Incomplete data: non-convex optimization
    - Variants of the EM algorithm

Grounded Inference

- Grounded models
  - Bayesian networks
  - Markov networks
- Common property
  - Joint distribution is a product of factors
- Inference queries: Pr(X|E)
  - Variable elimination

Grounded Inference

- Inference query: Pr(α|β)?
  - α and β are first order formulas
- Grounded inference:
  - Convert Markov Logic Network to ground Markov network
  - Convert α and β into grounded clauses
  - Perform variable elimination as usual
- This defeats the purpose of having a compact representation based on first-order logic… Can we exploit the first-order representation?

Lifted Inference

- Observation: first order formulas in Markov Logic Networks specify templates of identical potentials.
- Question: can we speed up inference by taking advantage of the fact that some potentials are identical?

Caching

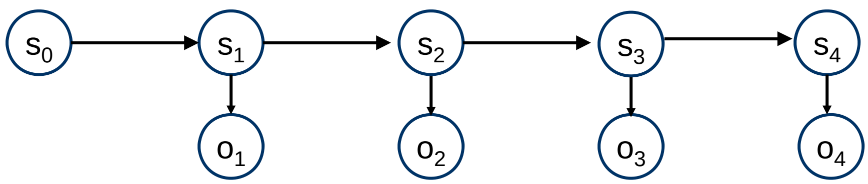
- Idea: cache all operations on potentials to avoid repeated computation
- Rational: since some potentials are identical, some operations on potentials may be repeated.
- Inference with caching: Pr($\alpha|\beta$)?
  - Convert Markov logic network to ground Markov network
  - Convert α and β to grounded clauses
  - Perform variable elimination with caching
    - Before each operation on factors, check answer in cache
    - After each operation on factors, store answer in cache

Caching

- How effective is caching?
- Computational complexity
  - Still exponential in the size of the largest intermediate factor
  - But, potentially sub-linear in the number of ground potentials/features
    - This can be significant for large networks
- Savings depend on the amount of repeated computation
  - Elimination order influences amount of repeated computation

Example: Hidden Markov Model

- Conditional distributions:
  - $Pr(S_0)$, $Pr(S_{t+1}|S_t)$, $Pr(O_t|S_t)$
  - Identical factors at each time step



Hidden Markov Models

Markov Logic Network encoding

```
obs = { Obs1, … , ObsN }
state = { St1, … , StM }
time = { 0, … , T }

State(state!,time)
Obs(obs!,time)

State(+s,0)
State(+s,t) ^ State(+s',t+1)
Obs(+o,t) ^ State(+s,t)
```

State Prediction

- Common task: state prediction
  - Suppose we have a belief at time t: $Pr(S_t|\mathbf{O}_{1..t})$
  - Predict state k steps in the future: $Pr(S_{t+k}|\mathbf{O}_{1..t})$?
- $P(S_{t+k}|\mathbf{O}_{1..t}) = \Sigma_{S_t..S_{t+k-1}} P(S_t|\mathbf{O}_{1..t}) \Pi_i P(S_{t+i+1}|S_{t+i})$
- In what order should we eliminate the state variables?

Common Elimination Orders

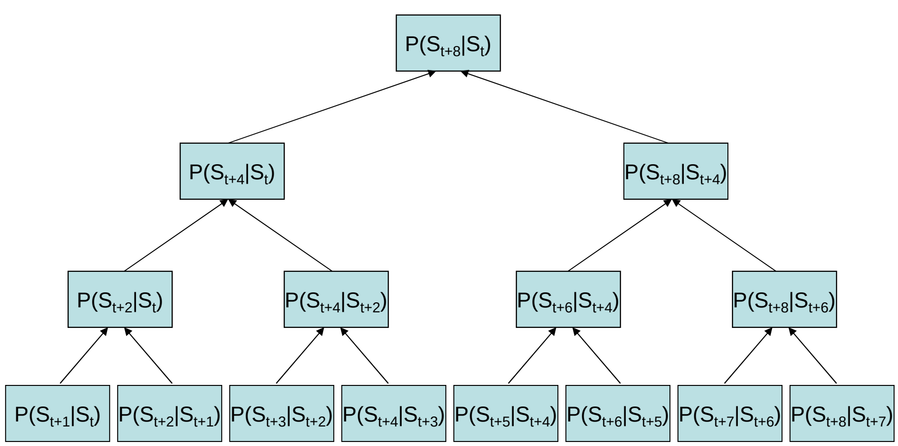
- Forward elimination
  - $P(S_{t+i+1}|\mathbf{O}_{1..t}) = \Sigma_{S_{t+i}} P(S_{t+i}|\mathbf{O}_{1..t}) P(S_{t+i+1}|S_{t+i})$
  - $P(S_{t+i}|\mathbf{O}_{1..t})$ is different for all i's, so no repeated computation
- Backward elimination
  - $P(S_{t+k}|S_{t+i}) = \Sigma_{S_{t+i+1}} P(S_{t+k}|S_{t+i+1}) P(S_{t+i+1}|S_{t+i})$
  - $P(S_{t+k}|\mathbf{O}_{1..t}) = \Sigma_{S_t} P(S_{t+k}|S_t) P(S_t|\mathbf{O}_{1..t})$
  - $P(S_{t+k}|S_{t+i})$ is different for all i's, so no repeated computation
- Any saving possible?

Pyramidal elimination

- Repeat until all variables are eliminated
  - Eliminate every other variable in order

- Example:
  - Eliminate S_{t+1}, S_{t+3}, S_{t+5}, S_{t+7}, …
  - Eliminate S_{t+2}, S_{t+6}, S_{t+10}, S_{t+14}, …
  - Eliminate S_{t+4}, S_{t+12}, S_{t+20}, S_{t+28}, …
  - Eliminate S_{t+8}, S_{t+24}, S_{t+40}, S_{t+56}, …
  - Etc.

Pyramidal elimination



Pyramidal elimination

- Observation: all operations at the same level of the pyramid are identical
  - Only one elimination per level needs to be performed
- Computational complexity:
  - log(k) instead of linear(k)

Automated elimination

- Question: how do we find an effective ordering automatically?
  - This is an area of active research
- Possible heuristic:
  - Before each elimination, examine operations that would have to be performed to eliminate each remaining variable
  - Eliminate variable that involves computation identical to the largest number of other variables (greedy heuristic)

Lifted Inference

- Variable elimination with caching still requires conversion of the Markov logic network to a ground Markov network, can we avoid that?
- Lifted inference:
  - Perform inference directly with first-order representation
  - Lifted variable elimination is an area of active research
    - Complicated algorithms due to first-order representation
    - Overhead due to the first-order representation often greater than savings in repeated computation
- Alchemy
  - Does not perform exact inference
  - Uses lifted approximate inference
    - Lifted belief propagation
    - Lifted MC-SAT (variant of Gibbs sampling)

Next Class

- Course wrap-up