Markov Logic Networks

Matt Richardson and Pedro Domingos (2006), Markov Logic Networks, *Machine Learning*, 62, 107-136, 2006.

CS 486/686
University of Waterloo
Lecture 21: Nov 20, 2012

Outline

- Markov Logic Networks
- Alchemy

Markov Logic Networks

- Bayesian networks and Markov networks:
  - Model uncertainty
  - But propositional representation (e.g., we need one variable per object in the world)
- First-order logic:
  - First-order representation (e.g., quantifiers allow us to reason about several objects simultaneously)
  - But we can't deal with uncertainty
- Markov logic networks: combine Markov networks and first-order logic

Markov Logic

- A logical KB is a set of **hard constraints** on the set of possible worlds
- Let's make them **soft constraints**: when a world violates a formula, it becomes less probable, not impossible
- Give each formula a **weight**: (higher weight ➔ stronger constraint)

$$P(\text{world}) \propto e^{\Sigma \text{ weights of formulas it satisfies}}$$

Markov Logic: Definition

- A Markov Logic Network (MLN) is a set of pairs (F, w) where
  - F is a formula in first-order logic
  - w is a real number
- Together with a set of constants, it defines a Markov network with
  - One node for each grounding of each predicate in the MLN
  - One feature for each grounding of each formula F in the MLN, with the corresponding weight w

Example: Friends & Smokers

Smoking causes cancer.
Friends have similar smoking habits.

Example: Friends & Smokers

$$\forall x\ Smokes(x) \Rightarrow Cancer(x)$$
$$\forall x, y\ Friends(x, y) \Rightarrow \left(Smokes(x) \Leftrightarrow Smokes(y)\right)$$

Example: Friends & Smokers

1.5	$\forall x\ Smokes(x) \Rightarrow Cancer(x)$
1.1	$\forall x, y\ Friends(x, y) \Rightarrow \left(Smokes(x) \Leftrightarrow Smokes(y)\right)$

Example: Friends & Smokers

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Two constants: **Anna** (A) and **Bob** (B)

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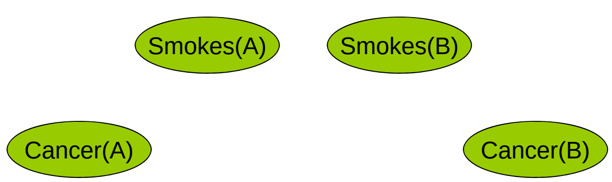
Two constants: **Anna** (A) and **Bob** (B)



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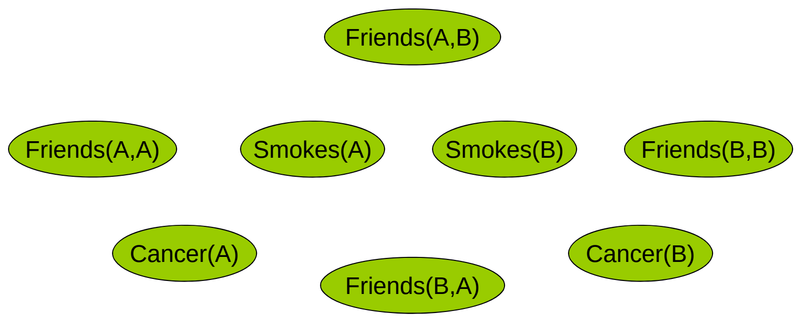

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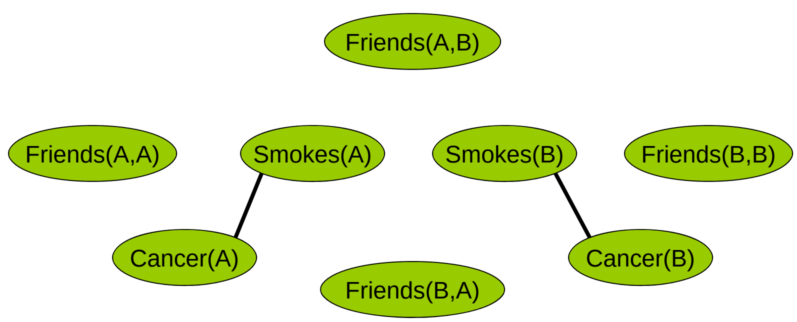

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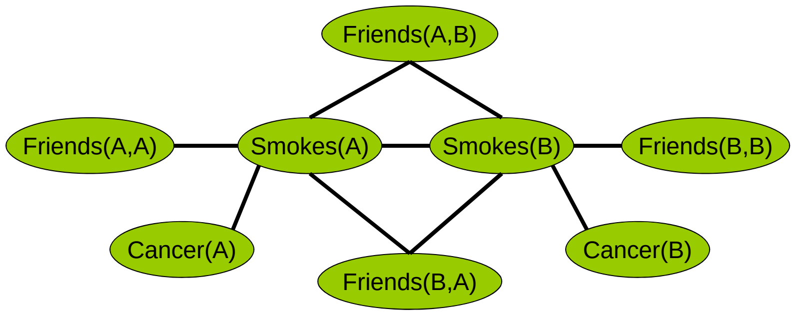

Markov Logic Networks

- MLN is **template** for ground Markov nets
- Probability of a world x:

$$P(x) = \frac{1}{Z} \exp\left(\sum_i w_i n_i(x)\right)$$

Weight of formula *i*

No. of true groundings of formula *i* in *x*

- **Typed** variables and constants greatly reduce size of ground Markov net

Alchemy

- Open Source AI package
- http://alchemy.cs.washington.edu
- Implementation of Markov logic networks

- Problem specified in two files:
  - File1.mln (Markov logic network)
  - File2.db (database / data set)

- Learn weights and structure of MLN
- Inference queries

Markov Logic Encoding

- File.mln

- Two parts:
  - Declaration
    - Domain of each variable
    - Predicates
  - Formula
    - Pairs of weights with logical formula

Markov Logic Encoding

- Example declaration
  - Domain of each variable
    - `person = {Anna, Bob}`
  - Predicates:
    - `Friends(person,person)`
    - `Smokes(person)`
    - `Cancer(person)`

- Example formula
  - `8 Smokes(x) => Cancer(x)`
  - `5 Friends(x,y) => (Smokes(x)<=>Smokes(y))`

NB: by default, formulas are universally quantified in Alchemy

Dataset

- File.db
- List of facts (ground atoms)
- Example:
  - `Friends(Anna,Bob)`
  - `Smokes(Anna)`
  - `Cancer(Bob)`

Syntax

- Logical connective:
  - `!` (not), `^` (and), `v` (or), `=>` (implies), `<=>` (iff)
- Quantifiers:
  - `forall` (∀), `exist` (∃)
  - By default unquantified variables are universally quantified in Alchemy
- Operator precedence:
  - `!` > `^` > `v` > `=>` > `<=>` > `forall` = `exist`

Syntax

- Short hand for predicates
  - `!` operator: indicates that the preceding variable has exactly one true grounding
  - Ex: `HasPosition(x,y!)`: for each grounding of `x`, exactly one grounding of `y` satisfies `HasPosition`
- Short hand for multiple weights
  - `+` operator: indicates that a different weight should be learned for each grounding of the following variable
  - Ex: `outcome(throw,+face)`: a different weight is learned for each grounding of face

Multinomial Distribution

Example: Throwing dice

Types: `throw = { 1, … , 20 }`
`face = { 1, … , 6 }`
Predicate: `Outcome(throw,face)`
Formulas: `Outcome(t,f) ^ f!=f' => !Outcome(t,f').`
`Exist f Outcome(t,f).`

Too cumbersome!

Multinomial Distrib.: ! Notation

Example: Throwing dice

Types: `throw = { 1, … , 20 }`
`face = { 1, … , 6 }`
Predicate: `Outcome(throw,face!)`
Formulas:

Semantics: Arguments without "!" determine args with "!".
Only one face possible for each throw.

Multinomial Distrib.: + Notation

Example: Throwing biased dice

Types: `throw = { 1, … , 20 }`
`face = { 1, … , 6 }`
Predicate: `Outcome(throw,face!)`
Formulas: `Outcome(t,+f)`

Semantics: Learn weight for each grounding of args with "+".

Text Classification

```
page = { 1, … , n }
word = { … }
topic = { … }

Topic(page,topic!)
HasWord(page,word)
Links(page,page)

HasWord(p,+w) => Topic(p,+t)
Topic(p,t) ^ Links(p,p') => Topic(p',t)
```

Next Class

- Applications of Markov Logic Networks