Outline

• Markov Logic Networks
• Alchemy

• Readings:
Markov Logic Networks

• Bayesian networks and Markov networks:
  - Model uncertainty
  - But propositional representation (e.g., we need one variable per object in the world)

• First-order logic:
  - First-order representation (e.g., quantifiers allow us to reason about several objects simultaneously)
  - But we can’t deal with uncertainty

• Markov logic networks: combine Markov networks and first-order logic
Markov Logic

• A logical KB is a set of hard constraints on the set of possible worlds
• Let’s make them soft constraints: when a world violates a formula, it becomes less probable, not impossible
• Give each formula a weight: (higher weight $\Rightarrow$ stronger constraint)

$$P(\text{world}) \propto e^{\sum \text{weights of formulas it satisfies}}$$
Markov Logic: Definition

• A Markov Logic Network (MLN) is a set of pairs \((F, w)\) where
  - \(F\) is a formula in first-order logic
  - \(w\) is a real number

• Together with a set of constants, it defines a Markov network with
  - One node for each grounding of each predicate in the MLN
  - One feature for each grounding of each formula \(F\) in the MLN, with the corresponding weight \(w\)
Example: Friends & Smokers

Smoking causes cancer.
Friends have similar smoking habits.
Example: Friends & Smokers

\[ \forall x \ Smokes(x) \Rightarrow Cancer(x) \]
\[ \forall x, y \ Friends(x, y) \Rightarrow (\Smokes(x) \iff \Smokes(y)) \]
## Example: Friends & Smokers

<table>
<thead>
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Two constants: **Anna** (A) and **Bob** (B)
Example: Friends & Smokers

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Example: Friends & Smokers

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Two constants: Anna (A) and Bob (B)
Markov Logic Networks

- **MLN is template** for ground Markov nets
- Probability of a world $x$:

  $$P(x) = \frac{1}{Z} \exp\left(\sum_i w_i n_i(x)\right)$$

  - Weight of formula $i$
  - No. of true groundings of formula $i$ in $x$

- **Typed** variables and constants greatly reduce size of ground Markov net
Alchemy

- Open Source AI package
- http://alchemy.cs.washington.edu
- Implementation of Markov logic networks

- Problem specified in two files:
  - File1.mln (Markov logic network)
  - File2.db (database / data set)

- Learn weights and structure of MLN
- Inference queries
Markov Logic Encoding

• File.mln

• Two parts:
  - Declaration
    • Domain of each variable
    • Predicates
  - Formula
    • Pairs of weights with logical formula
Markov Logic Encoding

• Example declaration
  - Domain of each variable
    • person = {Anna, Bob}
  - Predicates:
    • Friends(person,person)
    • Smokes(person)
    • Cancer(person)

• Example formula
  - 8 Smokes(x) => Cancer(x)
  - 5 Friends(x,y) => (Smokes(x)<=>Smokes(y))

NB: by default, formulas are universally quantified in Alchemy
Dataset

• File.db

• List of facts (ground atoms)

• Example:
  - Friends(Anna,Bob)
  - Smokes(Anna)
  - Cancer(Bob)
Syntax

- Logical connective:
  - ! (not), ^ (and), v (or), => (implies), <=> (iff)

- Quantifiers:
  - forall (\forall), exist (\exists)
  - By default unquantified variables are universally quantified in Alchemy

- Operator precedence:
  - ! > ^ > v > => > <=> > forall = exist
Syntax

• Short hand for predicates
  - ! operator: indicates that the preceding variable has exactly one true grounding
  - Ex: $\text{HasPosition}(x,y!)$: for each grounding of $x$, exactly one grounding of $y$ satisfies $\text{HasPosition}$

• Short hand for multiple weights
  - + operator: indicates that a different weight should be learned for each grounding of the following variable
  - Ex: $\text{outcome}(\text{throw}, +\text{face})$: a different weight is learned for each grounding of face
Multinomial Distribution

Example: Throwing dice

Types:  
\begin{align*}
\text{throw} & = \{ 1, \ldots, 20 \} \\
\text{face} & = \{ 1, \ldots, 6 \}
\end{align*}

Predicate: \( \text{Outcome}(\text{throw}, \text{face}) \)

Formulas:  
\begin{align*}
\text{Outcome}(t, f) \land f \neq f' & \Rightarrow \neg \text{Outcome}(t, f'). \\
\exists f \text{ Outcome}(t, f).
\end{align*}

Too cumbersome!
Multinomial Distrib.: ! Notation

Example: Throwing dice

Types:  
\[ \text{throw} = \{ 1, \ldots, 20 \} \]
\[ \text{face} = \{ 1, \ldots, 6 \} \]

Predicate: \text{Outcome} (\text{throw}, \text{face}!)

Formulas:

Semantics: Arguments without ‘!’ determine args with ‘!’.
Only one face possible for each throw.
Multinomial Distrib.: + Notation

Example: Throwing biased dice

Types:  
  throw = \{ 1, \ldots, 20 \}  
  face = \{ 1, \ldots, 6 \}

Predicate:  Outcome(\text{throw},\text{face})

Formulas:  Outcome(t,+f)

Semantics: Learn weight for each grounding of args with “+”. 
Text Classification

\[
\text{page} = \{1, \ldots, n\} \\
\text{word} = \{\ldots\} \\
\text{topic} = \{\ldots\}
\]

\text{Topic}(\text{page},\text{topic})! \\
\text{HasWord}(\text{page},\text{word}) \\
\text{Links}(\text{page},\text{page})

\text{HasWord}(p,+w) \Rightarrow \text{Topic}(p,+t) \\
\text{Topic}(p,t) \land \text{Links}(p,p') \Rightarrow \text{Topic}(p',t)
Next Class

• Applications of Markov Logic Networks