CS485/685
Lecture 2: January 5th, 2012

Decision Trees

Readings: [B] Sec. 14.4

Inductive Learning (recap)

- Induction
  - Given a training set of examples of the form $(x, f(x))$
    - x is the input, $f(x)$ is the output
  - Return a function h that approximates f
    - h is called the hypothesis

Supervised Learning

- Two types of problems
  1. **Classification:**

  2. **Regression**

- NB: The nature (categorical or continuous) of the domain (input space) of f does not matter

Classification Example

- Problem: Will you enjoy an outdoor sport based on the weather?
- Training set:

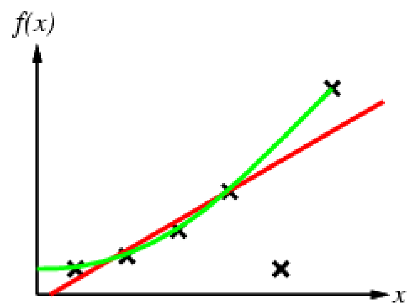
Sky	Humidity	Wind	Water	Forecast	EnjoySport
Sunny	Normal	Strong	Warm	Same	yes
Sunny	High	Strong	Warm	Same	yes
Sunny	High	Strong	Warm	Change	no
Sunny	High	Strong	Cool	Change	yes

x $f(x)$

- Possible Hypotheses:
  - $h_1: S = sunny \rightarrow ES = yes$
  - $h_2: Wa = cool$ or $F = same \rightarrow enjoySport$

Regression Example

- Find function h that fits f at instances x



More Examples

Problem	Domain	Range	Classification / Regression
Spam Detection			
Stock price prediction			
Speech recognition			
Digit recognition			
Housing valuation			
Weather prediction			

Hypothesis Space

- Hypothesis space H
  - Set of all hypotheses h that the learner may consider
  - Learning is a search through hypothesis space
- Objective: find h that minimizes
  - Misclassification
  - Or more generally some error function

  with respect to the training examples
- But what about unseen examples?

Generalization

- A good hypothesis will generalize well
  - i.e., predict unseen examples correctly
- Usually …
  - Any hypothesis h found to approximate the target function f well over a **sufficiently large set of training examples** will also approximate the target function well over any unobserved examples

Inductive learning

- Goal: find an h that agrees with f on training set
  - h is **consistent** if it agrees with f on all examples
- Finding a consistent hypothesis is not always possible
  - Insufficient hypothesis space:
    - E.g., it is not possible to learn exactly $f(x) = ax + b + xsin(x)$ when $H =$ space of polynomials of finite degree
  - Noisy data
    - E.g., in weather prediction, identical conditions may lead to rainy and sunny days

Inductive Learning

- A learning problem is if the hypothesis space contains the true function otherwise it is **unrealizable**.
  - Difficult to determine whether a learning problem is realizable since the true function is not known
- It is possible to use a very large hypothesis space
  - For example: H = class of all Turing machines
- But there is a **tradeoff** between **expressiveness** of a hypothesis class and the **complexity** of finding a good hypothesis

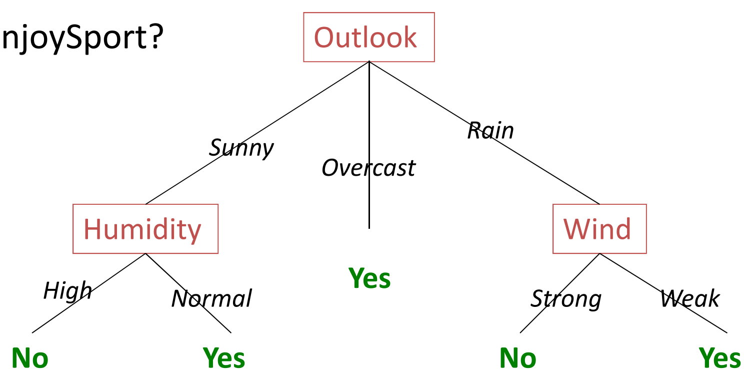
CART (Classification and Regression Trees)

- Tree
  - Nodes: labeled with attributes
  - Edges: labeled with attribute values
  - Leaves: labeled with
    - Classes (classification tree)
    - Values (Regression tree)

- Label an instance by following the branch consistent with the attribute values and returning the label stored in the resulting leaf.

Example: classification

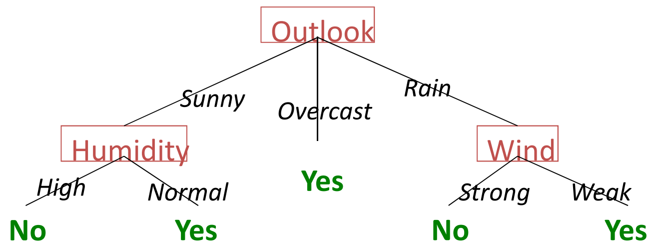
- EnjoySport?



- Instance:
- Classification:

Decision tree representation

- Decision trees can represent disjunctions of conjunctions of constraints on attribute values



Disjunction:

Decision tree representation

- Decision trees are fully expressive within the class of propositional languages
  - Any Boolean function can be written as a decision tree
    - Trivially by allowing each row in a truth table correspond to a path in the tree
    - Can often use small trees
    - Some functions require exponentially large trees (majority function, parity function)
  - However, there is no compact representation for all functions

Inducing a decision tree

- Aim: find a small tree consistent with the training examples
- Idea: (recursively) choose "most significant" attribute as root of (sub)tree

Decision Tree Learning

function DTL(*examples, attributes, default*) **returns** a decision tree
  if *examples* is empty **then return** *default*
  else if all *examples* have the same classification **then return** the classification
  else if *attributes* is empty **then return** MODE(*examples*)
  else
    best ← CHOOSE-ATTRIBUTE(*attributes, examples*)
    tree ← a new decision tree with root test *best*
    for each value v_i of *best* **do**
      $examples_i$ ← {elements of *examples* with *best* $= v_i$}
      subtree ← DTL($examples_i$, *attributes* − *best*, MODE(*examples*))
      add a branch to *tree* with label v_i and subtree *subtree*
    return *tree*

Choosing attribute tests

- The central choice is deciding which attribute to test at each node
- We want to choose an attribute that is most useful for classifying examples

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Example: Restaurant

Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*Wait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

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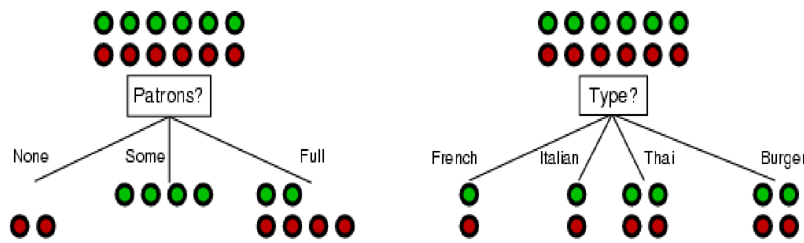
Choosing an attribute

- Idea: a good attribute splits the examples into subsets that are (ideally) "all positive" or "all negative"



- $Patrons$? is better choice

Residual error for classification

- Let τ denote a leaf
- Let Q_τ denote the residual error at leaf τ
- Some residual error functions for classification
  - Gini Index: $Q_\tau = \sum_k p_\tau(k)(1 - p_\tau(k))$
  - Entropy: $Q_\tau = -\sum_k p_\tau(k) \log_2 p_\tau(k)$

Here k denotes the k^{th} class
And $p_\tau(k)$ = relative frequency of class k at leaf τ

Residual Error for Classification

Gini Index:
$Q_\tau = \sum_k p_\tau(k)(1 - p_\tau(k))$

Expected misclassification when choosing the class according to $p_\tau(k)$

Entropy:
$Q_\tau = \sum_k p_\tau(k) \, [-\log_2 p_\tau(k)]$

Expected #of bits to encode the class of an instance chosen at random according to $p_\tau(k)$

Residual Error for Regression

- Let $t_n = f(x_n)$ be the target for the n^{th} example
- Let y_τ be the value returned by leaf τ
- Common residual error function for regression
  - Euclidean error: $E_\tau = \sum_{n \in R_\tau} (t_n - y_\tau)^2$

Choosing attribute tests

- Choose leaf τ and attribute A that reduces residual error the most when expanded
  $(A^*, \tau^*) = argmax_{A,\tau}\ Q_\tau - \sum_a p_\tau(A = a)\, Q_{\tau a}$

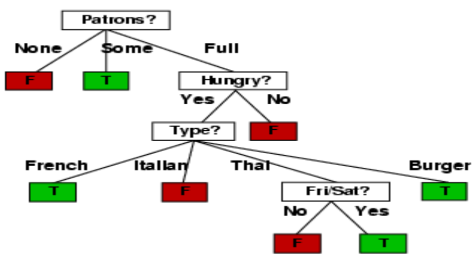
Where $p_\tau(A = a)$ is the relative frequency of examples with $A = a$ in leaf τ.

Example

- Consider restaurant example (slide 19)
- Gini index reduction:
- Entropy reduction:

Example

- Decision tree learned from the 12 examples:

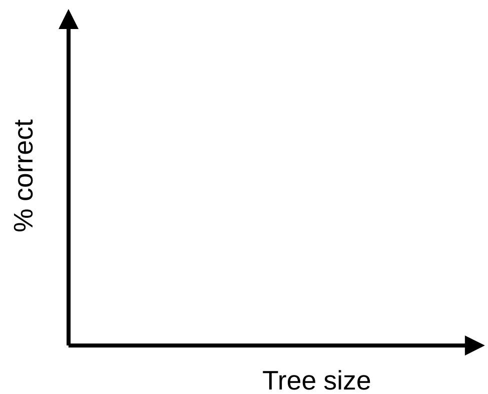


- Substantially simpler than "true" tree
  - a more complex hypothesis isn't justified by small amount of data

Performance of a learning algorithm

- A learning algorithm is good if it produces a hypothesis that does a good job of predicting classifications of unseen examples
- Verify performance with a **test set**
  1. Collect a large set of examples
  2. Divide into 2 disjoint sets: training set and test set
  3. Learn hypothesis h with training set
  4. Measure percentage of correctly classified examples by h in the test set
  5. Repeat 2-4 for different randomly selected training sets of varying sizes

Learning curves



Overfitting

- Decision-tree grows until all training examples are perfectly classified
- But what if…
  - Data is noisy
  - Training set is too small to give a representative sample of the target function
- May lead to **Overfitting!**
  - Common problem with most learning algorithms

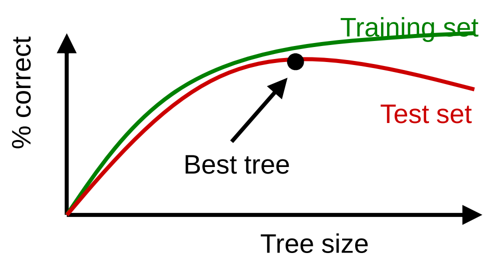
Overfitting

- **Definition**: Given a hypothesis space H, a hypothesis h ∈ H is said to overfit the training data if there exists some alternative hypothesis h' ∈ H such that h has smaller error than h' over the training examples but h' has smaller error than h over the entire distribution of instances

- Overfitting has been found to decrease accuracy of decision trees by 10-25%

Avoid overfitting

- Two popular techniques
  - Stop growing tree when test set performance starts decreasing
    - Use cross-validation
  - Prune statistically irrelevant nodes

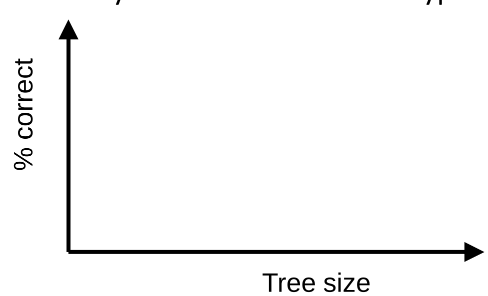


Cross-validation

- Split data in two parts, one for training, one for testing the accuracy of a hypothesis
- K-fold cross validation means you run k experiments, each time putting aside $1/k$ of the data to test on

Early stopping is difficult

- Performance curves:
  - Train accuracy: monotonically increasing curve, but not smooth
  - Test accuracy: curve is not concave typically

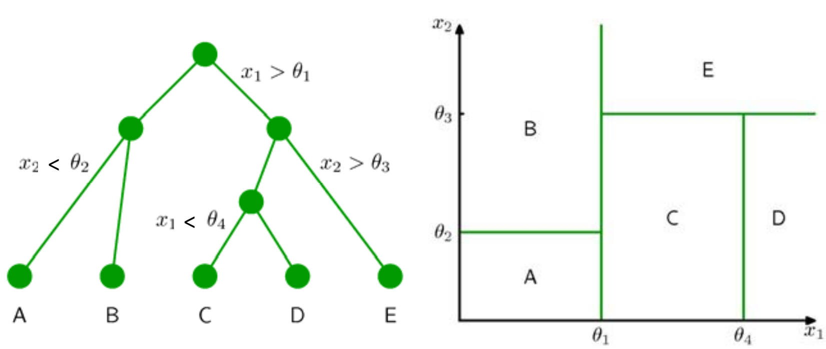


Pruning

- Pruning is more common in practice
- Prune nodes in a bottom up fashion
- Two approaches:
  - Remove nodes that improve test accuracy by less than some threshold based on cross-validation
  - Regularization: add penalty term that reflects tree complexity (e.g., $|T| = \#leaves$)
    - $Q_\tau - \sum_a p_\tau(A = a)Q_{\tau a} - \lambda|T|$
    - λ is a weight that adjusts the importance of the penalty
    - Remove leaves with negative "regularized" error reduction

Decision tree with continuous attributes

- Tree partitions the input space



Decision tree with continuous attributes

- How do we come up with good partitions?
- Common approach: thresholding
  - Single attribute: $x_j > \theta_j$
  - Multi-attribute: $f(x_1, \ldots, x_M) > \theta$
    - Where f can be linear or non-linear
- Alternative: nearest neighbour (next class)