# Lecture 12: Convolutional Neural Networks
# CS480/680 Intro to Machine Learning

2023-2-16

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# Large networks

- What kind of neural networks can be used for large or variable length input vectors (e.g., time series)?

- Common networks:
  - Convolutional networks
  - Recurrent networks
  - Transformers

# Convolution

- Convolution: mathematical operation on two functions $x()$ and $w()$ that produces a third function $y()$ that can be viewed as a modified version of the original function $x()$

$$y(i) = \int_t x(t)w(i-t)dt$$

$$y(i) = (x * w)(i)$$

where $*$ is an operator denoting a convolution

# Example Smoothing

# Discrete convolution

- Discrete convolution

$$y(i) = \sum_{t=-\infty}^{\infty} x(t)w(i-t)$$

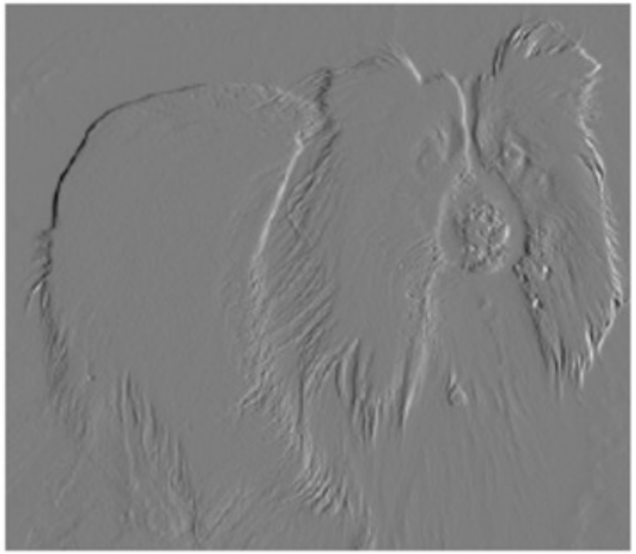
- Multidimensional convolution

$$y(i,j) = \sum_{t_1=-\infty}^{\infty} \sum_{t_2=-\infty}^{\infty} x(t_1, t_2)w(i-t_1, j-t_2)$$

# Example: Edge Detection

- Consider a grey scale image
- Detect vertical edges: $y(i,j) = x(i,j) - x(i-1,j)$

$$w(i-t_1, j-t_2) = \begin{cases} 1 & t_1 = i, t_2 = j \\ -1 & t_1 = i-1, t_2 = j \\ 0 & \text{otherwise} \end{cases}$$



# Convolutions for feature extraction

- In neural networks
  - A **convolution** denotes the linear combination of a **subset of units** based on a **specific pattern of weights.**

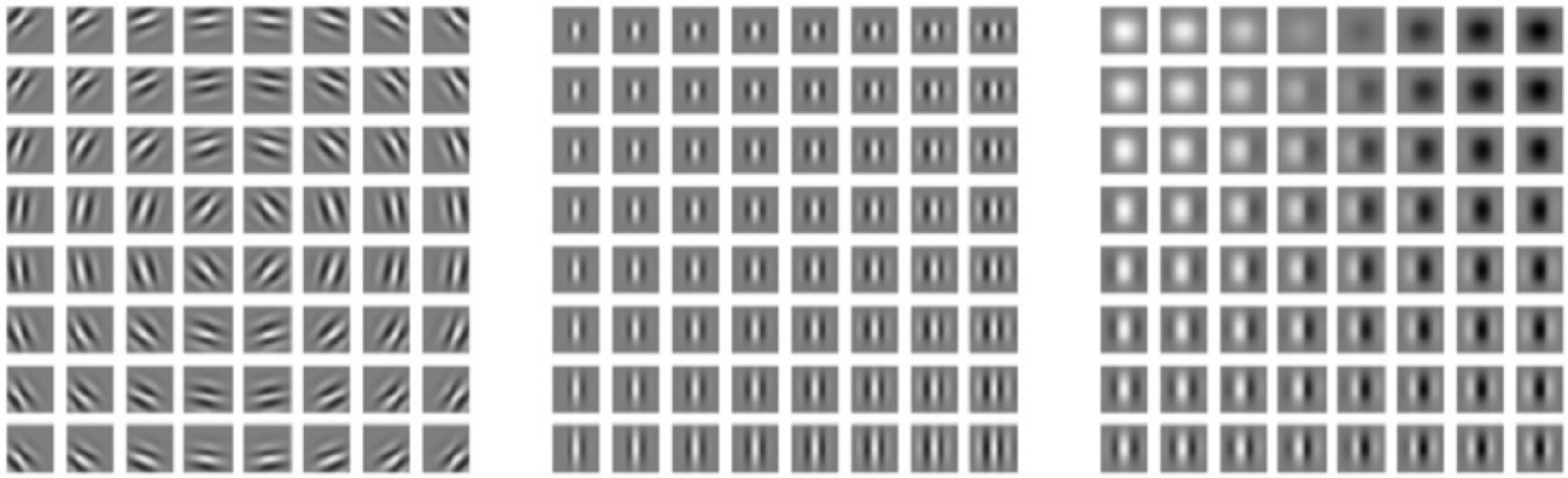
$$a_j = \sum_i w_{ji} z_i$$

  - Convolutions are often combined with an activation function to produce a feature

$$z_j = h(a_j) = h\left(\sum_i w_{ji} z_i\right)$$

# Gabor filters

- Gabor filters: common feature maps inspired by the human vision system



- Weights:

Grey: zero White: positive Black: negative

# Convolution Neural Network

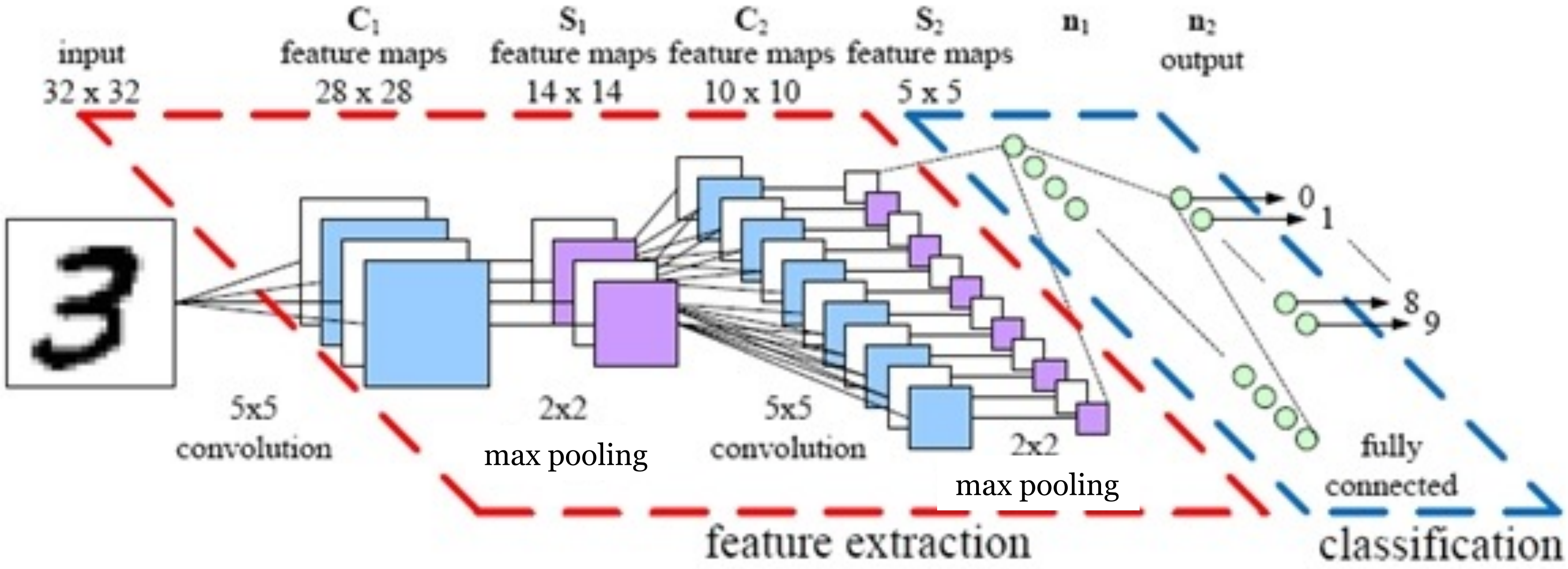
- A **convolutional neural network** refers to any network that includes an **alternation of convolution and pooling layers**, where **some of the convolution weights are shared.**
- Architecture:

# Pooling

- Pooling: **commutative** mathematical operation that combines several units
- Examples:
  - max, sum, product, average, Euclidean norm, etc.
- Commutative property (order does not matter):

Ex.: $\max(a, b) = \max(b, a)$

# Example: Digit Recognition



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# Benefits

- Sparse interactions
  - Fewer connections
- Parameter sharing
  - Fewer weights
- Locally equivariant representation
  - Locally invariant to translations
  - Handle inputs of varying length

# Parameters

- **# of filters**: integer indicating the # of filters applied to each window.
- **kernel size:** tuple (width, height) indicating the size of the window.
- **Stride:** tuple (horizontal, vertical) indicating the horizontal and vertical shift between each window.
- **Padding:** “valid” or ”same”.  Valid indicates no input padding. Same indicates that the input is padded with a border of zeros to ensure that the output has the same size as the input.

# Examples

# Training

- Convolutional neural networks are trained in the same way as other neural networks
  - E.g., backpropagation

- Weight sharing:
  - Combine gradients of shared weights into a single gradient

# Architecture design

- What is the preferred filter size?
- VGG (Visual Geometry Group at Oxford, 2014): stack of small filters is often preferred to a single large filter
  - Fewer parameters
  - Deeper network
- Picture



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# Residual Networks

- Problem: very deep networks can perform worse than shallower networks (due to local optima & other stationary non-optimal points)
- Solution [He et al., 2015]: introduce **residual connections** (a.k.a. skip connections) to make blocks optional
- Picture:

# Residual Learning

- Consider a block $b(\boldsymbol{x})$ that ends with a linear layer
  - i.e., $b(\boldsymbol{x}) = \boldsymbol{w}^T g(\boldsymbol{x})$ where $g(\boldsymbol{x})$ can be anything
- We can nullify $b(\boldsymbol{x})$ simply by setting $\boldsymbol{w}$ to $\boldsymbol{0}$
  - i.e., $b(\boldsymbol{x}) = \boldsymbol{0}^T g(\boldsymbol{x}) = \boldsymbol{0}$
- Hence, when $b(\boldsymbol{x}) = \boldsymbol{0}$,
  then $f(\boldsymbol{x}) = b(\boldsymbol{x}) + \boldsymbol{x} = \boldsymbol{x}$ computes the identity

# Applications

- Image processing
- Data with sequential, spatial, or tensor patterns