Online Algorithms for Sum-Product Networks with Continuous Variables

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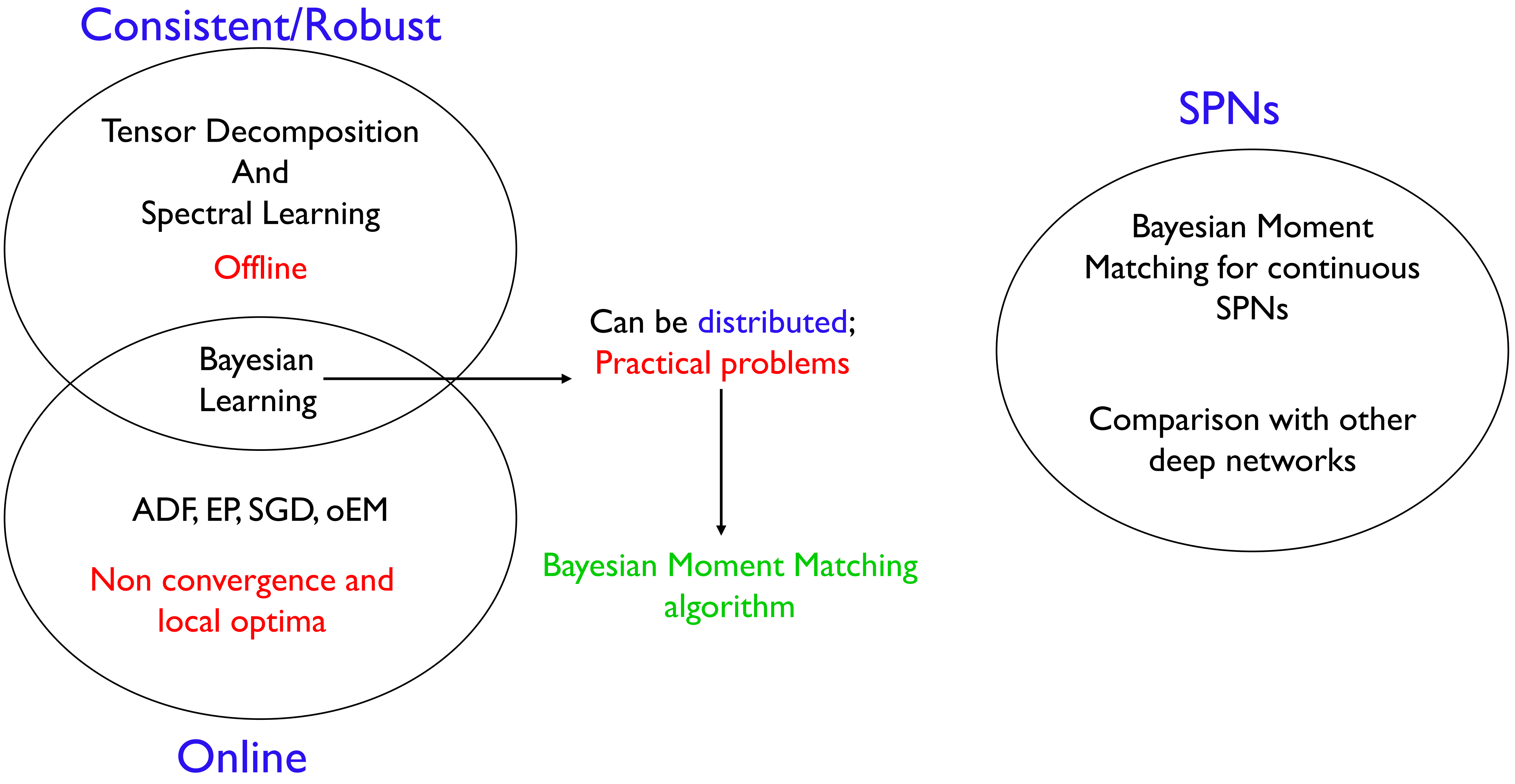
Ph.D. Seminar

Mixture Models



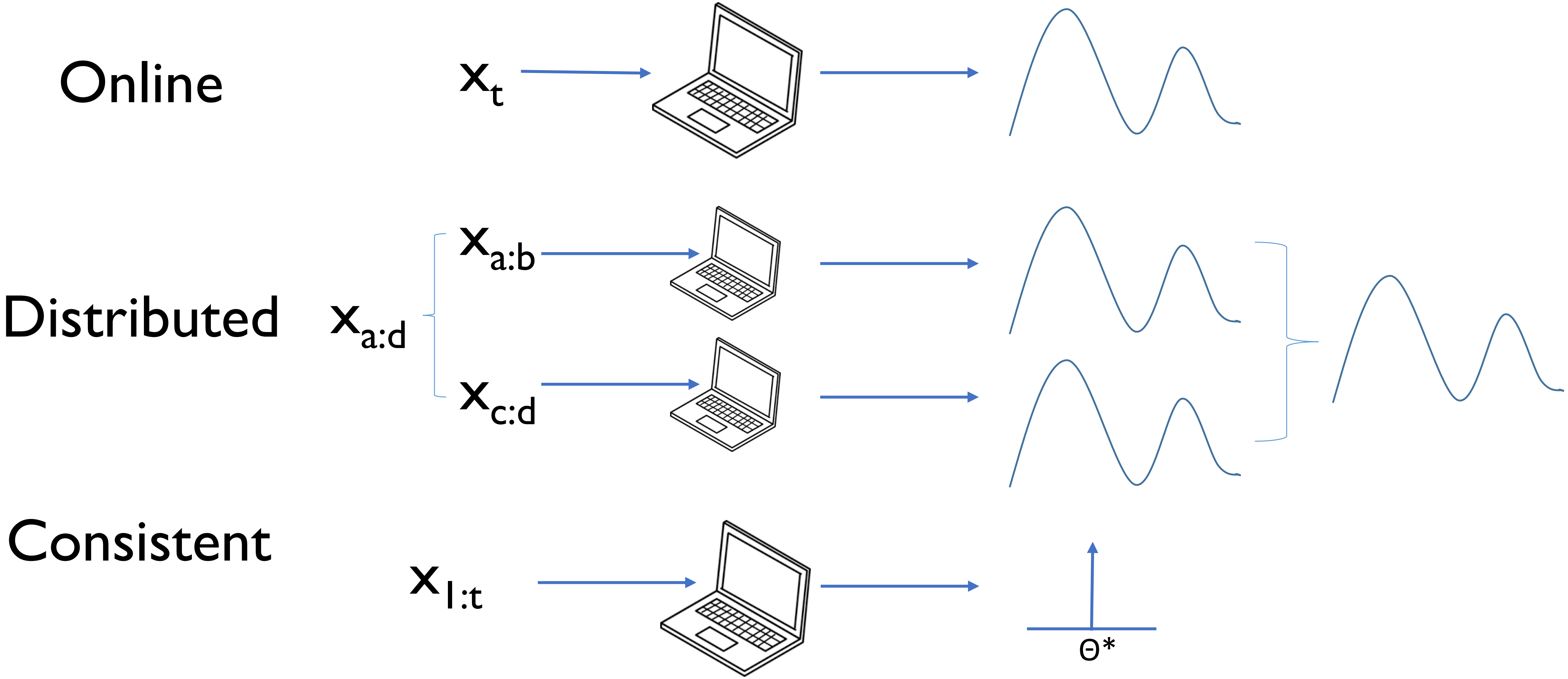
Streaming Data

Activity Recognition

Recommendation

Challenge : update model after each observation

Algorithm's characteristics



How can we learn mixture models robustly from streaming data ?

Learning Algorithms

- **Robust** : Tensor Decomposition(Anandkumar et.al, 2014),Spectral Learning(Hsu et al, 2012, Parikar and Xing, 2011); offline

- **Online** :
  - Assumed Density Filtering (Maybeck 1982; Lauritzen 1992; Opper & Winther 1999); not robust
  - Expectation Propagation (Minka 2001); does not converge
  - Stochastic Gradient Descent (Zhang 2004)
  - online Expectation Maximization (Cappe 2012)

  SGD and oEM : local optimum and cannot be distributed

Learning Algorithms

- **Exact Bayesian Learning** : Dirichlet Mixtures(Ghosal et al 1999), Gaussian Mixtures(Lijoi et al, 2005), Non-parametric Problems (Barron et al, 1999), (Freedman, 1999)

Exact Bayesian Learning → Online

Exact Bayesian Learning → Distributed

Exact Bayesian Learning → Consistent

In theory; practical problems!

Bayesian Learning

Thomas Bayes
(c. 1700-1761)

- Uses Bayes' Theorem

$$P(\Theta|x) = \frac{P(\Theta)P(x|\Theta)}{P(x)}$$

Prior Belief	→	New Information	→	Bayes' Theorem	→	Updated Belief
$P(\Theta)$		x		$\frac{P(\Theta)P(x\|\Theta)}{P(x)}$		$P(\Theta\|x)$

Bayesian Learning – Mixture models

Data : $x_{1:n}$ where $x_i \sim \sum_{j=1}^{M} w_j \, N(x_i; \mu_j, \Sigma_j)$

$$P_n(\Theta) = \Pr(\Theta | x^{1:n})$$
$$\propto P_{n-1}(\Theta) \Pr(x_n | \Theta)$$
$$\propto P_{n-1}(\Theta) \sum_{j=1}^{M} w_j \, N(x_i; \mu_j, \Sigma_j)$$

Intractable!!!

Solution : Bayesian Moment Matching Algorithm

Method of Moments

Karl Pearson
(c. 1837-1936)

- Probability distributions defined by set of parameters
- Parameters can be estimated by a set of moments

$$X \sim N(X; \mu, \sigma^2)$$
$$E[X] = \mu$$
$$E[(X-\mu)^2] = \sigma^2$$

Make Bayesian Learning Great Again

Bayesian Learning

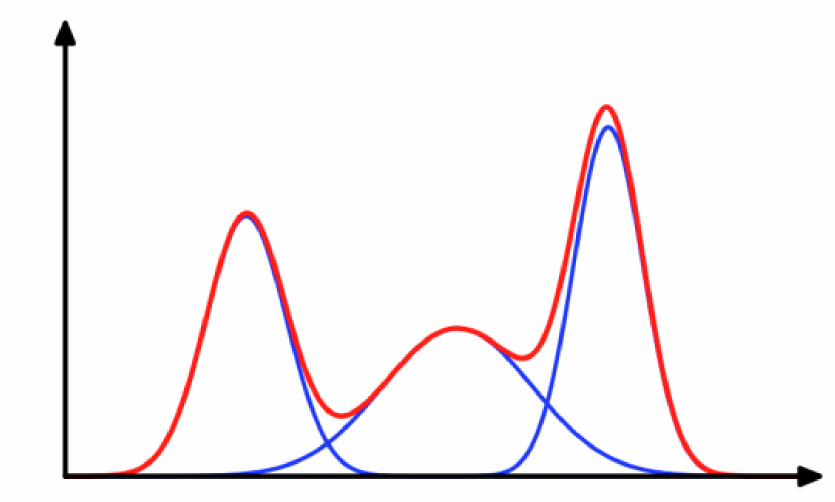
And

Method of Moments

STRONGER
TOGETHER

Gaussian Mixture Models

$$x_i \sim \sum_{j=1}^{M} w_j \; N(x_i; \mu_j, \Sigma_j)$$



Parameters : weights, means and precisions (inverse covariance matrices)

Bayesian Moment Matching for Gaussian Mixture Models

Prior　　Likelihood

$$P(\Theta|x) = \frac{P(\Theta)P(x|\Theta)}{P(x)}$$

Parameters

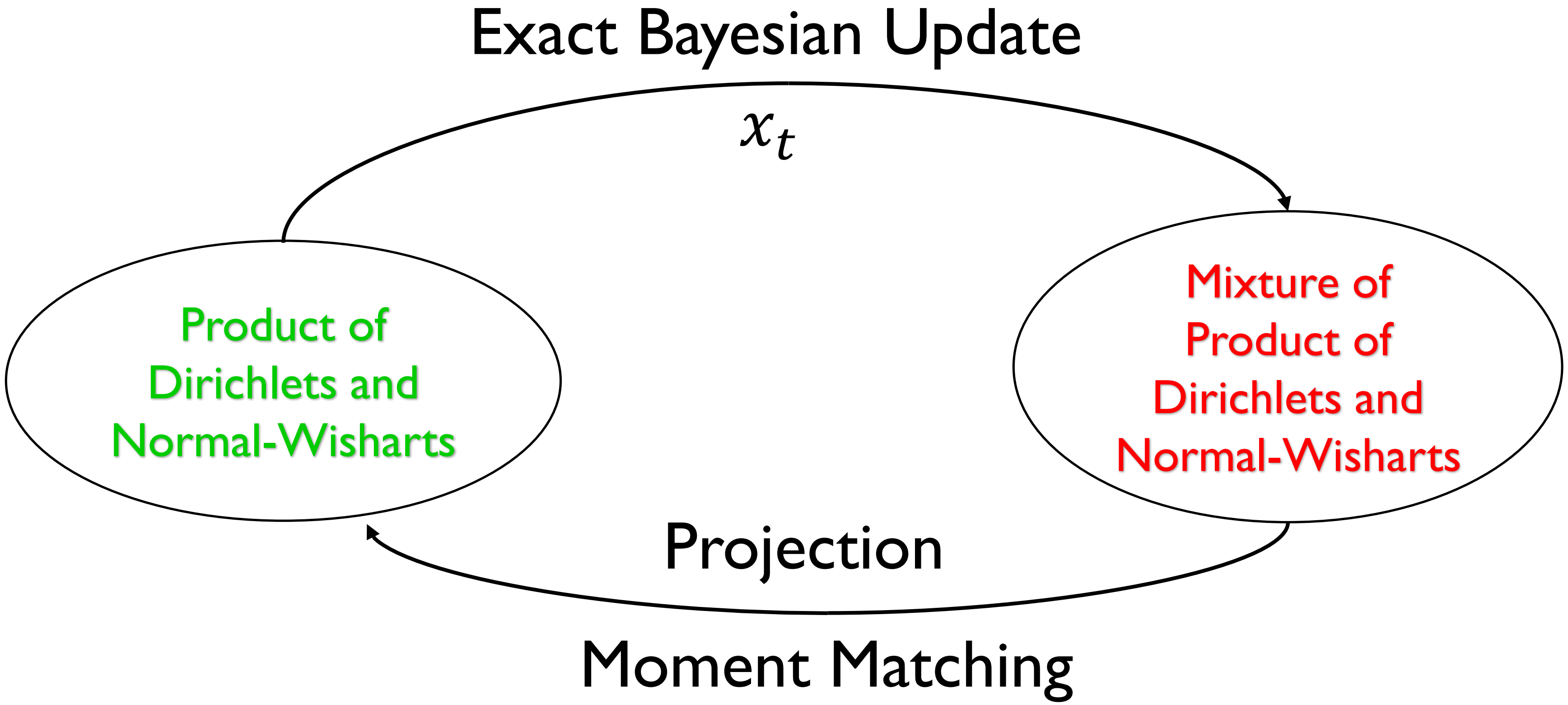
Parameters : weights, means and precisions (inverse covariance matrices)

Prior : $P(\boldsymbol{w}, \mu, \boldsymbol{\Lambda})$; product of Dirichlets and Normal-Wisharts

Likelihood :

$P(\boldsymbol{x}\,; \boldsymbol{w}, \mu, \boldsymbol{\Lambda}) = \sum_{j=1}^{M} w_j\, N(\boldsymbol{x}; \mu_j, \boldsymbol{\Lambda}_j^{-1})$

Bayesian Moment Matching Algorithm



Sufficient Moments

Dirichlet : $Dir(w_1, w_2 \dots w_M; \alpha_1, \alpha_2 \dots, \alpha_M)$

$$E[w_i] = \frac{\alpha_i}{\sum_j \alpha_j}; \qquad E[w_i^2] = \frac{\alpha_i(\alpha_i+1)}{(\sum_j \alpha_j)(1+\sum_j \alpha_j)}$$

Normal-Wishart : $NW(\mu, \Lambda; \mu_0, \kappa, W, v)$

$\Lambda \sim Wi(W, v)$ and $\mu|\Lambda \sim N_d(\mu_0, (\kappa\Lambda)^{-1})$

$$E[\mu] = \mu_0$$

$$E[(\mu - \mu_0)(\mu - \mu_0)^T] = \frac{\kappa+1}{\kappa(v-d-1)} W^{-1}$$

$$E[\Lambda] = vW$$

$$Var(\Lambda_{ij}) = v(W_{ij}^2 + W_{ii}W_{jj})$$

Overall Algorithm

- Bayesian Step
  - Compute posterior $P_t(\Theta)$based on observation x_t
- Sufficient Moments
  - Compute set of sufficient moments S for $P_t(\Theta)$
- Moment Matching
  - System of linear equations
  - Linear complexity in the number of components

Make Bayesian Learning Great Again

Bayesian Moment Matching Algorithm

- Uses Bayes' Theorem + Method of Moments
- Analytic solutions to Moment matching (unlike EP, ADF)
- One pass over data

Bayesian Moment Matching

- Online
- Distributed
- Consistent ?

Experiments

Data Set	Instances	oEM	oBMM
Abalone	4177	-2.65	**-1.82**
Banknote	1372	-9.74	**-9.65**
Airfoil	1503	**-15.86**	-16.53
Arabic	8800	-15.83	**-14.99**
Transfusion	748	-13.26	**-13.09**
CCPP	9568	-16.53	**-16.51**
Comp.Ac	8192	-132.04	**-118.82**
Kinematics	8192	-10.37	**-10.32**
Northridge	2929	-18.31	**-17.97**
Plastic	1650	-9.46	**-9.01**

Experiments

Data (Features)	Instances	oEM	oBMM	oDMM
Heterogeneity(16)	3,930,257	-176.2	**-174.3**	-180.7
Magic (10)	19,000	-33.4	**-32.1**	-35.4
Year MSD (91)	515,345	-513.7	**-506.5**	-513.8
Miniboone (50)	130,064	-58.1	**-54.7**	-60.3

Avg. Log-Likelihood

Data (Features)	Instances	oEM	oBMM	oDMM
Heterogeneity(16)	3,930,257	77.3	81.7	17.5
Magic (10)	19,000	7.3	6.8	1.4
Year MSD (91)	515,345	336.5	108.2	21.2
Miniboone (50)	130,064	48.6	12.1	2.3

Running Time

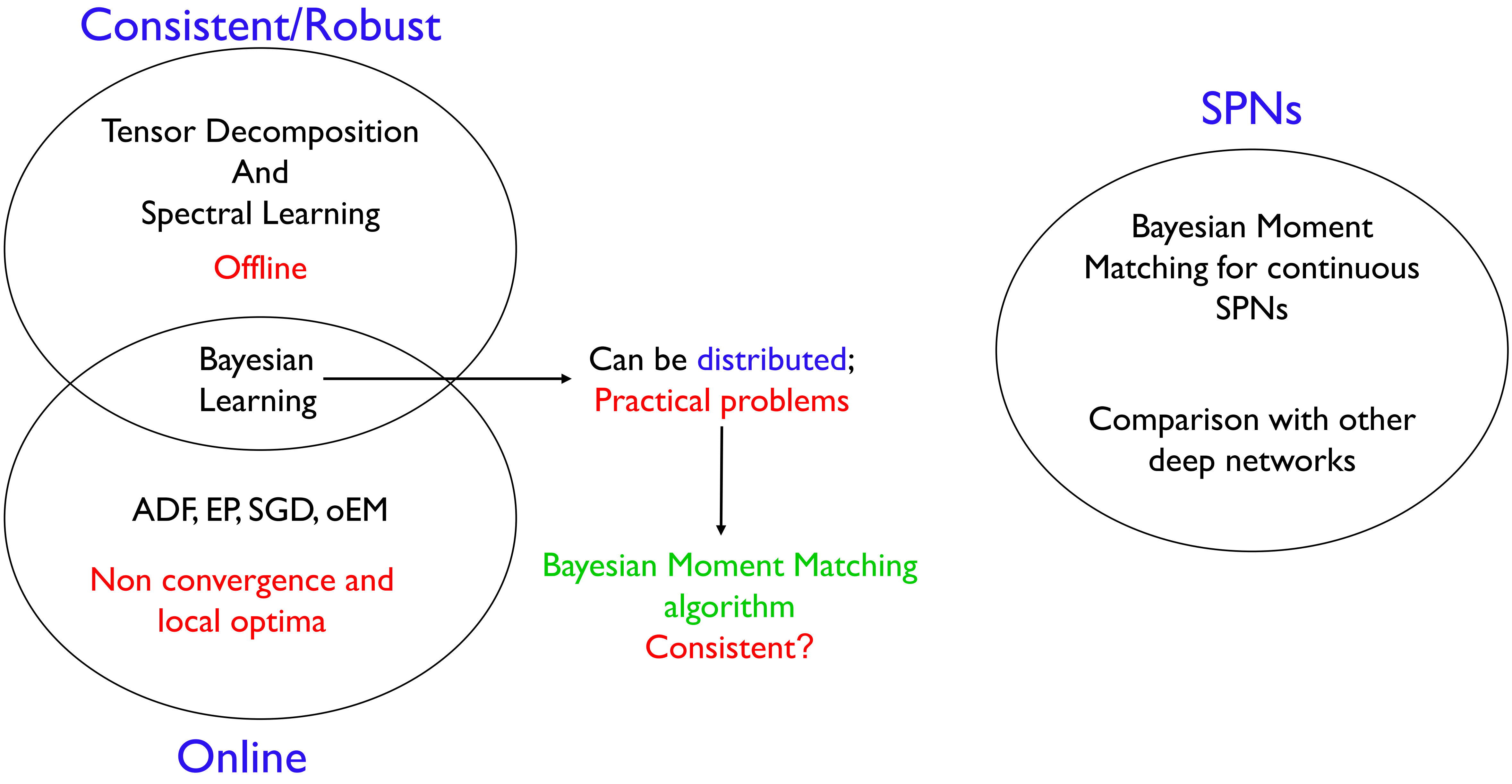
Bayesian Moment Matching

- **Discrete Data** : Omar (2015, PhD Thesis) for Dirichlets; Rashwan, Zhao & Poupart (AISTATS'16) for SPNs; Hsu & Poupart (NIPS'16) for Topic Modelling

- **Continuous Data** : Jaini & Poupart, 2016 (*arxiv*); Jaini, Rashwan et al, (PGM'16) for SPNs; Poupart, Chen, Jaini et al (NetworksML'16)

- **Sequence Data and Transfer Learning** : Jaini, Poupart et al, (submitted to ICLR'17)

Mixture Models



What is a Sum-Product Network?

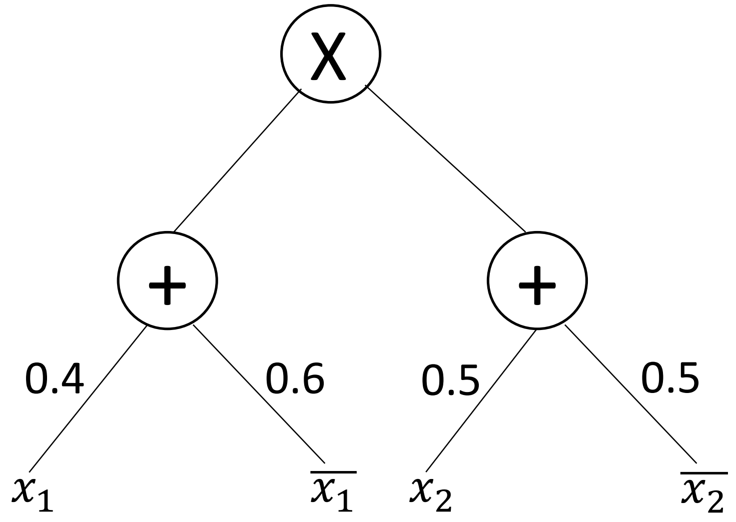
Proposed by Poon and Domingos (UAI 2011)
equivalent to Arithmetic Circuits (Darwiche 2003)

Deep architecture with clear semantics

Tractable Probabilistic Graphical Models

What is a Sum-Product Network?

$$P(x_1, x_2) = P(x_1, x_2)x_1x_2 + P(x_1, \overline{x_2})x_1\overline{x_2} + P(\overline{x_1}, x_2)\overline{x_1}x_2 + P(\overline{x_1}, \overline{x_2})\overline{x_1 x_2}$$

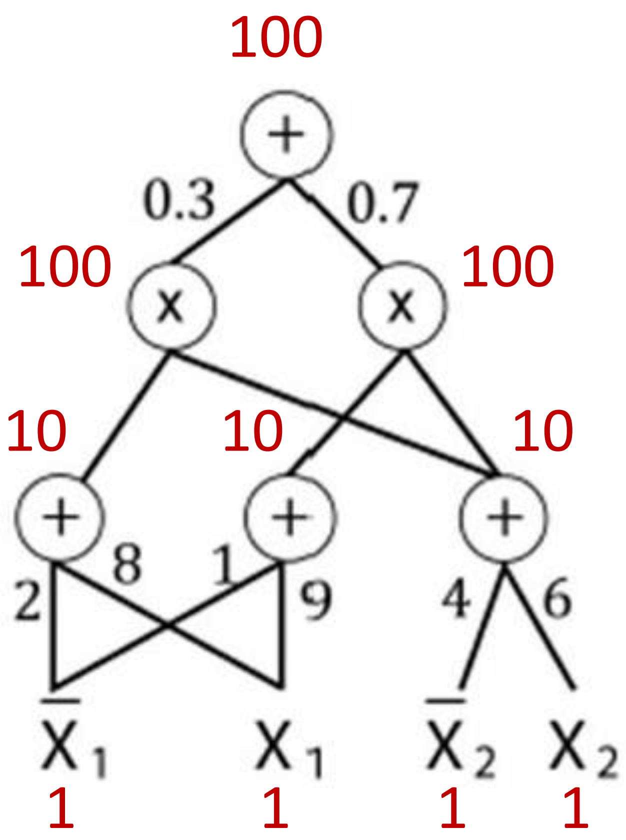


SPNs - Directed Acyclic Graphs

A valid SPN is

- Complete : Each sum node has children with same scope
- Decomposable : Each product node has children with disjoint scope

Probabilistic Inference - SPN



SPN represents a joint distribution over a set of random variables

Example :
Query : $\Pr(X_1 = 1, X_2 = 0)$

$Pr(X_1 = 1, X_2 = 0)$ = 34.8/100

Linear Complexity – two bottom passes for any query

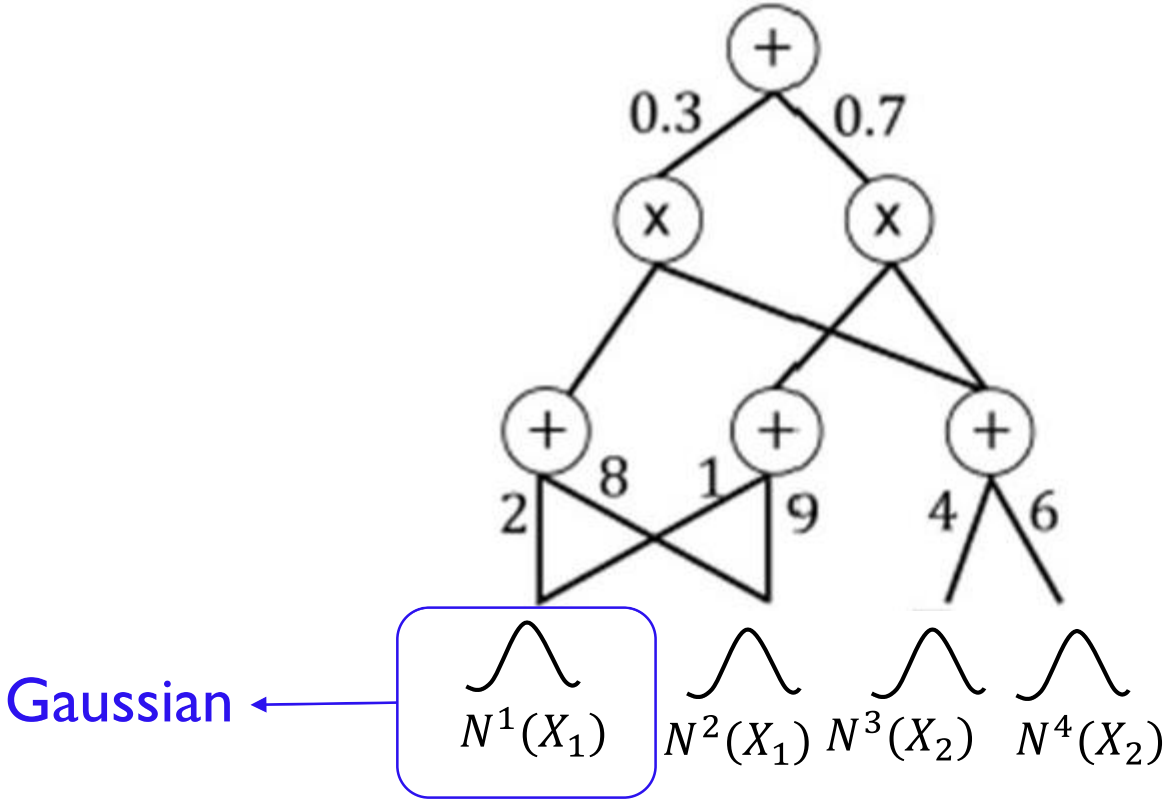
Learning SPNS

- Parameter Learning :

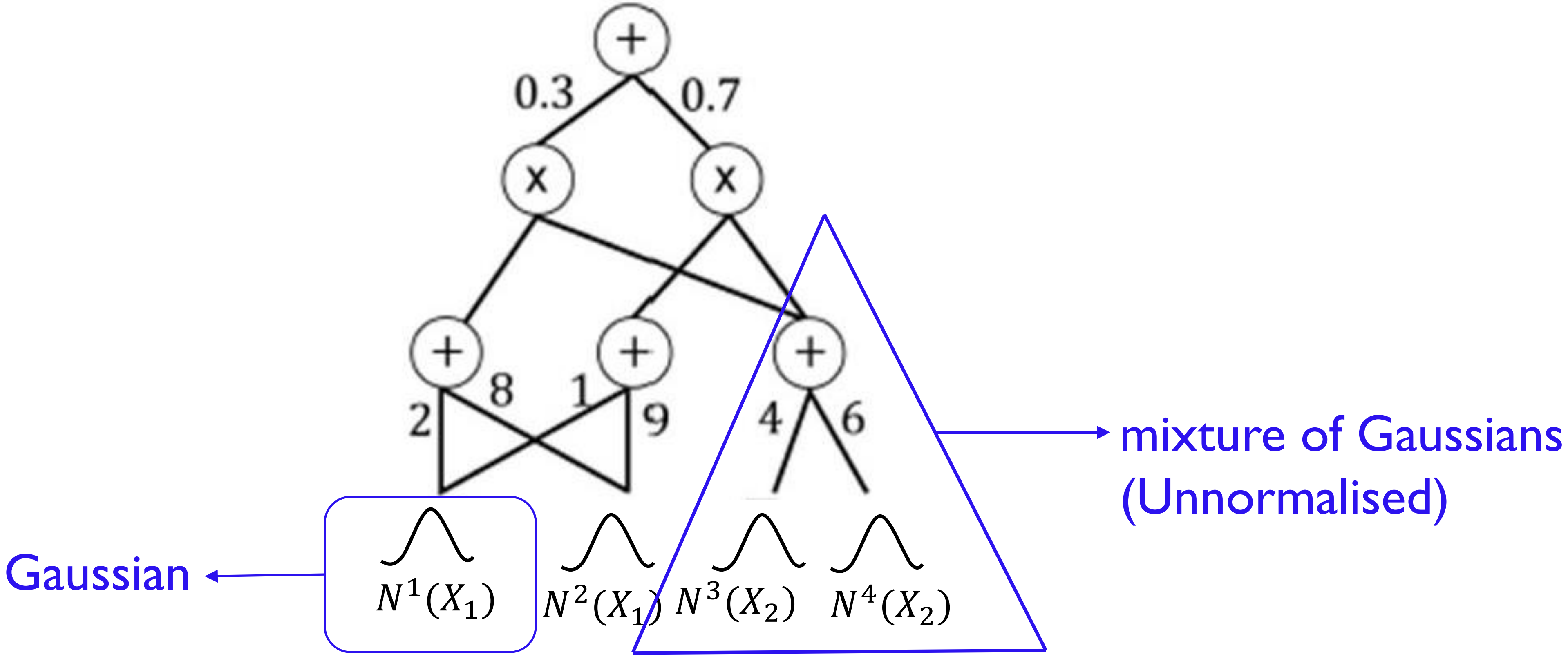
Discrete
- **Maximum Likelihood**: SGD (Poon & Domingos, 2011) slow convergence, inaccurate; EM(Perharz, 2015) inaccurate; Signomial Programming (Zhao & Poupart, 2016)
- **Bayesian Learning**: BMM(Rashwan et al., 2016) accurate; Collapsed Variational Inference (Zhao et al., 2016) accurate

- Online parameter learning for continuous SPNs (Jaini et al.2016)
  - extend oBMM to Gaussian SPNS

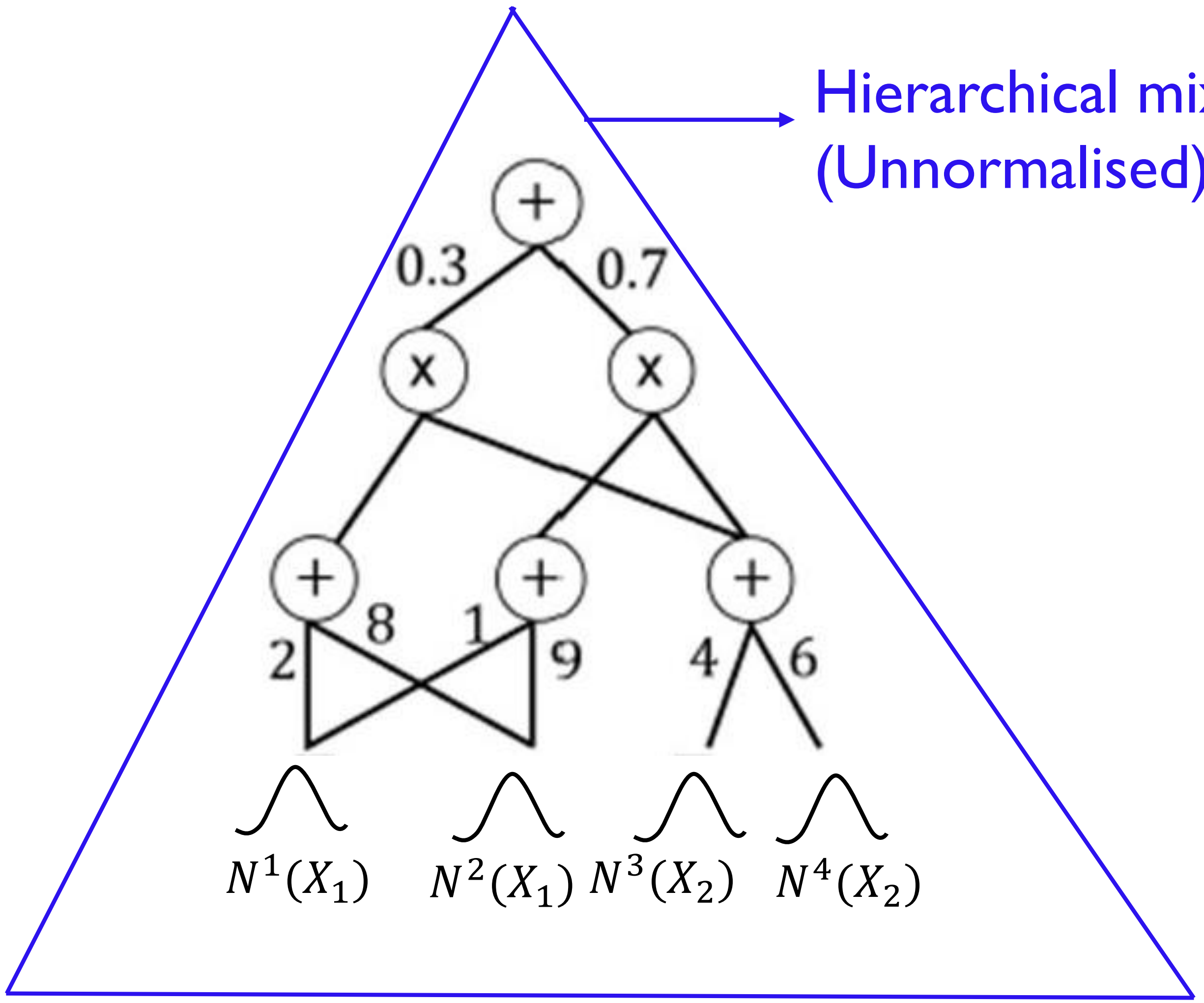
Continuous SPNS



Continuous SPNS

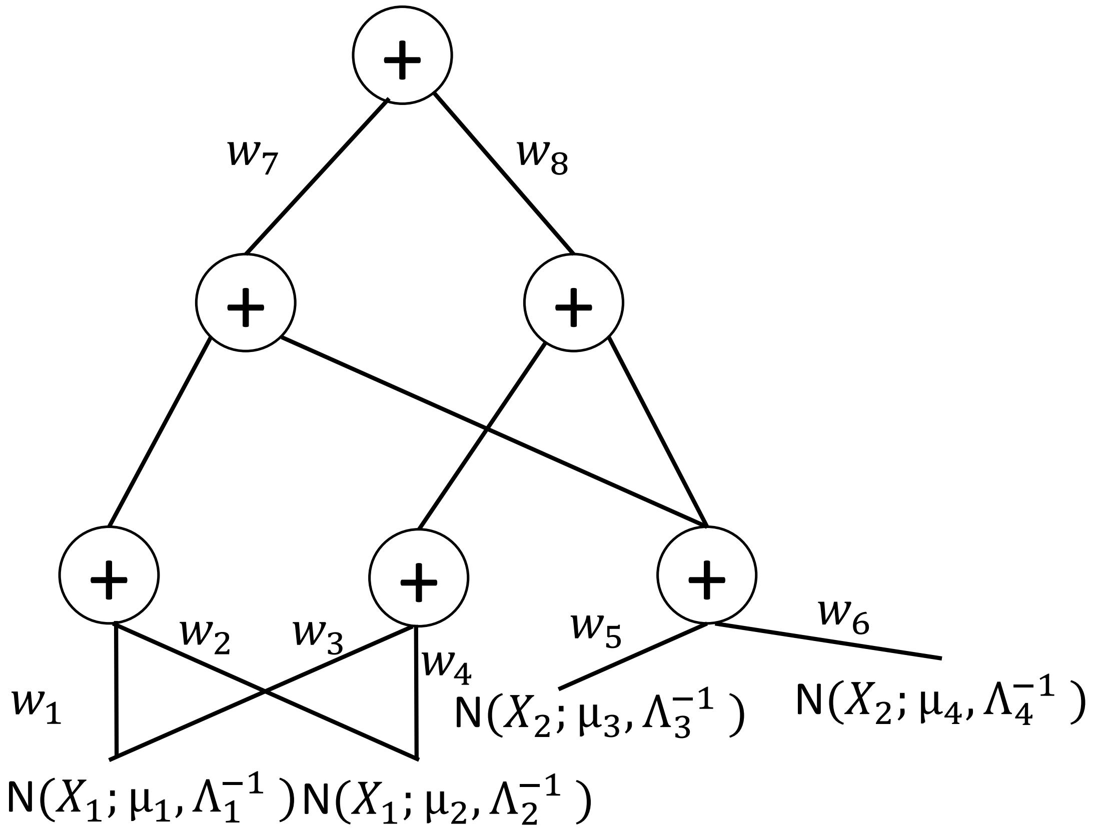


Continuous SPNS



Hierarchical mixture of Gaussians
(Unnormalised)

Bayesian Learning of SPNs



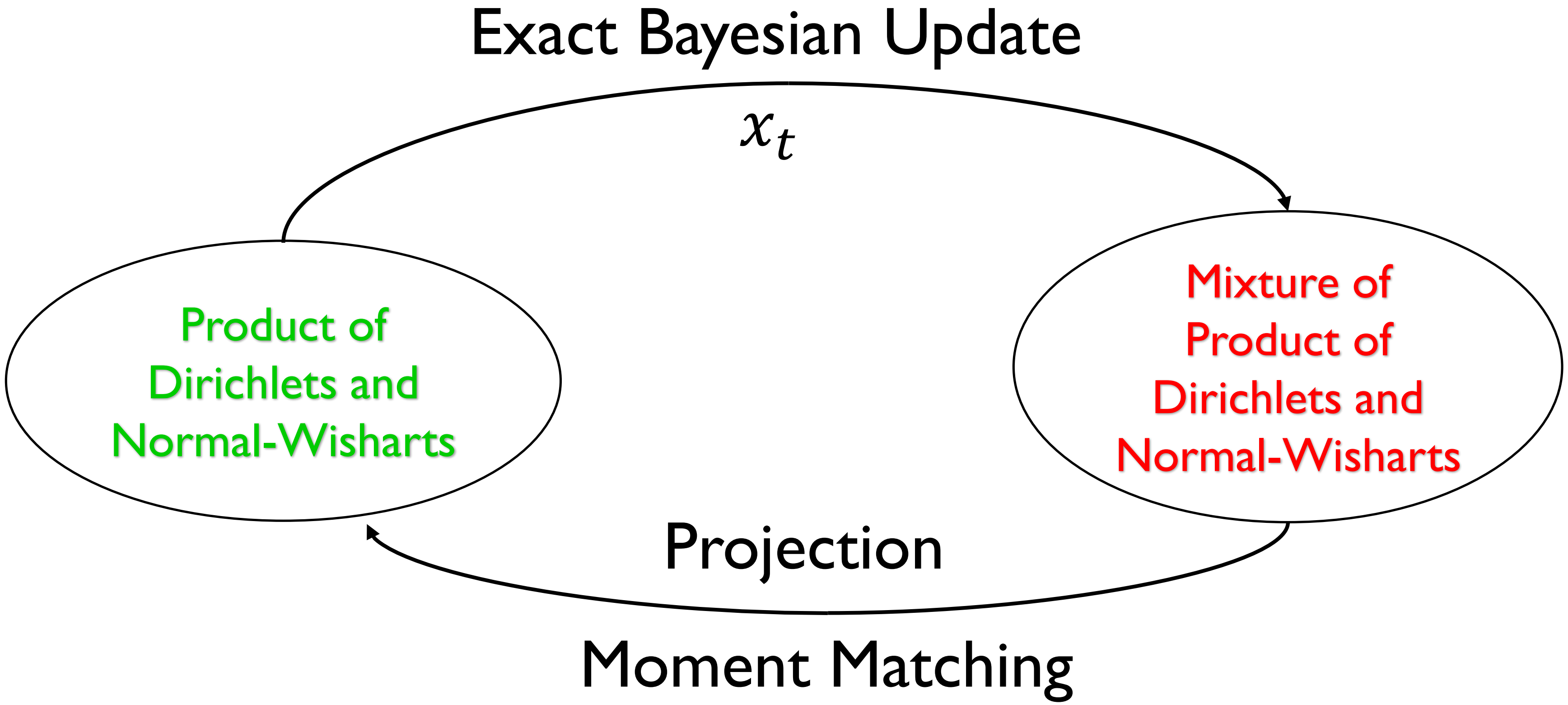
Parameters : weights, means and precisions (inverse covariance matrices)

Prior : $P(\boldsymbol{w}, \mu, \boldsymbol{\Lambda})$; product of Dirichlets and Normal-Wisharts

Likelihood :
$P(\boldsymbol{x}\,; \boldsymbol{w}, \mu, \boldsymbol{\Lambda}) = SPN(\boldsymbol{x}\,; \boldsymbol{w}, \mu, \boldsymbol{\Lambda})$

Posterior :
$P(\boldsymbol{w}, \mu, \boldsymbol{\Lambda}; \boldsymbol{data}) \propto$
$P(\boldsymbol{w}, \mu, \boldsymbol{\Lambda}) \prod_n SPN(\boldsymbol{x_n}\,; \boldsymbol{w}, \mu, \boldsymbol{\Lambda})$

Bayesian Moment Matching Algorithm



Overall Algorithm

- Recursive computation of all constants
  - Linear complexity in the size of SPN

- Moment Matching
  - System of linear equations
  - Linear complexity in the size of SPN

- Streaming Data
  - Posterior update : constant time w.r.t. amount of data

Empirical Results

Avg. log-likelihood and standard error based on 10-fold cross-validation

Dataset	Flow Size	Quake	Banknote	Abalone	Kinematics	CA	Sensorless Drive
Attributes	3	4	4	8	8	22	48
oBMM (random)	-	-	-	**-1.82** ∓ 0.19	**-11.19**∓ 0.03	**-2.47** ∓ 0.56	**1.58** ∓ 1.28
oEM (random)	-	-	-	-11.36 ∓ 0.19	-11.35 ∓ 0.03	-31.34 ∓ 1.07	-3.40 ∓ 6.06
oBMM (GMM)	**4.80** ∓ 0.67	**-3.84** ∓ 0.16	**-4.81** ∓ 0.13	**-1.21**∓ 0.36	**-11.24** ∓ 0.04	**-1.78** ∓ 0.59	-
oEM (GMM)	-0.49 ∓ 3.29	-5.50 ∓ 0.41	-4.81 ∓ 0.13	-3.53 ∓ 1.68	-11.35 ∓ 0.03	-21.39 ∓ 1.58	-

oBMM performs better than oEM

Empirical Results

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SRBM	-0.79 $\mp$ 0.01	**-2.38** $\mp$ 0.01	-2.76 $\mp$ 0.01	-2.28$\mp$ 0.01	**-5.55** $\mp$ 0.02	-4.95 $\mp$ 0.01	-26.91$\mp$ 0.03
GenMMN	-0.40 $\mp$ 0.01	-3.83 $\mp$ 0.01	**-1.70** $\mp$ 0.03	-3.29 $\mp$ 0.10	-11.36 $\mp$ 0.02`	-5.41 $\mp$ 0.14	-29.41 $\mp$ 1.19

oBMM is competitive with SBRM and GenMMN

Mixture Models

Consistent/Robust

Tensor Decomposition
And
Spectral Learning

Offline

Bayesian
Learning

ADF, EP, SGD, oEM

Non convergence and
local optima

Online

Can be distributed;
Practical problems

Bayesian Moment Matching
algorithm
Consistent?

SPNs

Bayesian Moment
Matching for continuous
SPNs

Comparison with other
deep networks

oBMM performs better
than oEM and
comparable to other
techniques

Conclusion and Future Work

Contributions :

- Online and distributed Bayesian Moment Matching algorithm
- Performs better than oEM w.r.t time and accuracy
- Extended it to continuous SPNs
- Comparative analysis with other deep learning methods

Future Work :

- Theoretical properties of BMM – consistent?
- Generalize oBMM to exponential family
- Extension to sequential data and transfer learning
- Online structure learning of SPNs

Thank you!