

Is Rebalancing Bad for Your Wealth?

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Abstract

The conventional wisdom suggests that retail investors should rebalance their portfolios back to a constant weight stock-bond mix. Although this sounds reasonable, empirical tests based on block bootstrap resampling of historical data do not support this claim. At best, the evidence for rebalancing the stock-bond split (as compared with buy and hold) for time horizons of less than ten years, is weak.

Only in the case where the investor wishes to avoid high instantaneous volatility (even though the CDF of the terminal wealth is arguably superior for buy and hold) will rebalancing be the better choice.

Keywords: Rebalancing, buy and hold, volatility pumping

1 Introduction

It is considered axiomatic in wealth management that investors should periodically rebalance their portfolios back to a target asset mix. The usual rationale is that this keeps the portfolio consistent with the investor's risk preferences. Rebalancing is fundamentally a contrarian strategy, selling winners and buying losers. This also allows the investor to *buy low and sell high*. There is a plethora of academic literature which seems to support this idea.

However, a classic example of a *buy and hold* investment is a capitalization weighted stock index exchange traded fund (ETF). The weights of each stock in the index are not constant, but drift in proportion to their market capitalization. After the initial purchase of the capitalization weighted ETF, the investor is basically following a buy and hold strategy. Consequently, anyone who holds a capitalization weighted index is actually following a buy and hold strategy for a significant proportion of their portfolio.

A sharp eyed reader will perhaps object to my characterization of a capitalization weighted index ETF as a pure buy and hold. Some rebalancing does take place. This is due to stocks being dropped from the index (as a result of not meeting market capitalization constraints). In addition, the implementation of dividend payments may also result in a small amount of rebalancing. For example, dividend payments may be distributed to ETF holders, or reinvested in new units of the total index. This can be viewed as a partial rebalancing. Nevertheless, a capitalization weighted index, at least to a first order approximation, can be considered to be a buy and hold strategy.

Consider a basket of stocks which have (i) low pair-wise correlation and (ii) high volatility. In this case, both theoretical and empirical analysis suggests that rebalancing to a constant weight in each

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32 underlying stock in the basket will produce superior investment results (Farago and Hjalmarsson,
33 2023; Forsyth, 2024), compared to a buy and hold (capitalization weighted) basket.

34 However, for most retail investors, the main investment decision is the split between bonds and
35 stocks, usually implemented in terms of ETFs. Bond ETFs have low volatility (at least based on
36 historical averages), hence the results concerning baskets of high volatility stocks may not apply to
37 this case.

38 Traditional measures of risk and reward (e.g. Sharpe ratios) seem to favour rebalancing. How-
39 ever, many authors have criticized Sharpe ratios as being too simplistic (variance penalizes upside
40 as well as downside). Some empirical studies even seem to favour buy and hold. For more discus-
41 sion, we refer the reader to (Perold and Sharpe, 1988; Wise, 1996; Dayanandan and Lam, 2015;
42 Dichtl et al., 2016; Edesess, 2017; Hilliard and Hilliard, 2018; Horn and Oehler, 2020; Bertrand and
43 Prigent, 2022) and the references cited therein.

44 Can it be the case that one of buy and hold or rebalancing is the superior strategy? Consider
45 the following example: an investment portfolio which consists of two assets: a stock index ETF
46 and a risk free, inflation protected bond index. To illustrate the nuances involved, we examine two
47 cases, where the investor has \$10,000 of initial wealth.

48 **Buy and hold:** The investor invests \$5,000 in the stock index, and \$5,000 in the inflation protected
49 bond index, and never rebalances.

50 **Rebalance yearly:** The initial allocation is the same as the buy and hold case. However, the
51 portfolio is rebalanced back to 50% in bonds and 50% in stocks annually.

52 Suppose the investment horizon is very long. Since stocks can be assumed to return more than
53 bonds over the very long term, eventually, the buy and hold portfolio will be almost all stocks (by
54 value). This will be undesirable for most investors, in terms of risk and reward.

55 On the other hand, the rebalanced portfolio has a positive probability of ending up (after a finite
56 time) with a value less than \$5,000 (this is the *catch a falling knife* scenario).¹ By assumption (the
57 bond fund is a government guaranteed, inflation protected index), the buy and hold portfolio has
58 zero probability of ever being less than \$5,000.

59 These two extreme cases illustrate that it is not possible, *a priori* to conclude that rebalancing
60 is superior to buy and hold. In the above example, we can see that we have to specify the time
61 frame of the investment, and the desire to avoid left tail risk, in order to make a recommendation.

62 A more mathematical statement of this example is that neither strategy stochastically dominates
63 the other ²

64 In this paper, we consider an investor who holds only two assets: a stock index fund and a bond
65 index fund.³ Initially, in order to gain intuition, we will make simplifying assumptions (the stock
66 index follows geometric Brownian motion, the bond index has zero volatility, and rebalancing is
67 continuous). We will then go on to more realistic assumptions: a jump diffusion for the stock index
68 and discrete rebalancing. We will examine the entire cumulative distribution function (CDF) of the
69 final wealth distribution, as well as the wealth percentiles during the investment horizon (based on
70 simulations) for both rebalancing and buy and hold.

71 Finally, we will again examine the wealth distributions for both strategies, but based on boot-
72 strap resampling of historical data (Politis and Romano, 1994; Politis and White, 2004; Dichtl et al.,

¹Suppose stocks trend down over a long period. Rebalancing will keep buying losers all the way down.

²First order stochastic dominance can be identified from the CDFs. If the CDF of strategy A plots below the CDF for strategy B, then A stochastically dominates (to first order) B. Any investor who prefers more rather than less will prefer strategy A. Stochastic dominance is rare. However, partial stochastic dominance can sometimes be observed (van Staden et al., 2021).

³This would be representative of the fundamental rebalancing issue for a retail investor.

2016; Anarkulova et al., 2022). For these simulations, we make no assumptions about the underlying stochastic processes of the stock and bond indexes. These results are purely data driven.

Based on historical US stock and bond data, and considering a ten year investment horizon, our main conclusion is that it is difficult to make the case that rebalancing is superior to buy and hold. The caveat here is that the buy and hold portfolio will show higher instantaneous volatility compared to the rebalanced portfolio, but only near the end of investment horizon, in cases where the buy and hold portfolio has large gains. Hence, this may be acceptable to many investors.

Only in the case where the investor wishes to avoid high instantaneous volatility (even though the CDF of the terminal wealth is arguably superior for buy and hold) will rebalancing be the better choice.

2 Intuition: geometric Brownian motion (GBM)

Let the value of the stock index ETF be denoted by $S(t)$ and the value of a bond index be denoted by $B(t)$. The stochastic process of the underlying stock index ETF with value S is assumed to follow geometric Brownian motion (GBM)

$$\begin{aligned} \frac{dS}{S} &= \mu dt + \sigma dZ \\ \mu &= \text{arithmetic return} \\ \sigma &= \text{volatility} \\ dZ &= \text{increment of a Wiener process} . \end{aligned} \tag{2.1}$$

The bond index is considered to be risk-free and non-volatile

$$dB = rB dt . \tag{2.2}$$

The value of a portfolio $W(t)$, which is continuously rebalanced to a weight of β in the stock index and $(1 - \beta)$ in the bond index then follows the process

$$\begin{aligned} \frac{dW}{W} &= \beta \left(\frac{dS}{S} \right) + (1 - \beta) \left(\frac{dB}{B} \right) \\ &= \left((1 - \beta)r + \beta\mu \right) dt + \beta\sigma dZ . \end{aligned} \tag{2.3}$$

The exact solution to equation (2.3) is

$$\begin{aligned} \frac{W(t)}{W(0)} &= e^{(1-\beta)rt - \beta^2\sigma^2t/2} e^{\beta\mu t + \beta\sigma(Z(t) - Z(0))} \\ (Z(t) - Z(0)) &\simeq \mathcal{N}(0, t) , \end{aligned} \tag{2.4}$$

where $\mathcal{N}(0, t)$ is a draw from a normal distribution with mean zero and variance t .

The exact solution to equation (2.1) is

$$\frac{S(t)}{S(0)} = e^{\mu t - \sigma^2 t/2} e^{\sigma(Z(t) - Z(0))} \tag{2.5}$$

or

$$\left(\frac{S(t)}{S(0)} \right)^\beta e^{\beta\sigma^2 t/2} = e^{\beta\mu t + \beta\sigma(Z(t) - Z(0))} \tag{2.6}$$

94 Substitute equation (2.6) into equation (2.4) to obtain

$$\frac{W(t)}{W(0)} = e^{(1-\beta)rt + \beta(1-\beta)\sigma^2 t/2} \left(\frac{S(t)}{S(0)} \right)^\beta \quad (2.7)$$

95 Assume that $W(0) = S(0)$, so that this becomes

$$\text{Rebalanced portfolio} = \frac{W^{rebal}(t)}{S(0)} = e^{(1-\beta)rt + \beta(1-\beta)\sigma^2 t/2} \left(\frac{S(t)}{S(0)} \right)^\beta \quad (2.8)$$

96 Now consider a buy and hold portfolio with $\beta W^{bh}(0)$ in the stock index, $(1-\beta)W^{bh}(0)$ in the
97 bond index at $t = 0$, never rebalanced, and liquidated at time t . Assume $W^{bh}(0) = S(0)$. Then

$$\text{Buy and hold} = \frac{W^{bh}(t)}{S(0)} = \frac{\beta S(t) + (1-\beta)e^{rt}S(0)}{S(0)}. \quad (2.9)$$

98 Finally, the value of the pure stock ETF, denoted by W^s , at time t is simply (initial value $S(0)$)

$$\text{All Stock ETF} = \frac{W^s(t)}{S(0)} = \frac{S(t)}{S(0)}. \quad (2.10)$$

99 3 Derivative contracts

100 If $\beta < 1$ in equation (2.8) we can see that rebalancing results in a nonlinear, option-like payoff.
101 For example if $\beta = 0.5$, then rebalancing produces a square root payoff. We can think of W^{rebal} ,
102 W^{bh} , W^s as derivative contracts, each costing $S(0)$ to purchase, with payoffs given by equations
103 (2.8-2.10).

104 We consider the stock index to be the Center for Research in Securities Prices (CRSP) capital-
105 ization weighted index. We take the bond index to be based on the return of 30-day T-bills. The
106 GBM stock index parameters are obtained by fitting to the inflation adjusted CRSP capitalization
107 weighted index, 1926:1-2023:12 (see Appendix A) using maximum likelihood. The interest rate r is
108 based on the average (real) return of 30-day T-bills from CRSP as well.

109 Note that equations (2.8-2.10) are independent of the stock drift rate μ (see equation (2.1)).
110 Figure 3.1(a) compares the payoffs, as a function of $(S(t)/S(0))$ for all three contracts. Figure
111 3.1(b) plots

$$\frac{W^{rebal} - W^{bh}}{S(0)}. \quad (3.1)$$

112 Observe from Figure 3.1(b), that for values of $(S_T/S_0) \in (.5, 1.75)$, the payoff of the rebalanced
113 portfolio is above the buy and hold strategy, but by most $\simeq 6\%$. Outside these ranges for (S_T/S_0) ,
114 buy and hold has a superior payoff, sometimes by very large amounts. This payoff diagram confirms
115 our intuition. If stocks trend up for long periods, then rebalancing gives up some of these large stock
116 gains. If stocks trend down for long periods, then rebalancing will erode the bond protection, since
117 bonds are sold and losing stocks purchased. If stocks trade in a limited range, then rebalancing
118 does generate a *volatility pumping* return. But, for reasonable market parameters, this effect is not
119 large.

120 In Bertrand and Prigent (2022), it is demonstrated that for a wide range of μ, σ, T (T up to 30
121 years), the probability of falling inside the region where rebalancing has a higher payoff than buy
122 and hold is $< 70\%$. In summary, we can say that, in general, the payoff of rebalancing is slightly

123 better than buy and hold at most 70% of the time. On the other hand, the payoff of buy and hold
 124 can be much larger than rebalancing $\simeq 30\%$ of the time.

125 The CRSP index, fit to GBM and adjusted for inflation, gives an arithmetic return of $\mu = .0818$,
 126 with a median value of $(S_T/S(0))$ at $T = 10$ of about 1.91. From Figure 3.1(b) this suggests that
 127 for this data, at least 50% of the time, buy and hold is superior to rebalancing, sometimes by a
 128 large amount.

129 At this point, we really cannot say much more here unless we know the probabilities of being in
 130 the various regions of the payoff diagrams.

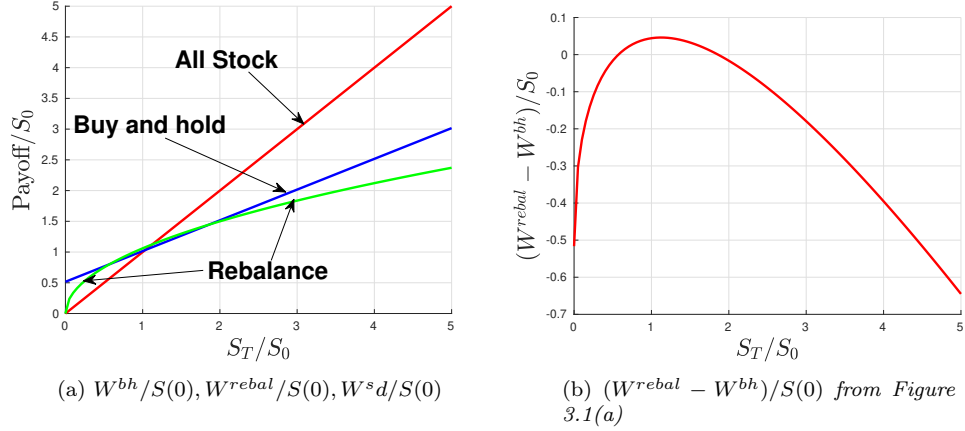


FIGURE 3.1: *Payoff diagrams comparing $W^{bh}/S(0), W^{rebal}/S(0), W^s/S(0)$. $T = 10.0$ years, from equations (2.8-2.10). Fraction in equities $\beta = 0.5$. Stock data fit of equation (2.1) to inflation adjusted CRSP index, 1926:1-2023:12. Interest rate r from T-bills, inflation adjusted, 1926:1-2023:12. $\sigma = .1849, r = .0032$.*

131 4 Jump diffusion, discrete rebalancing

132 Clearly, it is simplistic to assume that (i) stocks follow GBM and (ii) rebalancing is continuous.
 133 In this Section, we remove these two assumptions. Assume that the stock index follows a jump
 134 diffusion process, which allows for non-normal returns. If a jump occurs $S(t) = \xi S(t^-)$, and

$$\begin{aligned} \frac{dS}{S(t^-)} &= (\mu - \lambda\kappa) dt + \sigma dZ + (\xi - 1)dQ \\ dQ &= \begin{cases} 0 & ; \text{probability } (1 - \lambda dt) \\ 1 & ; \text{probability } \lambda dt \end{cases} \\ \kappa &= E[\xi - 1] \\ \lambda &= \text{intensity of the Poisson process} . \end{aligned} \tag{4.1}$$

135 Assume that $y = \log \xi$ follows a double exponential process(Kou, 2002), with density $g(y)$ given by

$$g(y) = p_{up}\eta_1 e^{-\eta_1 y} \mathbf{1}_{y \geq 0} + (1 - p_{up})\eta_2 e^{\eta_2 y} \mathbf{1}_{y < 0}. \tag{4.2}$$

136 where p_{up} is the probability of an upward jump. Note as well that

$$E[\xi] = \frac{p_{up}\eta_1}{\eta_1 - 1} + \frac{(1 - p_{up})\eta_2}{\eta_2 + 1}. \tag{4.3}$$

137 As before, we assume that the bond index is non-volatile, and follows equation (2.2). We use a
 138 filtering method (Cont and Mancini, 2011; Dang and Forsyth, 2016) to estimate the jump diffusion
 139 parameters, based on the CRSP data 1926:1-2023:12 (see Appendix A). The parameters are listed
 140 in Appendix B. Our basic scenario is given in Table 4.1. We will use Monte Carlo simulation to
 141 determine the CDFs for these strategies.

T	10 years
Initial Investment	1000
Rebalancing frequency	1 month
T-bill return r	0.0031
Jump diffusion parameters	Table B.1

TABLE 4.1: *Data for example payoffs.*

142 Table 4.2 shows the summary statistics for these simulations. $ES(5\%)$ is the expected shortfall
 143 at the five per cent level, i.e. the mean of the worst 5% of the outcomes. The Omega ratio (Keating
 144 and Shadwick, 2002) at level L is defined as

$$\text{Omega}(L) = \frac{E[\max(W_T - L, 0)]}{E[\max(L - W_T, 0)]}. \quad (4.4)$$

145 The Omega ratio is a measure of upside versus downside, with respect to the level L . Since both
 146 rebalancing and buy and hold have similar median values, Table 4.2 shows the Omega ratio at level
 147 $L = 1481$, the median of the rebalanced portfolio.

148 We can see that the buy and hold portfolio has a higher expected terminal wealth W_T compared
 149 to the rebalanced portfolio. The median values of W_T are essentially the same, for both strategies.
 150 However, the 5th percentile is smaller (worse) than the rebalanced portfolio by about 6%, but
 151 the expected shortfalls (the tail risk measure) are essentially the same. The standard deviation of
 152 the buy and hold portfolio is much larger (947 versus 574) compared to the rebalanced portfolio.
 153 However, the Omega ratio indicates that this is primarily due to more upside variation, i.e. more
 154 extreme values above the median. Hence, we can see here that the standard deviation is not a good
 155 measure of risk.

	$E[W_T]$	Median[W_T]	$W_T : 5^{th}$ percentile	$W_T : 95^{th}$ percentile	ES(5%)	std[W_T]	Omega(1481)
Rebalance	1572 (2.2)	1481	870	2563	754	574	1.58
Buy and Hold	1710 (3.7)	1484	827	3302	746	947	2.29
All stocks	2392 (7.3)	1937	624	5599	462	1894	n/a

TABLE 4.2: *Jump diffusion model for stocks. Statistics for: rebalanced monthly, $\beta = 0.5$, buy and hold (initial stock fraction $\beta = 0.5$), and an all stock portfolio (i.e. buy and hold with $\beta = 1.0$). 2.56×10^5 Monte Carlo simulations. Numbers in brackets are the standard error estimate at the 95% confidence level. Stocks follow the jump diffusion model 4.1-4.2. Parameters fit to value-weighted CRSP index deflated by the CPI. Sample period 1926:1 to 2023:12, see Table B.1. The average real return of a 30 day T-bill in the same period was $r = .00031$. $ES(5\%)$ is the mean of the worst 5% of the outcomes.*

156 Figure 4.1(a) compares the CDFs of W_T for rebalancing at monthly intervals, buy and hold, and
 157 the 100% stock portfolio. We can see from Figure 4.1(a) that buy and hold underperforms compared

158 to rebalancing, below the median, and outperforms rebalancing above the median. However, the
 159 extreme left tail performance for both strategies is about the same (from Table 4.2).

160 Figure 4.1(b) compares yearly and monthly rebalancing strategies. The CDF curves are virtually
 161 indistinguishable, indicating that frequent rebalancing appears unnecessary.

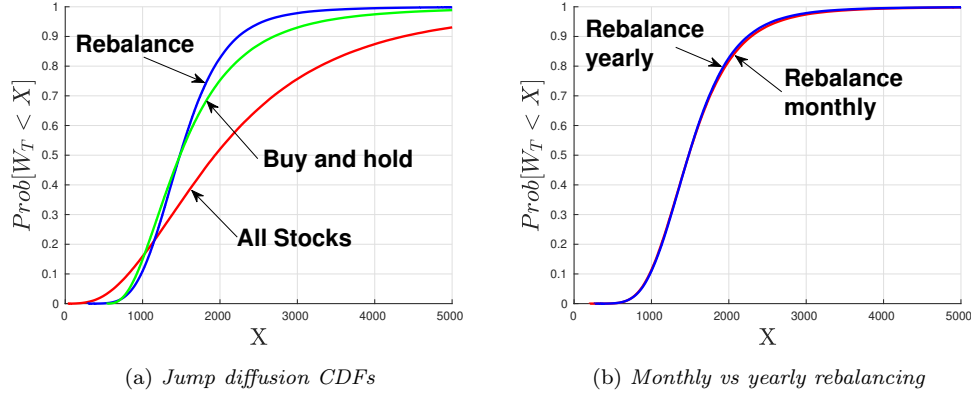


FIGURE 4.1: *Jump diffusion model (4.1). Data in Appendix B. Scenario in Table 4.1. Rebalancing fraction in equities $\beta = 0.5$. Stock data fit of equation (4.1) to inflation adjusted CRSP index, 1926:1-2023:12. Interest rate r from T-bills, inflation adjusted, 1926:1-2023:12.*

162 5 Bootstrap resampling

163 Our last test consists of examining the performance of rebalancing and buy and hold using a
 164 pure data driven approach. We will use bootstrap resampling of the inflation adjusted CRSP
 165 capitalization weighted index, and the inflation adjusted 30-day T-bill index (see Appendix A). The
 166 data set covers the historical range 1926:1-2023:12.

167 A ten year investment scenario consists of 120 consecutive one month returns. A single scenario
 168 is constructed as follows. We select a month at random from the historical data, and use this as
 169 our first month's return. Then, we select another month at random (with replacement) which is
 170 the second month's return in our ten year scenario. We keep doing this until we have a set of 120
 171 returns (one thirty-year path). We then repeat this procedure many times, to produce many 30-year
 172 return paths.

173 However, this bootstrapping approach does not take into account possible serial correlation in
 174 the returns. This is just another way of saying that next month's returns may be affected by the
 175 returns of the past few months or years.

176 To take this into account, we select an initial month at random, but use b consecutive monthly
 177 returns (starting at the initial random month). We repeat this $(120/b)$ times to generate a single
 178 10 year path. We call b the blocksize.

179 But we are not done yet. It turns out that a better approach is to not use a fixed blocksize, but
 180 to specify an average blocksize b , and randomly vary the blocksize within each ten year path. This
 181 is called the stationary block bootstrap method.

182 For more details about this method, see (Politis and Romano, 1994; Politis and White, 2004;
 183 Patton et al., 2009; Dichtl et al., 2016; Forsyth and Vetzal, 2019; Anarkulova et al., 2022). Detailed
 184 pseudo-code for block bootstrap resampling is given in Ni et al. (2022).

185 We will block bootstrap the returns for both the CRSP capitalization index, and for the CRSP
 186 30 day T-bill index (both inflation adjusted), based on the historical data over the period 1926:1-

187 2023:12. We will simultaneously draw returns from both the stock index and the bond index
 188 (preserving any possible correlations. We use an expected blocksize of one year. Experiments with
 189 expected blocksizes ranging from 3 months to two years do not change the results significantly. The
 190 basic scenario is shown in Table 5.1.

191 For the buy and hold case, we initially invest $0.5W(0)$ in the stock index and $0.5W(0)$ in the bond
 192 index, and never rebalance. In the rebalancing case, we start off with the same initial investment
 193 as buy and hold, but then rebalance to a weight of 0.50 in stocks annually.

T	10 years
Initial Investment	$W_0 = 1000$
Rebalancing frequency	1 year
T-bill returns	CRSP data
Stock returns	CRSP data
Rebalancing fraction	$\beta = 0.5$

TABLE 5.1: *Data for the bootstrap simulations.*

194 Table 5.2 shows summary statistics for the various strategies. Note that for the rebalancing
 195 case, the summary statistics for rebalancing monthly are quite similar to the statistics for annual
 196 rebalancing. This is not unexpected, in view of Figure 4.1(b).

197 From now on, we will focus exclusively on annual rebalancing. Annual rebalancing will be
 198 straightforward to implement for a retail investor.

	$E[W_T]$	Median $[W_T]$	$W_T : 5^{th}$ percentile	$W_T : 95^{th}$ percentile	ES(5%)	std $[W_T]$
Rebalance monthly						
Rebalance	1534	1477	870	2386	758	476
Rebalance yearly						
Rebalance	1552	1487	878	2438	768	491
Never rebalance						
Buy and Hold	1656	1501	854	2970	764	706
All stocks	2272	1969	671	4869	510	1400

TABLE 5.2: *Bootstrap simulations, rebalanced yearly (monthly) $\beta = 0.5$, buy and hold (initial stock fraction $\beta = 0.5$), and an all stock portfolio (i.e. buy and hold with $\beta = 1.0$). 10^5 block bootstrap simulations, expected blocksize one year. CRSP data, 1926:1-2023:12. Stock index: CRSP capitalization weighted index. Bond index: 30 day US T-bills. Indexes inflation adjusted. See Table 5.1. ES(5%) is the mean of the worst 5% of the outcomes.*

199 From Table 5.2 and Figure 5.1(a) we can see that

- 200 • Rebalancing outperforms buy and hold, below the median of W_T , by a small amount, however
 201 the extreme left tail statistic $ES(5\%)$ is similar for both strategies
- 202 • Buy and hold outperforms rebalancing, sometimes by a large amount, above the median of
 203 W_T .

204 Figure 5.1(b) shows the wealth percentiles for rebalancing and buy and hold, at each rebalancing
 205 time. We can observe that the 5th percentile and median values of wealth for both strategies is

206 almost identical. On the other hand, the 95th percentile wealth for buy and hold is significantly
 207 larger than for the rebalanced portfolio, as $t \rightarrow T$.

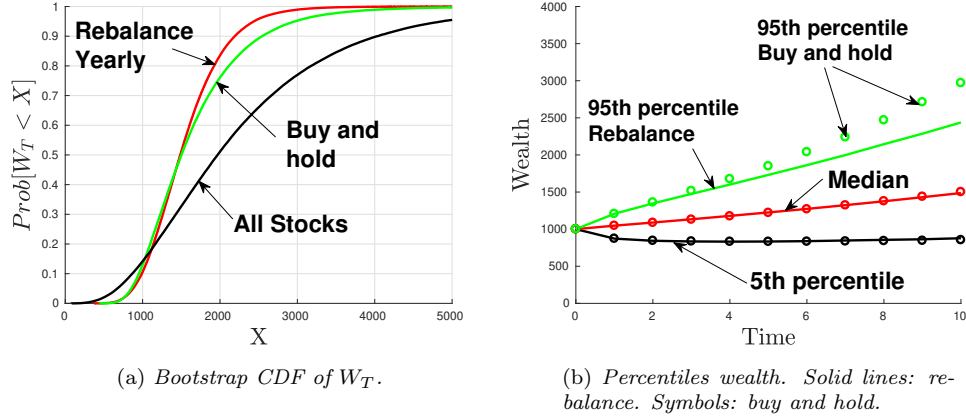


FIGURE 5.1: *Bootstrap simulations: rebalanced yearly $\beta = 0.5$, buy and hold (initial stock fraction $\beta = 0.5$), and an all stock portfolio (i.e. buy and hold with $\beta = 1.0$). 10^5 block bootstrap simulations, expected blocksize one year. CRSP data, 1926:1-2023:12. Stock index: CRSP capitalization weighted index. Bond index: 30 day US T-bills. See Table 5.1.*

208 6 Pathwise comparison

209 A more rigorous comparison of buy and hold and rebalancing can be determined by examining the
 210 pathwise comparison of rebalancing and buy and hold. To this end, we will consider the statistics
 211 of (W^{bh}/W^{rebal}) along each path. Buy and hold outperformance, along each path, is indicated if
 212 $(W^{bh}/W^{rebal}) > 1.0$

213 Table 6.1 shows the statistics for the ratio (W^{bh}/W^{rebal}) at $t = T$. The Omega ratio for this
 214 pathwise test is defined as

$$\text{Omega}(L) = \frac{E[\max(R_T - L, 0)]}{E[\max(L - R_T, 0)]}$$

$$R_T = \frac{W_T^{bh}}{W_T^{rebal}} . \quad (6.1)$$

215 We will examine the Omega ratio for $L = 1$, i.e. we consider the expected value of buy and hold
 216 outperforming rebalancing, compared to underperforming.

217 We can see from Table 6.1 that, along any path, the 5th percentile of the wealth of the buy and
 218 hold portfolio is about 92% of the rebalanced portfolio, while at the 95th percentile, (W_T^{bh}/W_T^{rebal})
 219 is 125%. This indicates a better upside, compared to the downside, for buy and hold. This is also
 220 reflected in the Omega ratio.

221 Figure 6.1(a) shows the CDF of (W^{bh}/W^{rebal}) at $t = T$. We can see that the probability that
 222 buy and hold will outperform rebalancing is about 55%, with greater upside compared to downside,
 223 consistent with the Omega ratio in Table 6.1. Figure 6.1(b) shows that the 5th and 95th percentiles
 224 of (W^{bh}/W^{rebal}) show a larger deviation from the median as time goes on. However, at each time
 225 in $[0, T]$, the upside is larger than the downside.

$E[(W_T^{bh}/W_T^{rebal})]$	(W_T^{bh}/W_T^{rebal}) Median	(W_T^{bh}/W_T^{rebal}) 5 th percentile	(W_T^{bh}/W_T^{rebal}) 95 th percentile	ES(5%)	Omega (L=1)
1.04	1.02	.924	1.25	.889	3.53

TABLE 6.1: Statistics for (W^{bh}/W^{rebal}) at $t = T$. W^{bh} : buy and hold (initial stock fraction $\beta = 0.5$). W^{rebal} : rebalance yearly ($\beta = 0.5$). 10^5 block bootstrap simulations, expected blocksize one year. CRSP data, 1926:1-2023:12. Stock index: CRSP capitalization weighted index. Bond index: 30 day US T-bills. Both indexes inflation adjusted. See Table 5.1. ES(5%) is the mean of the worst 5% of the outcomes of (W^{bh}/W^{rebal}) . The Omega ratio for (W^{bh}/W^{rebal}) is defined in equation (6.1).

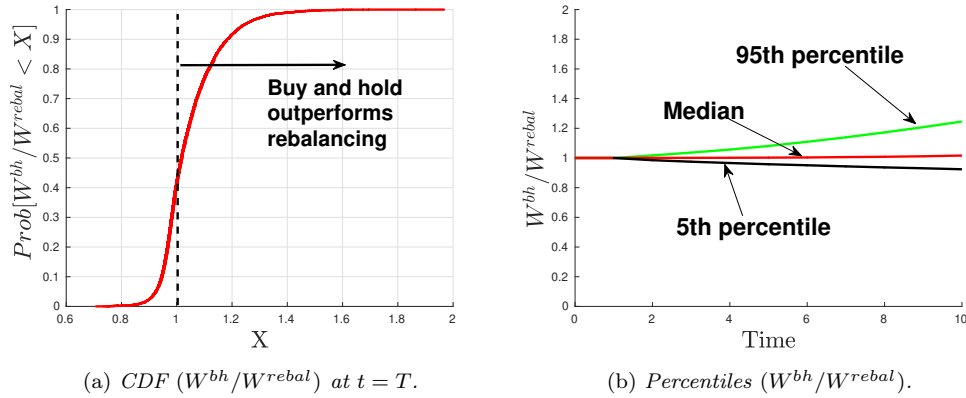


FIGURE 6.1: Ratio (W^{bh}/W^{rebal}) . Bootstrap simulations: rebalanced yearly $\beta = 0.5$, buy and hold (initial stock fraction $\beta = 0.5$), and an all stock portfolio (i.e. buy and hold with $\beta = 1.0$). 10^5 block bootstrap simulations, expected blocksize one year. CRSP data, 1926:1-2023:12. Stock index: CRSP capitalization weighted index. Bond index: 30 day US T-bills. Both indexes inflation adjusted. See Table 5.1.

226 7 Instantaneous volatility: buy and hold

227 As a first approximation, we can ignore the volatility of the bond index. Consequently, the instanta-
 228 neous volatility of the buy and hold portfolio will be proportional to fraction in equities, compared
 229 to the rebalanced portfolio which maintains the fraction 0.5 in stocks.

230 Figure 7.1 shows the percentiles of the fraction in stocks, through time, for the buy and hold
 231 policy. We can observe that the buy and hold strategy has a median fraction in stocks which increases
 232 steadily as time goes on, ending up with about 66% stocks at $t = T$. The terminal fraction in stocks,
 233 at the 95th percentile is about 0.80, indicating that at this percentile, the volatility of buy and hold
 234 is $(.8/.5) \simeq 1.60$ times larger than the rebalanced portfolio. However, this large volatility will only
 235 occur at large values of wealth. On the other hand, at the 5th percentile, the terminal fraction in
 236 stocks is about 0.4, indicating a smaller volatility compared to rebalancing, along paths with poor
 237 stock performance.

238 In summary, it is clear that when stocks perform well, the fraction in stocks for buy and hold
 239 will increase, and the volatility of the portfolio will be larger compared to rebalancing. We can also
 240 expect larger drawdowns. However, from Table 6.1 and Figure 6.1(b), the pathwise worst case 5th
 241 percentile of (W^{bh}/W^{rebal}) is about 92%. This pathwise criteria is very strict. Looking at the 5th
 242 percentiles of wealth for rebalancing and buy and hold (not pathwise), Figure 5.1(b) indicates that
 243 the 5th percentiles for the wealth are almost the same for both strategies.

244 It would appear then that there is at least a behavioral argument in favour of rebalancing, since
 245 it is probable that the instantaneous volatility of the buy and hold portfolio will be larger than the
 246 rebalanced portfolio. This smaller volatility, of the rebalanced portfolio, comes at the cost of giving
 247 up potential upside. However, note that if stocks do poorly, then the buy and hold portfolio will
 248 have less volatility than the rebalanced portfolio.

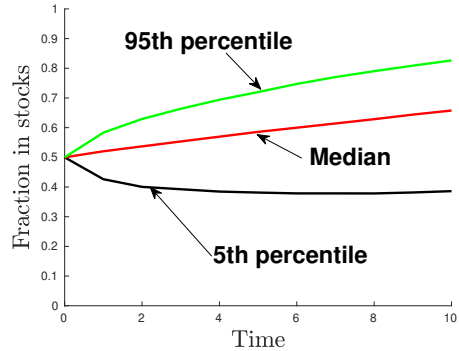


FIGURE 7.1: *Fraction of wealth in stocks, buy and hold. 10^5 block bootstrap simulations, expected blocksize one year. CRSP data, 1926:1-2023:12. Stock index: CRSP capitalization weighted index. Bond index: 30 day US T-bills. See Table 5.1.*

249 8 Alternate assets

250 In order to verify that there is nothing special about our choice of assets, we carry out block
 251 bootstrap resampling using the CRSP equal weighted stock index, and for the bond index, we use
 252 the 10 year US Treasury index. Both indexes are constructed for the period 1926:1-2023:12, and
 253 deflated using the CPI. The results are reported in Appendix C, and are qualitatively similar to
 254 those with carried out the capitalization weighted CRSP index, and the 30-day T-bill index.

255 9 Other strategies

256 A popular alternative to constant weight rebalancing is a deterministic glide path. A simple example
 257 of a glide path is

$$\text{fraction in equities} = \frac{100 - \text{your age}}{100} . \quad (9.1)$$

258 However, we know that for any glide path, there is a constant weight strategy which has almost
 259 the same CDF of the final wealth (Forsyth and Vetzal, 2019; Ni et al., 2022). This implies that, in
 260 terms of the final wealth CDF, if we replaced constant weight rebalancing by (any) glide path, our
 261 conclusions would be similar.

262 On the other hand, use of dynamic, adaptive strategies would generally be superior to buy and
 263 hold, in terms of meeting the specified objective function (van Staden et al., 2021; Forsyth, 2022;
 264 Forsyth and Vetzal, 2022).

265 In other words, compared to buy and hold, there is little to be gained by using a constant
 266 weight strategy, or a deterministic glide path. Significant improvements can only be obtained using
 267 dynamic, adaptive strategies.

268 10 Summary

269 We have carried out a detailed analysis of the CDFs of the final wealth for buy and hold compared to
270 annually rebalancing over a ten year period. Consistent results are obtained using various levels of
271 modelling: (i) GBM models of stock index returns (ii) jump diffusion models of stocks (iii) bootstrap
272 resampling of historical data.

273 Generally, rebalancing outperforms buy and hold by a small amount, below the median of the
274 final wealth, but underperforms buy and hold by larger amounts above the median. In addition,
275 the extreme left tail, as measured by the average of the worst 5% of the outcomes, is similar for
276 both buy and hold and rebalancing. In other words, buy and hold has more upside than downside,
277 compared to rebalancing, with similar worst case performance.

278 We also note that, considering a ten year investment horizon for rebalancing investors, it is
279 unnecessary to rebalance more frequently than annually.

280 The bootstrap results were qualitatively similar for the cases (i) stock index: CRSP capitalization
281 weighted index; bond index: 30-day T-bills and (ii) stock index: CRSP equal weight index; bond
282 index: 10 year US treasuries.

283 The negative aspect of buy and hold is that the instantaneous volatility of this strategy will be
284 generally larger than the rebalanced portfolio. This will be particularly pronounced when buy and
285 hold has large returns in stocks, meaning that the wealth of the buy and hold portfolio will be large.
286 It is possible that this may be a behavioral reason to recommend rebalancing to some investors.

287 11 Conclusions

288 Based on the CDFs of the terminal wealth, the percentiles of wealth through time, and considering
289 the extreme left tail of the final wealth, it is difficult to recommend that investors should rebalance
290 a stock-bond portfolio, over ten year horizons. Buy and hold has more upside than rebalancing,
291 with slightly worse downside, but similar risk in the extreme left tail. Buy and hold is also generally
292 more volatile than rebalancing, but this effect is large in cases where the buy and hold wealth is
293 also large. This may not be a problem for many investors.

294 We have not considered taxes or transaction costs in this paper. Transaction costs for infre-
295 quently traded index ETFs are negligible. However, in a taxable account, rebalancing can trigger
296 capital gains taxes (winners are sold). Buy and hold, of course, defers taxes. Consequently, our
297 results would favour buy and hold in a taxable account.

298 Appendices

299 A Data

300 We use data from the Center for Research in Security Prices (CRSP) on a monthly basis over the
301 1926:1-2023:12 period.⁴ Our base case tests use the CRSP US 30 day T-bill for the bond asset
302 and the CRSP value-weighted total return index for the stock asset. This latter index includes all
303 distributions for all domestic stocks trading on major U.S. exchanges. All of these various indexes
304 are in nominal terms, so we adjust them for inflation by using the U.S. CPI index, also supplied by
305 CRSP. We use real indexes since investors should be focused on real (not nominal) wealth goals. We
306 also include examples using the CRSP equal weighted index, and the CRSP 10-year U.S. Treasury
307 index.⁵

308 B Jump diffusion parameters

The parameters for equations (4.1) and (4.2) are fit to the CRSP data, with results in Table B.1.

	μ	σ	λ	p_{up}	η_1	η_2
CRSP Index (real)	0.08732	0.1477	0.3163	0.2258	4.3591	5.5337

TABLE B.1: *Estimated annualized parameters for double exponential jump diffusion model. Value-weighted CRSP index deflated by the CPI. Sample period 1926:1 to 2023:12. The average real return of a 30 day T-bill in the same period was $r = .00031$.*

309

310 C Alternate assets

311 In order to verify that our findings are robust to the choice of assets, we carry out block bootstrap
312 resampling using the CRSP equal weighted stock index, and for the bond index, we use the 10 year
313 US Treasury index. Both indexes are constructed for the period 1926:1-2023:12. As before, we
314 deflate these indexes using the CPI.

315 Table C.1 shows that median returns for all methods are larger than for the case where the
316 underlying assets are the capitalization weighted CRSP index, and the 30-day T-bill index. This is
317 hardly unexpected. However, the qualitative results are similar to that reported in Section 4. In
318 particular, compare Figure C.1 with Figure 5.1.

⁴More specifically, results presented here were calculated based on data from Historical Indexes, ©2023 Center for Research in Security Prices (CRSP), The University of Chicago Booth School of Business. Wharton Research Data Services (WRDS) was used in preparing this article. This service and the data available thereon constitute valuable intellectual property and trade secrets of WRDS and/or its third-party suppliers.

⁵The 10-year Treasury index was calculated using monthly returns from CRSP dating back to 1941. The data for 1926-1941 were interpolated from annual returns in Homer and Sylla (2005). The bond index is constructed by (i) purchasing a 10-year Treasury at the start of each month, (ii) collecting interest during the month and (iii) selling the Treasury at the end of the month.

	$E[W_T]$	Median $[W_T]$	5 th percentile	95 th percentile	ES(5%)	std $[W_T]$
Rebalance	2026	1811	895	3900	758	1023
Buy and Hold	2229	1796	872	5047	756	1617
All stocks	3222	2340	613	8800	435	3205

TABLE C.1: *Statistics for bootstrap simulations: rebalanced yearly $\beta = 0.5$, buy and hold (initial stock fraction $\beta = 0.5$), and an all stock portfolio (i.e. buy and hold with $\beta = 1.0$). 10^5 block bootstrap simulations, expected blocksize one year. CRSP data, 1926:1-2023:12. Stock index: CRSP equal weighted index. Bond index: 10 year Treasuries. See Table 5.1. ES(5%) is the mean of the worst 5% of the outcomes.*

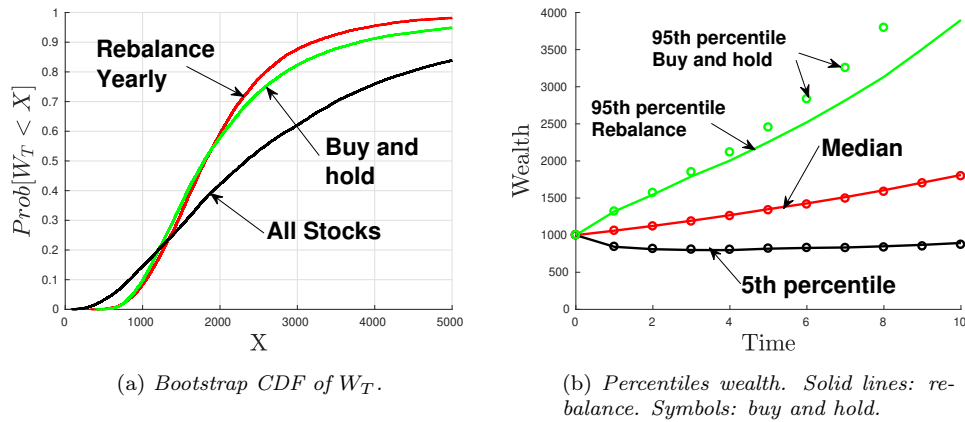


FIGURE C.1: *Bootstrap simulations: rebalanced yearly $\beta = 0.5$, buy and hold (initial stock fraction $\beta = 0.5$), and an all stock portfolio (i.e. buy and hold with $\beta = 1.0$). 10^5 block bootstrap simulations, expected blocksize one year. CRSP data, 1926:1-2023:12. Stock index: CRSP equal weighted index. Bond index: 10 year US Treasuries.*

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