Strategic interactions and uncertainty in decisions to curb greenhouse gas emissions *

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Abstract

This paper examines the optimal choice of carbon emissions for two asymmetric regions, both contributing to rising atmospheric carbon stocks which damage the economy. Optimal decisions are modelled as a differential Stackelberg pollution game with closed loop strategies. The global average temperature is modelled as a mean reverting stochastic process driven by the stock of atmospheric carbon. A numerical solution of a coupled system of HJB equations is implemented. In a numerical example we examine the trade-offs for regions making strategic decisions about greenhouse gas emissions in the face of the uncertainty and compare the results to the case of a Social Planner. We demonstrate a classic tragedy of the commons which is made worse if regions experience asymmetric damages or if temperature volatility increases. We also consider the effect of asymmetric preferences over emissions reduction.
Keywords: regional climate change policies, differential game, uncertainty

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1 Introduction

Climate change caused by human activity represents a particularly intractable tragedy of the commons, which can only be addressed by cooperative actions of individual decision makers at both national and regional levels. The likely success of cooperative actions is hampered by the large incentives for free riding by decision makers who may delay making deep cuts in carbon emissions in hopes that others will do the "heavy lifting". Further complicating the problem are the enormous uncertainties inherent in predicting climate responses to the buildup in atmospheric carbon stocks and resulting impacts on human welfare, including the prospects for adaptation and mitigation. These huge uncertainties and the need for cooperative global action have been used by some as justification for delaying aggressive unilateral policy actions. And yet, many nations and sub-national jurisdictions have acted on their own to adopt policies to reduce carbon emissions even without national agreements or legislation in place. As a prominent example, since the Trump administration has reneged on the Paris Climate Accord, several states have vowed to go it alone and continue with strong climate policies. Other examples of jurisdictions taking unilateral carbon pricing initiatives are given in Table 1.

The observation that national or regional governments implement environmental regulations sooner or more aggressively than required by international agreements or national legislation has been studied by various researchers.[1] Local circumstances including voter preferences, local damages from emissions, and strategic considerations regarding the actions of other jurisdictions may play a role. A nation or region may be motivated to act ahead of others if it experiences relatively more severe local damages from emissions. Differences in environmental preferences may prompt some jurisdictions to take early action (Bednar-Friedl 2012). California and British Columbia (a province in Canada), both early adopters of carbon pricing, appear to have residents who are more environmentally aware,

[1] Urpelainen (2009) and Williams (2012) examine the puzzle at a sub-regional level.

Table 1: Unilateral CO_2 Pricing Initiatives

Jurisdiction	Policy	Implementation Date
Finland	carbon tax	1990
Sweden	carbon tax	1991
Estonia	carbon tax	2000
Alberta (CAN)	Specified Gas Emitters Regulations	2007
Switzerland	carbon tax	2008
British Columbia (CAN)	carbon tax	2008
Quebec(CAN)	carbon tax	2008
Ireland	carbon tax	2010
Japan	carbon tax	2012
California (US)	cap and trade	2013
Quebec(CAN)	cap and trade	2013
Portugal	carbon tax	2015
Alberta (CAN)	carbon tax	2017
Ontario (CAN)	cap and trade	2017

Source: Kossey et al. (2015).

implying these governments acted in accordance with the preferences of a large segment of their voters. A survey of stakeholders involved in the introduction of the B.C. carbon tax concluded that a number of factors were at work including a high priority given to environmental stewardship by B.C. residents and the fact that several other regional jurisdictions appeared to be poised in 2008 to take climate change more seriously (Clean Energy Canada 2015). Governments may choose environmental policies strategically to gain a competitive advantage or to shift emissions to other regions (Barcena-Ruiz 2006).

This paper examines the strategic interactions of decision makers responding to climate change, focusing on three central features of the problem: uncertainty, the incentive for free riding, and asymmetric characteristics of decision makers. We develop a model of a differential Stackelberg game involving two regions, each large emitters of green house gases who benefit from their individual emissions, but also face costs from the impact on global temperature of the cumulative emissions of both players. The modelling of the linkage between carbon emissions and global temperature is based on the assumptions of the well-known DICE model (Nordhaus & Sztorc 2013). To capture uncertainty, average global temperature is modelled as a stochastic differential equation. We allow for differing damages of climate change for each region as well as differing preferences for reducing green house gas emissions. We explore the impact of these features on the optimal choice of emissions for each player and contrast with the choices made by a Social Planner.

It is well known that for differential games with closed loop strategies, only special classes of models result in well-posed mathematical problems for which it is possible to characterize Nash equilibria.[2] These include linear-quadratic games where the feedback controls depend linearly on the state variable, as well as certain forms of stochastic differential games where the state evolves according to an Ito process. This paper analyzes a stochastic differential game which we solve using numerical techniques. We make no prior assumption about the existence of a Nash equilibrium. Our approach is to discretize time to approximate the dynamic game as a series of one-shot games (which occur at discrete points in time) and solve for a Stackelberg equilibrium for each of these one-shot games. We can check if the Stackelberg solution also represents a Nash equilibrium (See 4.2.2).

There is a significant prior literature which examines the tragedy of the commons caused by polluting emissions in a differential game setting. The relevant differential game literature is reviewed in Section 2, but we note here two papers most closely related to our paper in their focus on asymmetry of players' utilities. Both employ economic models in a deterministic setting. Zagonari (1998) analyzes cooperative and non-cooperative games when the two players (countries) differ in the utility derived from a consumption good, the disutility caused by the pollution stock, and their concern for future generations as reflected in their discount rate. For the non-cooperative game, Zagonari examines the conditions under which the steady state stock of pollution might be less than under the cooperative game. Interestingly, he finds equilibria for which the steady state pollution stock is lower than in the cooperative game. In particular, this result holds if the country with stronger environmental preferences (the "eco-country") has sufficiently large disutility from pollution and either a relatively strong concern for future generations or relatively small utility from consumption goods.

[2]Bressan (2011) provides an excellent overview of the mathematical theory of differential games.

Wirl (2011) also examines whether differences in environmental sentiments can mitigate the tragedy of the commons associated with a problem such as global warming. He characterizes a multi-player game with green and brown players. Green players are distinguished from brown players by a penalty term in their objective function which depends on the extent to which their emissions exceed the social optimum. Wirl finds the presence of green players introduces discontinuous strategies with some interesting features. In the examples chosen, the effect of green players on total emissions is modest, as their actions increase the free riding of brown players. Wirl notes the possibility of a type of green paradox in which the increasing numbers of green players causes increased emissions, because brown players increase their emissions and more than offset the impact of green players' decisions.

This paper examines some similar issues but using a more general model which includes uncertainty and closed loop strategies in a dynamic setting. The paper contributes to our understanding of the effect of increasing uncertainty in future temperatures and of asymmetry in damages and environmental preferences on emission choices and the evolution of the atmospheric carbon stock. We also contrast the non-cooperative outcome with the outcome assuming a central planner empowered to make choices which maximize the sum of the welfare of both players.

The paper also makes a contribution in terms of the numerical methodology for solving a Stackelberg game under uncertainty with path dependent variables. We describe the method used to determine the closed loop optimal Stackelberg solution (which always exists) and then show how to determine if a Nash equilibrium exists. Our numerical solution procedure involves use of a finite difference discretization of the system of HJB equations. We determine a timestep condition which preserves the monotonicity property of the exact solution. In contrast to much of the previous literature, the choice of damage function can be any desired function of state variables.

To preview our results, we highlight the crucial role of the damage function which specifies the harm from rising temperature, as has been noted by others (Weitzman 2012, Pindyck 2013). Very little reduction in carbon emissions occurs in the Stackelberg game or with the central planner using a conventional quadratic damage function. Exponentially increasing damages better reflects the catastrophic nature of damages anticipated if average global temperature should increase beyond 3 °C above preindustrial levels. We also find that temperature uncertainty plays a key role. With a larger temperature volatility, the tragedy of the commons is exacerbated. Asymmetric costs are also found to have an important effect on strategic interactions of players. Another main finding is the effect of differing preferences for environmental actions. If the preference for emissions reductions increases in one player, the beneficial effect on emissions is offset to some extent by an increase in the free riding of the other player. As noted, this has been explored previously in Wirl (2011) in a deterministic setting.

The remainder of the paper proceeds as follows. In Section 2, we provide a more detailed literature review. The formulation of the climate change decision problem is described in Section 3. Section 4 provides a detailed description of the dynamic programming solution. Section 5 describes the detailed modelling assumptions and parameter values. Numerical results are described in Section 6, while Section 7 provides concluding comments.

2 Literature

This paper contributes to the literature on differential games dealing with trans-boundary pollution problems as well as to the developing literature on accounting for uncertainty in optimal policies to address climate change.

Economic models of climate change have long been criticized for arbitrary assumptions regarding functional forms and key parameter values as well as unsatisfactory treatment of key uncertainties including the possibility of catastrophic events.[3] Of course, this is not surprising given the intractable nature of the climate change problem. Policies to address climate change have been extensively studied using the DICE (Dynamic Integrated Model of Climate and the Economy) model, a deterministic model developed in the 1990s, which has been revised and updated several times since then (Nordhaus 2013). Initially uncertainty was

[3] SeePindyck (2013) for a harsh critique.

addressed through sensitivities or Monte Carlo analysis, but there has since been a significant research effort to address uncertainty using more robust methodologies. We mention only a sample of that literature. Kelly & Kolstad (1999) and Leach (2007) embed a model of learning into the DICE model to examine active learning by a Social Planner about key climate change parameters. More recent papers which incorporate stochastic components into one or more state variables in the DICE model include Crost & Traeger (2014), Ackerman et al. (2013) and Traeger (2014). Lemoine & Traeger (2014) extend the work of Traeger (2014) by incorporating the possibility of sudden shifts in system dynamics once parameters cross certain thresholds. Policy makers learn about the thresholds by observing the evolution of the climate system over time. Hambel et al. (2017) present a stochastic equilibrium model for optimal carbon emissions with key state variables, including carbon concentration, temperature and GDP, modelled as stochastic differential equations. Chesney et al. (2017) examine optimal climate polices using a model in which global temperature is stochastic and assuming there is a known temperature threshold which will result in disastrous consequences if it is exceeded for a sustained period of time.

This previous work considers uncertainty in models with a single decision maker, abstracting from the strategic interactions of multiple decision makers which is a key feature of policy making for climate change. Differential game models have been used extensively to examine strategic interactions between players who benefit individually from polluting emissions but are also harmed by the cumulative emissions of all players. The literature addresses the strategic interactions of decision makers over time in deterministic and stochastic settings. Key assumptions, such as the information known to each player, determine whether the game can be described by a closed form mathematical solution.[4] For example, open loop strategies, which depend solely on time, result when players know only the initial state of the system. Nash and Stackelberg equilibria for open loop strategies are well understood. In contrast, when players can directly observe the state of the system at every instant in time, feedback strategies (also called closed-loop or Markovian strategies) which depend on

[4]See Bressan (2011) for a discussion of the challenges of finding appropriate mathematical models which result in closed form solutions.

the state of the system may be employed. The resulting value functions satisfy a system of highly non-linear HJB partial differential equations. From the theory of partial differential equations it is known that if the system is non-stochastic, it should be hyperbolic in order for it to be well posed, in that it admits a unique solution depending continuously on the initial data (Bressan & Shen 2004).

In games with feedback strategies only special classes of models are known to result in well-posed mathematical problems. These include zero-sum games, as well as linear-quadratic games. Linear-quadratic games have been used extensively in the economics literature to study pollution games, and some relevant papers, which admit closed form solutions, are detailed below. In this class of games, utility is a quadratic function of the state variable, while the state variable is linear in the control. Robust game models are also found with Nash feedback equilibria for stochastic differential games where the state evolves according to an Ito process such as

$$dx = f(t, x, u_1, u_2)dt + \sigma(t, x)dZ \tag{1}$$

where x represents the state variable, t is time, u_1 and u_2 represent the controls of players 1 and 2, f and σ are known functions, and dZ is the increment of a Wiener process. As noted by Bressan (2011), for this case the value functions can be found by solving a Cauchy problem for a system of parabolic equations. The Cauchy problem is well posed if the diffusion tensor σ has full rank. Additional discussion of the complexities of solving problems involving differential games can be found in Salo & Tahvonen (2001), Ludkovski & Sircar (2015), Harris et al. (2010), Cacace et al. (2013), Amarala (2015), and Ledvina & Sircar (2011).

Long (2010) and Dockner et al. (2000) provide surveys of the sizable literature addressing strategic interactions in the optimal control of pollution or natural resource exploitation using games, much of it in a deterministic setting. This literature is particularly interested in whether players are better off with cooperative behaviour and also how the steady state level of pollution compares under cooperative versus non-cooperative games. Examples of dynamic differential pollution games in a non-stochastic setting include Dockner & Long

(1993), Zagonari (1998), Wirl (2011), and List & Mason (2001). Under certain conditions, analytical closed-form solutions are found for linear and non-linear closed-loop strategies.

A few papers derive analytical solutions to differential pollution models in stochastic settings. Xepapadeas (1998) models global warming policy as a stochastic dynamic game in which damage is linear in the pollution stock and uncertainty in damage is described by geometric Brownian motion. An analytical solution is derived assuming exclusively linear strategies.

Wirl (2008) considers a dynamic pollution game with n symmetric players and differentiates between the cases of reversible and irreversible emissions. He characterizes the accumulation of pollution as an Ito process. The case of irreversible emissions requires a non-negativity constraint such that an explicit analytical solution is no longer possible. However, Wirl is able to analyze Nash equilibria in continuous Markov strategies using value matching and smooth pasting conditions between the value functions when optimal emissions are positive or zero. He characterizes feasible stopping thresholds for the pollution stock at which emissions will cease. He notes that multiple Nash equilibria cannot be ruled out. As expected, an increase in uncertainty reduces optimal emissions and this effect is more pronounced when emissions are irreversible. In the non-cooperative game, the stock of pollution grows in an unlimited fashion as the number of players increases. This contrasts with the cooperative game in which optimal emissions and pollution levels decline as the number of players increases.

Finally, Nkuiya (2015) analyzes a pollution control game with n symmetric players when there is a risk of sudden jump in damages or a catastrophe at an unknown future date. The switch from the low damage to high damage state is modelled as a Poisson process. This allows the stochastic differential game to be transformed into a deterministic one which admits an analytic solution. He distinguishes two cases: one in which damages may jump to a level such that all economic activity ceases (doomsday scenario) and the other in which damages jump to a higher level, but economic activity continues (non-doomsday scenario). For the non-doomsday scenario, linear strategies, and an exogenous hazard rate, an increase in the hazard rate causes players to reduce emissions. In the doomsday case under linear strategies, an increase in the hazard rate adds to the effective discount rate and causes players to increase emissions. These results do not necessarily hold if the probability of jumping to the bad state depends endogenously on players' emission decisions. The results also do not necessarily hold if players commit to non-linear strategies, even when the jump probability is exogenous.

There is a developing literature on the numerical solution of dynamic games in the context of non-renewable resource markets. Some earlier papers developed models where two or more players extract from a common stock of resource. Examples include van der Ploeg (1987) and Dockner et al. (1996). Salo & Tahvonen (2001) were among the first to explore oligopolistic natural resource markets in a differential Cournot game using closed loop strategies. Prior to that, the focus had been on open-loop strategies, because of their tractability. Of course, in general, open loop strategies are not optimal compared to closed loop strategies. This paper developed an analytical solution of affine-quadratic specifications and demonstrated a numerical method for other functional forms using a Markov chain approximation.

Harris et al. (2010) study the extraction of an exhaustible resource as an N-player continuous time Cournot game when players have heterogeneous costs. They note that existence, uniqueness and regularity of value functions are not well understood and that numerical solutions represent a major challenge. They present an asymptotic approximation for a low exhaustible case (i.e. small cost of the alternate technology) and a numerical solution given a restriction on the cost of the alternate technology. Ludkovski & Sircar (2012) extend the work of Harris et al. (2010) by adding exploration to the model. Ludkovski & Yang (2015) includes both exploration and stochastic demand.

3 Problem Formulation

This section provides an overview of the climate change decision model. Details of functional forms and parameter values are provided in Section 5. A summary of variable names is given in Table 2. We model the optimal timing and stringency of environmental regulations (in

Table 2: List of Model Variables

Variable	Description
$E_p(t)$	Emissions in region p
e_1^+, e_1^-	optimal controls for players 1 and 2
$S(t)$	Stock of pollution at time t
$\bar{S}$	preindustrial level of carbon
$\rho(t)$	Rate of natural removal of the pollution stock
$X(t)$	Average global temperature
$\bar{X}$	long run equilibrium level of carbon
$B_p(t)$	Benefits from emissions
$C_p(t)$	Damages from pollution
$g_p(t)$	Emissions reduction in region p relative to a target
θ_p	Willingness to pay in region p for emissions reduction from a target
$R_p(t)$	Green reward benefits from emissions reductions
π_p	Flow of net benefits to region p
r	Discount rate
ρ	removal rate of atmospheric carbon

terms of the reduction of greenhouse gas emissions) as a stochastic optimal control problem. We consider two cases: a Stackelberg game and a Social Planner. The players in the Stackelberg game are two regions, each contributing to the atmospheric stock of greenhouse gases - which, for simplicity, we will refer to as the carbon stock. These regions may be thought of as single nations or groups of nations acting together, but each is a major contributor to the global carbon stock. Each region seeks to maximize discounted expected utility by making emission choices taking into account the optimal actions of the other region. The Social Planner chooses emission levels in each region so as to maximize the expected sum of utilities from both regions.

Regions emit carbon in order to generate income. For simplicity we assume that there is a one to one relation between emissions and regional income. The two regions are indexed by $p = 1, 2$ and E_p refers to carbon emissions from region p. The stock of atmospheric carbon, S, is augmented by the emissions of each player and is reduced by a natural cycle whereby carbon is removed from the atmosphere and absorbed into other carbon sinks. The removal of carbon from the atmosphere can be described by decay function, $\rho(X,S,t)$, which in theory may depend on the average surface temperature, X, the stock of carbon, S, and time, t. $\rho(X,S,t)$ is referred to as the removal rate. For simplicity, as described in Section 5, we will later drop the dependence on X and S, assuming that ρ is a function only of time. However, our solution technique can easily accommodate more general functional forms for ρ. The evolution of the carbon stock over time is described by the deterministic differential equation:

$$\frac{dS(t)}{dt} = E_1 + E_2 + (\bar{S} - S(t))\rho(X,S,t); \; S(0) = S_0 \quad S \in [s_{min}, \; s_{max}] \, . \tag{2}$$

$\bar{S}$ is the pre-industrial equilibrium level of atmospheric carbon.

The mean global increase in temperature above the pre-industrial level, denoted by X, is described by an Ornstein Uhlenbeck process:

$$dX = \eta(t)\Big[\bar{X}(S,t) - X(t)\Big]dt + \sigma dZ. \tag{3}$$

where $\eta(t)$ represents the speed of mean reversion and is a deterministic function of time. $\bar{X}$ represents the long run mean of global average temperature which depends on the stock of carbon and time. The detailed specification of these functions and parameters is given in Section 5. dZ is the increment of a standard Weiner process, intended to capture the volatility in the earth's temperature due to random effects.

The net benefits from carbon emissions are represented as a general function π_p:

$$\pi_p = \pi_p(E_1, E_2, X, S, t) \tag{4}$$

More specifically, π is composed of the benefits from emissions, $B(E_p,t)$, the damages from increasing temperature, $C_p(X,t)$, and a green reward that results from reducing emissions relative to a given target or baseline level, $R_p(g_p(t))$:

$$\pi_p = B_p(E_p,t) - C_p(X,t) + R_p(g_p(t)) \quad p = 1,2; \tag{5}$$

where $g_p(t)$ refers to emissions reduction. The detailed specification of benefits, damages, and the green reward is left to Section 5

It is assumed that the control (choice of emissions) is adjusted at discrete decision times denoted by:

$$\mathcal{T} = \{t_0 = 0 < t_1 < ...t_m... < t_M = T\}. \tag{6}$$

Let t_m^- and t_m^+ denote instants just before and after t_m, with $t_m^- = t_m - \epsilon$ and $t_m^+ = t_m + \epsilon$, where $\epsilon \to 0^+$, where T is the time horizon of interest.

$e_1^+(X, S, t_m)$ and $e_2^+(X, S, t_m)$ denote the controls implemented by the players 1 and 2 respectively, which are contained within the set of admissible controls: $e_1^+ \in Z_1$ and $e_2^+ \in Z_2$. In general $e_p^+ = e_p^+(E_1, E_2, X, S, t_m)$, however in our examples we assume no costs to change emissions levels so there is no dependence of the controls on the states (E_1, E_2). This assumption could be changed, but for ease of exposition we assume this simpler form of the control functions.

We can specify a control set which contains the optimal controls for all t_m.

$$K = \left\{(e_1^+, e_2^+)_{t_0=0},\ (e_1^+, e_2^+)_{t_1=1},\ \dots, (e_1^+, e_2^+)_{t_M=T}\right\}. \tag{7}$$

In this paper we will consider three possibilities for selection of the controls (e_1^+, e_2^+) at $t \in \mathcal{T}$: Stackelberg, Nash, and Social Planner. We delay the precise specification of how these controls are determined until Section 4.2.

Regardless of the control strategy, the value function for player p, $V_p(e_1, e_2, x, s, t)$ is defined as:

$$\begin{aligned} V_p(e_1, e_2, x, s, t) \quad &= \quad \mathcal{E}_K \left[\int_{t'=t}^{T} e^{-rt'} \pi_p(E_1(t'), E_2(t'), X(t'), S(t'))\ dt' + e^{-r(T-t)} V(0, 0, x, s, T) \right. \\ &\qquad \left. \Big| E_1(t) = e_1, E_2(t) = e_2, X(t) = x,\ S(t) = s \right], \end{aligned} \tag{8}$$

where $\mathcal{E}_K[\cdot]$ is the expectation under control set K. Note that lower case letters e_1, e_2, x, s have been used to denote realizations of the state variables E_1, E_2, X, S. The value in the final time period, T, is assumed to be the present value of a perpetual stream of expected net benefits at given carbon stock, S, and temperature levels, X, with emissions set to zero. This is reflected in the term $V(0,0,x,s,T)$ and is described in Section 4.1 as a boundary condition.

4 Dynamic Programming Solution

Using dynamic programming, we solve the problem represented by Equation (8) backwards in time, breaking the solution phases up into two components for $t \in (t_m^-, t_m^+)$ and (t_m^+, t_{m+1}^-). In the interval (t_m^-, t_m^+), we determine the optimal controls, while in the interval (t_m^+, t_{m+1}^-), we solve a system of PDEs. As a visual aid, Equation (9) shows the noted time intervals going forward in time,

$$t_m^- \rightarrow t_m^+ \rightarrow t_{m+1}^- \rightarrow t_{m+1}^+ \tag{9}$$

4.1 Advancing the solution from $t_{m+1}^- \rightarrow t_m^+$

The solution proceeds going backward in time from $t_{m+1}^- \rightarrow t_m^+$. Define the differential operator, $\mathcal{L}$ for player p, in Equation (10). The arguments in the V_p function have been suppressed when there is no ambiguity.

$$\mathcal{L}V_p \equiv \frac{(\sigma)^2}{2}\frac{\partial^2 V_p}{\partial x^2} + \eta(\bar{X} - x)\frac{\partial V_p}{\partial x} + [(e_1 + e_2) + \rho(\bar{S} - s)]\frac{\partial V_p}{\partial s} - rV_p; \quad p = 1, 2, \tag{10}$$

where r is the discount rate. Then using standard techniques (Dixit & Pindyck 1994), the equation satisfied by the value function, V_p is expressed as:

$$\frac{\partial V_p}{\partial t} + \pi_p(e_1, e_2, x, s, t) + \mathcal{L}V_p = 0, \quad p = 1, 2; \tag{11}$$

The computational domain of Equation (11) is defined as $(e_1, e_2, x, s, t) \in \Omega$, where $\Omega \equiv Z_1 \times Z_2 \times [x_{min}, x_{\max}] \times [\bar{S}, s_{\max}] \times [0, T]$. T, $\bar{S}$, $s_{\max}$, Z_1, Z_2, x_{min}, and $x_{\max}$ are specified based on reasonable values for the climate change problem, and are specified in Section 5.

Boundary conditions for the PDEs are specified below.

- As $x \to x_{max}$, it is assumed that $\frac{\partial^2 V_p}{\partial x^2} \to 0$. This boundary condition is commonly used in the literature and implies that the impact of volatility at very high temperature levels is unimportant relative to the size of the damages. Assuming that $x_{max} > \bar{X}$, Equation (11) has outgoing characteristics with $V_{xx} = 0$ at $x = x_{max}$ and hence no other boundary conditions are required.

- As $x \to x_{min}$, where x_{min} is below the pre-industrial temperature, the effect of volatility is small compared to the drift term. Hence we set $\sigma = 0$ at $x = x_{min}$. Assuming $x_{min} < \bar{X}$ then Equation (11) has outgoing characteristics at $x = x_{min}$ and no other boundary conditions are required. Note that we will show that $\pi_p \geq 0$ at $x = x_{min}$.

- As $s \to s_{max}$, it is assumed that emissions do not increase s beyond the limit of s_{max}. s_{max} is set to be a large enough value so that there is no impact on utility or optimal emission choices for s levels of interest. We have verified this in our computational experiments. This amounts to dropping the term $\frac{\partial V_P}{\partial S}(e_1 + e_2)$ from Equation (10). This can be justified by noting that if $s_{max} \gg \bar{S}$ then $\rho(\bar{S} - S) >> (e_1 + e_2)$ for reasonable values of e_1 and e_2.

- As $s \to \bar{S}$, no extra boundary condition is needed as we assume $e_1, e_2 \geq 0$ hence the Equation has outgoing characteristics at $s = \bar{S}$.

- At $t = T$, it is assumed that V_p is equal to the present value of the infinite stream of benefits associated with a given temperature when emissions are set to zero. Essentially, it is assumed that we receive the costs associated with that temperature in perpetuity and T is large enough that we assume the world has decarbonized.

More details of the numerical solution of the system of PDEs are provided in Appendix A

4.2 Advancing the solution from $t_m^+ \to t_m^-$

Going backward in time, the optimal control, is determined between $t_m^+ \to t_m^-$. We consider three possibilities for selection of the controls (e_1^+, e_2^+) at $t \in \mathcal{T}$: Stackelberg, Nash, and Social Planner. We include the Nash case for reference only. To avoid notational clutter in the following, we will fix (s, x, t_m), so that, without ambiguity, we will write (e_1^+, e_2^+) which will be understood to mean $(e_1^+(s, x, t_m), e_2^+(s, x, t_m))$. Note that (e_1^+, e_2^+) are independent of (e_1, e_2) at t_m^-, since we have ignored switching costs. Of course, there are indirect switching costs since the flow of benefits depends on the state variables (e_1, e_2).

4.2.1 Stackelberg Game

In the case of a Stackelberg game, suppose that, in forward time, player 1 goes first, and then player 2. Conceptually, we can then think of the time intervals (in forward time) as $(t_m^-, t_m]$, (t_m, t_m^+). Player 1 chooses control e_1^+ in $(t_m^-, t_m]$, then player 2 chooses control e_2^+ in (t_m, t_m^+).

We suppose at t_m^+, we have the value functions $V_1(e_1, e_2, s, x, t_m^+)$ and $V_2(e_1, e_2, s, x, t_m^+)$.

Definition 1 (Response set of player 2)**.** *The best response set of player 2, $R_2(\omega_1, s, x, t_m)$ is defined to be the best response of player 2 to a control ω_1 of player 1.*

$$R_2(\omega_1, s, x, t_m) = \operatorname*{argmax}_{\omega_2' \in Z_2} V_2(\omega_1, \omega_2', s, x, t_m^+) \ . \tag{12}$$

Note that $R_2(\omega_1, s, x, t_m)$ may be a set with more than one element (i.e. there may be ties). Similarly, we define the best response set of player 1.

Definition 2 (Response set of player 1)**.** *The best response set of player 1, $R_1(\omega_2, s, x, t_m)$ is defined to be the best response of player 1 to a control ω_2 of player 2.*

$$R_1(\omega_2, s, x, t_m) = \operatorname*{argmax}_{\omega_1' \in Z_1} V_1(\omega_1', \omega_2, s, x, t_m^+) \tag{13}$$

Again, to avoid notational clutter, we will fix (s, x, t_m) so that we can write without ambiguity $R_1(\omega_2) = R_1(\omega_2, s, x, t_m)$ and $R_2(\omega_1) = R_2(\omega_1, s, x, t_m)$.

Definition 3 (Stackelberg Game: Player 1 first)**.** *The optimal controls (e_1^+, e_2^+) assuming player 1 goes first are given by*

$$\begin{aligned} e_1^+ &= \operatorname*{argmax}_{\omega_1' \in Z_1} V_1(\omega_1', R_2(\omega_1'), s, x, t_m^+) \ , \\ e_2^+ &= R_2(e_1^+) \ . \end{aligned} \tag{14}$$

Dynamic Programming Algorithm: Player 1 first

Input: $V_1(e_1, e_2, s, x, t_m^+)$, $V_2(e_1, e_2, s, x, t_m^+)$.

Step 1: Compute the best response set for player 2 assuming player 1 chooses control ω_1 first, $\forall \omega_1 \in Z_1$, using Equation (12), giving $R_2(\omega_1)$.

Step 2: Determine an optimal pair (e_1^+, e_2^+) using Equation (14).

Determine solution at t_m^-

$$\begin{aligned} V_1(e_1, e_2, s, x, t_m^-) &= V_1(e_1^+(s, x, t_m), e_2^+(s, x, t_m), s, x, t_m^+) \ ; \ \forall e_1 \in Z_1, \forall e_2 \in Z_2 \ \ , \\ V_2(e_1, e_2, s, x, t_m^-) &= V_2(e_1^+(s, x, t_m), e_2^+(s, x, t_m), s, x, t_m^+) \ ; \ \forall e_1 \in Z_1, \forall e_2 \in Z_2 \ . \end{aligned} \tag{15}$$

Output: $V_1(e_1, e_2, s, x, t_m^-), V_2(e_1, e_2, s, x, t_m^-)$

Remark 1. *Note that from Equation (15) we can see that the optimal state at t_m^+ has no dependence on the state variables (e_1, e_2) at t_m^-.*

Remark 2 (Tie-breaking)**.** *In the event that e_1^+ is not unique, we break the tie by choosing the control which maximizes $R_2(e_1^+)$. In the event that e_2^+ is not unique, we break ties arbitrarily.*

4.2.2 Nash Equilibrium

We again fix (s, x, t_m), so that we understand that $e_p^+ = e_p^+(s, x, t_m)$, $R_p(\omega) = R_p(\omega, s, x, t_m)$.

Definition 4 (Nash Equilibrium)**.** *Given the best response sets $R_2(\omega_1)$, $R_1(\omega_2)$ defined in Equations (12)-(13), then the pair (e_1^+, e_2^+) is a Nash equilibrium point if and only if*

$$\begin{aligned} e_1^+ &= \operatorname*{argmax}_{\omega_1' \in Z_1} V_1(\omega_1', R_2(\omega_1'), s, x, t_m^+) \ , \\ e_2^+ &= \operatorname*{argmax}_{\omega_2' \in Z_2} V_2(R_1(\omega_2'), \omega_2', s, x, t_m^+) \ , \ , \end{aligned} \tag{16}$$

with the additional constraint

$$\begin{aligned} e_1^+ &= R_1(e_2^+), \\ e_2^+ &= R_2(e_1^+) \ . \end{aligned} \tag{17}$$

From Definition 3 of a Stackelberg game, if player 1 goes first, we have the optimal pair $(\hat{e}_1^+, \hat{e}_2^+)$

$$\begin{aligned} \hat{e}_1^+ &= \operatorname*{argmax}_{\omega_1' \in Z_1} V_1(\omega_1', R_2(\omega_1'), s, x, t_m^+) \ , \\ \hat{e}_2^+ &= R_2(\hat{e}_1^+) \ . \end{aligned} \tag{18}$$

Similarly, we have the pair $(\bar{e}_1^+, \bar{e}_2^+)$ if player 2 goes first

$$\begin{aligned} \bar{e}_2^+ &= \operatorname*{argmax}_{\omega_2' \in Z_2} V_2(R_1(\omega_2'), \omega_2', s, x, t_m^+) \ , \\ \bar{e}_1^+ &= R_1(\bar{e}_2^+) \ . \end{aligned} \tag{19}$$

Define

$$\mathcal{S}_{12} = \left\{ (\omega_1, \omega_2) \middle| \omega_1 = \hat{e}_1^+, \omega_2 = \hat{e}_2^+ \right\} \quad ; \quad \mathcal{S}_{21} = \left\{ (\omega_1, \omega_2) \middle| \omega_1 = \bar{e}_1^+, \omega_2 = \bar{e}_2^+ \right\} \ . \tag{20}$$

Note that the $(\hat{e}_1^+, \hat{e}_2^+)$ and $(\bar{e}_1^+, \bar{e}_2^+)$ need not be unique. Suppose $\exists (e_1^+, e_2^+) \in \mathcal{S}_{12} \cap \mathcal{S}_{21} \neq \emptyset$, then for this pair we have $(e_1^+, e_2^+) = (\hat{e}_1^+, \hat{e}_2^+) = (\bar{e}_1^+, \bar{e}_2^+)$ and we replace the $\hat{e}_p^+$ by e_p^+ and $\bar{e_p}^+$ by $e_p{}^+$ in Equations (18) - (19) giving

$$\begin{aligned} e_1^+ &= \operatorname*{argmax}_{\omega_1' \in Z_1} V_1(\omega_1', R_2(\omega_1'), s, x, t_m^+) \ , \\ e_2^+ &= \operatorname*{argmax}_{\omega_2' \in Z_2} V_2(R_1(\omega_2'), \omega_2', s, x, t_m^+) \ , \\ & e_1^+ = R_1(e_2^+) \ ; \ e_2^+ = R_2(e_1^+) \ , \end{aligned} \tag{21}$$

which is identical to Definition 4. We can summarize this result in the following

Proposition 1 (Existence of a Nash Equilibrium). *A Nash equilibrium exists at a point* (s, x, t_m) *iff* $\mathcal{S}_{12} \cap \mathcal{S}_{21} \neq \emptyset$.

Remark 3 (Checking for a Nash equilibrium). *Proposition 1 gives a straightforward algorithm for determining if a Nash equilibrium exists (and determining the Nash controls). We simply compute the set of optimal Stackelberg controls* $\mathcal{S}_{12}$ *assuming player 1 goes first. We then compute the set of optimal Stackelberg controls* $\mathcal{S}_{21}$ *assuming player 2 goes first. If* $\mathcal{S}_{12} \cap \mathcal{S}_{21} \neq \emptyset$, *then a Nash equilibrium exists at this point* (s, x, t_m).

We will not consider Nash equilibria further in this work, since in our numerical tests, we hardly ever observe that a Nash control exits.

4.2.3 Social Planner

For the Social Planner case, we have that an optimal pair (e_1^+, e_2^+) is given by

$$(e_1^+, e_2^+) = \operatorname*{argmax}_{\substack{\omega_1 \in Z_1 \\ \omega_2 \in Z_2}} V_1(\omega_1, \omega_2, s, x, t_m^+) + V_2(\omega_1, \omega_2, s, x, t_m^+) \ . \tag{22}$$

and as a result

$$V_1(e_1, e_2, s, x, t_m^-) = V_1(e_1^+, e_2^+, s, x, t_m^+) \quad ; \quad V_2(e_1, e_2, s, x, t_m^-) = V_2(e_1^+, e_2^+, s, x, t_m^+) \ . \tag{23}$$

Table 3: Base Case Parameter Values

Parameter	Description	Equation Reference	Assigned Value
$\bar{S}$	Pre-industrial atmospheric carbon stock	(2)	588 GT carbon
s_{min}	Minimum carbon stock	(2)	588 GT carbon
s_{max}	Maximum carbon stock	(2)	3000 GT carbon
$\bar{\rho}$, ρ_0, ρ^*	Parameters for carbon removal Equation	(24)	0.0003, 0.01, 0.01
ϕ_1, ϕ_2, ϕ_3	Parameters of temperature Equation	(24)	0.02, 1.1817, 0.088
ϕ_4	Forcings at CO2 doubling	(25)	3.681
$F_{EX}(0)$ $F_{EX}(100)$	Parameters from forcing Equation	(25)	0.5 1
α_1, α_2	Ratio of the deep ocean to surface temp, $\alpha(t) = \alpha_1 + \alpha_2 \times t$, t is time in years	(24)	0.08, 0.0021
σ	Temperature volatility	(24)	0.1
x_{min}, x_{max}	Upper and lower limits on average temperature, °C	(24)	-3, 20
a_1, a_2	Parameter in benefit function, player p	(27)	10
Z_1, Z_2	Admissible controls	(7)	0, 3, 7, 10
$\bar{E}$	Baseline emissions	(30)	10
b_1,b_2	Cost scaling parameter, players 1 & 2 respectively	(28)	15, 15
κ_1	Linear parameter in cost function for both players	(28)	0.05
κ_2	Exponent in cost function for both players	(28)	2 or 3
κ_3	Term in exponential cost function for both players	(29)	1
θ_P	WTP for emissions reduction by player p	(5)	0 or 5
T	terminal time		150 years
r	risk free rate	(10)	0.01

Ties are broken by minimizing $|V_1(e_1^+, e_2^+, s, x, t_m^+) - V_2(e_1^+, e_2^+, s, x, t_m^+)|$. In other words, the Social Planner picks the emissions choices which give the most equal distribution of welfare across the two players.

5 Detailed model specification and parameter values

This section describes the functional forms and parameter values used in the numerical application. Assumed parameter values are summarized in Table 3.

5.1 Carbon stock details

The evolution of the carbon stock is described in Equation (2). In Integrated Assessment Models, there is typically a detailed specification of the exchange of carbon emissions between the various carbon reservoirs: the atmosphere, the terrestrial biosphere and different ocean layers (Nordhaus 2013, Lemoine & Traeger 2014, Traeger 2014, Golosov et al. 2014). In Equation 2 the removal function is given as $\rho(X, S, T)$. In our numerical application, we use a simplified specification, based on Traeger (2014), to avoid the creation of additional path dependent variables which increases computational complexity. We denote the rate at which carbon is removed from the atmosphere by $\rho(t)$ and assume it is a deterministic function of time which approximates the removal rates in the DICE 2016 model.

$$\rho(t) = \bar{\rho} + (\rho_0 - \bar{\rho})e^{-\rho^* t}$$

ρ_0 is the initial removal rate per year of atmospheric carbon, $\bar{\rho}$ is a long run equilibrium rate of removal, and ρ^* is the rate of change in the removal rate. Specific parameter assumptions for this Equation are given in Table 3. The resulting removal rate starts at 0.01 per year and falls to 0.0003 per year within 100 years.

The pre-industrial equilibrium level of carbon, $\bar{S}$ in Equation (2), is assumed to be 588 gigatonnes (GT) based on estimates used in the DICE (2016)[5] model for the year 1750. The allowable range of carbon stock is given by $s_{min} = 588$ and $s_{max} = 10000$. s_{max} is set well above the 6000 GT carbon in Nordhaus (2013) and will not be a binding constraint in the numerical examples.[6] A 2014 estimate of the atmospheric carbon level is 840 GT.[7]

[5] The 2013 version of the DICE model is described in Nordhaus & Sztorc (2013). GAMS and Excel versions for the updated 2016 version are available from William Nordhaus's website: http://www.econ.yale.edu/ nordhaus/homepage/.

[6] Golosov et al. (2014) chose a maximum atmospheric carbon stock of 3000 GT which is intended to reflect the carbon stock that results if most of the predicted stocks of fossil fuels are burned in "a fairly short period of time" (page 67).

[7] According to the Global Carbon Project, 2014 global atmospheric CO2 concentration was 397.15 ± 0.10 ppm on average over 2014. At 2.21 GT carbon per 1 ppm CO2, this amounts to 840 GT carbon.(www.globalcarbonproject.org)

5.2 Temperature details

Equation (3) specifies the stochastic differential Equation which describes temperature X and includes the parameters $\eta(t)$ and $\bar{X}(t)$. To relate Equation (3) to common forms used in the climate change literature, we rewrite it in the following format:

$$dX = \phi_1 \Big[F(S,t) - \phi_2 X(t) - \phi_3 [1 - \alpha(t)] X(t) \Big] dt + \sigma dZ \tag{24}$$

where ϕ_1, ϕ_2, ϕ_3 and σ are constant parameters.[8] The drift term in Equation (24) is a simplified version of temperature models typical in Integrated Assessment Models, based on Lemoine & Traeger (2014). $\alpha(t)$ represents the ratio of the deep ocean temperature to the mean surface temperature and, for simplicity, is specified as a deterministic function of time.[9] Equation (24) is equivalent to Equation (3) with $\eta(t) \equiv \phi_1 \Big(\phi_2 + \phi_3 (1 - \alpha(t) \Big)$ and the long run mean $\bar{X}(t) \equiv \frac{F(S,t)}{(\phi_2 + \phi_3(1-\alpha(t))}$.

$F(S,t)$ refers to radiative forcing, and it measures additional energy trapped at the earth's surface due to the accumulation of carbon in the atmosphere compared to preindustrial levels plus other greenhouse gases.

$$F(S,t) = \phi_4 \left(\frac{ln(S(t)/\bar{S})}{ln(2)} \right) + F_{EX}(t) \tag{25}$$

ϕ_4 indicates the forcing from doubling atmospheric carbon.[10] $F_{EX}(t)$ is forcing from causes other than carbon and is modelled as an exogenous function of time as specified in Lemoine & Traeger (2014) as follows:

$$F_{EX}(t) = F_{EX}(0) + 0.01\big(F_{EX}(100) - F_{EX}(0)\big) \min\{t, 100\} \tag{26}$$

The values for the parameters in Equation (24) are taken from the DICE (2016) model. Note that $\phi_1 = 0.02$ which is the value reported in Dice (2016) divided by five to convert

[8] ϕ_1, ϕ_2, ϕ_3 are denoted as ξ_1, ξ_2, and ξ_3 in Nordhaus (2013).

[9] We are able to get a good match to the DICE2016 results using a simple linear function of time.

[10] ϕ_4 translates to Nordhaus's η (Nordhaus & Sztorc 2013).

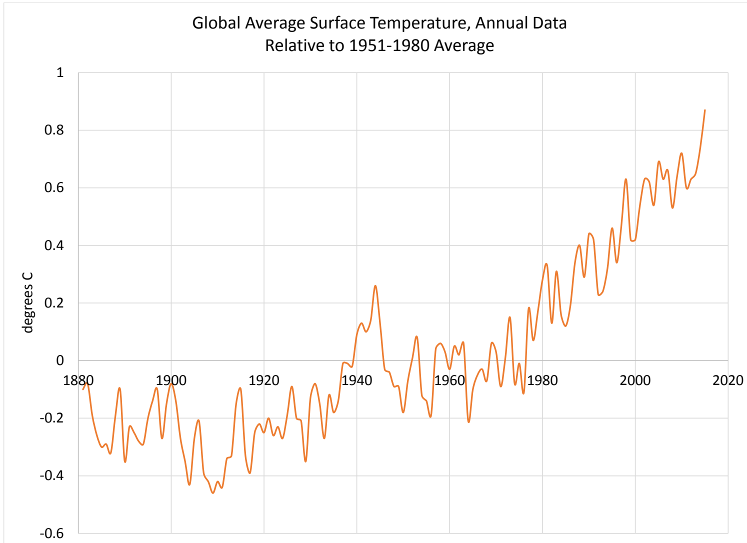


Figure 1: Global land-ocean temperature index, degrees C, annual averages since 1880 relative to the 1951-1980 average

to an annual basis from the five year time steps used in the DICE (2016) model. $F_{EX}(0)$ and $F_{EX}(100)$ (Equation (25)) are also from the DICE (2016) model. The ratio of the deep ocean temperature to surface temperature, $\alpha(t)$, is modelled as a linear function of time. This function approximates the average values from the DICE (2016) base and optimal tax cases.

Figure 1 shows the changes in global surface temperature relative to 1951 to 1980 averages.[11] Based on this data the volatility parameter was estimated using maximum likelihood to be approximately $\sigma = 0.1$. For the numerical solution we choose $x_{min} = -3$ and $x_{max} = 20$.

5.3 Benefits, Damages and the Green Reward

The term π_p in Equation (5) comprises benefits and damages from emissions as well as the green reward. This section describes these components.

[11]The data is from NASA's Goddard Institute for Space Studies and is available on NASA's web site Global Climate Change: *https://climate.nasa.gov/vital-signs/global-temperature/*.

5.3.1 Benefits

As is common in the pollution game literature, the benefits of emissions are quadratic according the following utility function:

$$B_p(E_p) = a_p E_p(t) - E_p^2(t)/2, \; p = 1, 2 \tag{27}$$

a_p is a constant parameter which may be different for different players. As in List & Mason (2001), $E_p \in [0, a_p]$ so that the marginal benefit from emissions is always positive. In the numerical example, there are four possible emissions levels for each player $E_p \in \{0, 3, 7, 10\}$ in GT of carbon and we set $a_1 = a_2 = 10$.

5.3.2 Damages

Assumptions regarding damages functions from increasing temperatures are speculative, and this is a highly criticized element of climate change models. The DICE model (and others) specify damages as a multiplicative quadratic function of temperature implying that damages never exceed 100% of output. For example, if carbon emissions cease, then no damages occur regardless of the mean temperature. This is clearly unrealistic, and ignores possible catastrophic effects. Damage function calibrations are generally based on estimates for the zero to 3 °C range above preindustrial temperatures. In this paper we adopt an additive damage function and explore results with quadratic, cubic and exponential forms. For quadratic or cubic forms, the cost of damages to player p from increasing temperature is given as:

$$C_p(X, t) = b_p(\kappa_1 X(t)^{\kappa_2}) \quad p = 1, 2. \tag{28}$$

where b_p, κ_1 and κ_2 are constants. The exponential damage function is give by:

$$C_p(t) = b_p(\kappa_1 e^{\kappa_3 X(t)}) \quad p = 1, 2, \tag{29}$$

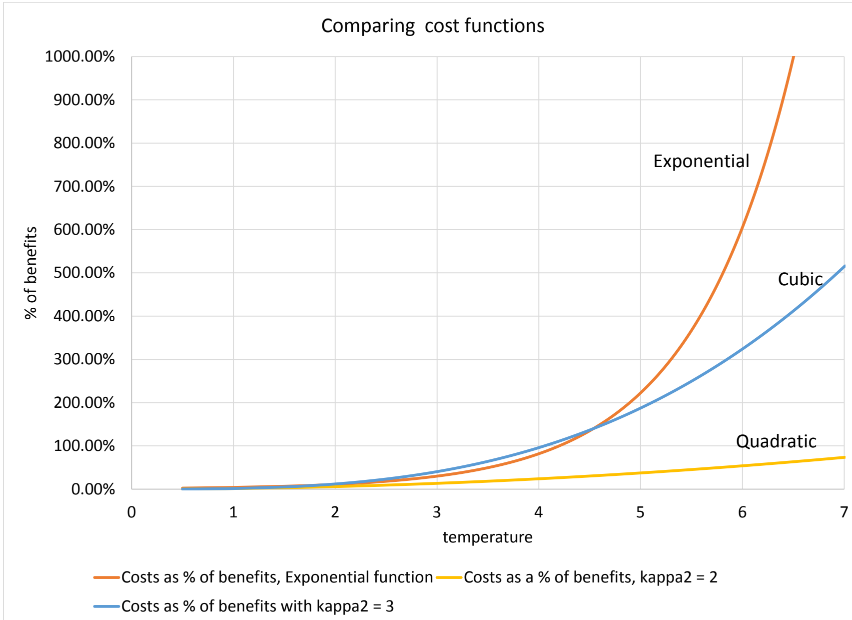


Figure 2: Comparing costs of increased temperatures as a % of benefits for different cost functions

where κ_3 is a constant. The parameter b_p allows the specification of different costs for each of the players, $p = 1, 2$.

We choose the parameters in the damage functions (Equation (28) and (29)) so that damages represent a reasonable portion of benefits at current temperatures levels (i.e. at 0.86 degrees C over preindustrial levels). Base case values for b_1, b_2, κ_1, κ_2 and κ_3 imply damages of about 1% of benefits at current temperature levels. Figure 2 compares the three cost functions as a percentage of benefits. The comparison is for the exponential function, Equation (29), compared with the power function, Equation (28), with the exponent set to 2 or 3. We observe that the three cost functions are virtually indistinguishable up to 3 °C above preindustrial levels. After 3 °C the cost functions diverge dramatically. We choose the exponential cost function as our base case as it implies that for temperature increases above 3 °C, damages from climate change would be disastrous, which seems a reasonable supposition.

5.3.3 Green Reward

We define emissions reduction, $g_p(t)$, relative to a baseline level of emissions level, $\bar{E}$, for each region.

$$g_p(t) = \bar{E}_p - E_p(t), \quad p = 1, 2 \tag{30}$$

Citizens of each region are assumed to value emissions reduction as contributing to the public good. We denote the degree of environmental awareness in a region by θ_p which represents a willingness to pay for emissions reduction because of a desire to be good environmental citizens, distinct from the expressions for the benefits and costs of emissions as defined in Equations (27) and (28) or (29).

The benefit from emissions reduction, called the green reward, R_p, depends on environmental awareness as well as emissions reduction in both regions:

$$R_p(t) = \theta_p g_p(t), \quad p = 1, 2;\ j \in [L, H]. \tag{31}$$

In our base case, $\theta_p = 0$ for both players initially. We then explore differential green preferences by setting $\theta_p = 2$ for one or the other player. In other work we explore the possibility that environmental preferences may evolve randomly over time and may depend on environmental actions taken in the other region.

6 Numerical results

6.1 Base case: identical players

We begin with consideration of the case in which both players have identical costs and benefits, the willingness to pay for emissions reduction is zero, and the damage function is exponential (Equation (29)). Figure 3 contrasts total discounted expected utility versus temperature at $S = 800$ for the game compared to the Social Planner. The figures show $V_p(e_1, e_2, x, s, t)$ for $p = 1, 2$ in the game, the sum of players 1 and 2 utilities in the game which we denote $V_{1+2}(e_1, e_2, x, s, t)$, and the sum of utilities for the players as chosen by the

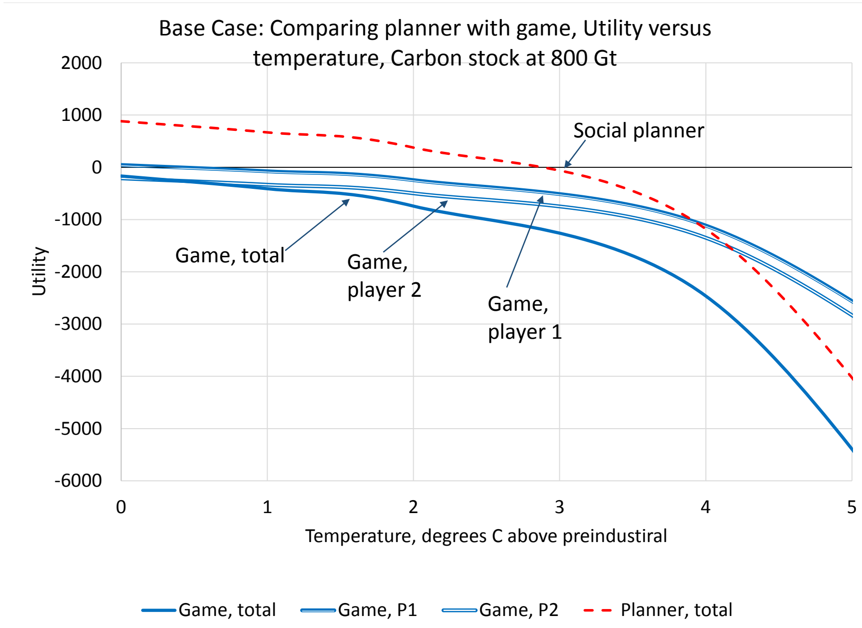
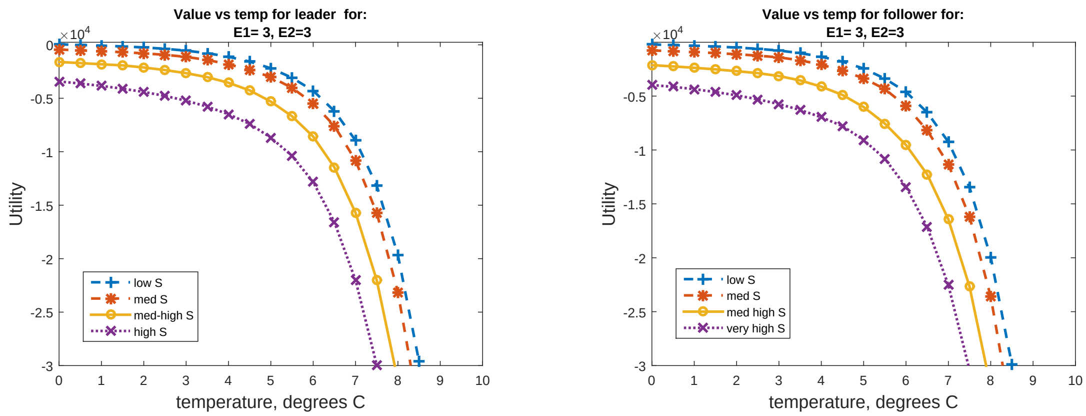


Figure 3: Base case utility for both players, comparing game and Social Planner. Exponential damage function. Stock of carbon at 800 GT.

Social Planner, which we denote by $V_{SP}(e_1, e_2, x, s, t)$. We observe, as expected, that utility declines with temperature and total utility is greater in the Social Planner case. Under the symmetric game, the leader is better off than the follower, which follows from the differing optimal strategies described below.

Figure 4 gives a different perspective on the game with plots of total discounted expected utility for the follower and leader versus temperature, with several different curves shown reflecting different levels of S. Utility is uniformly lower at higher carbon stocks.

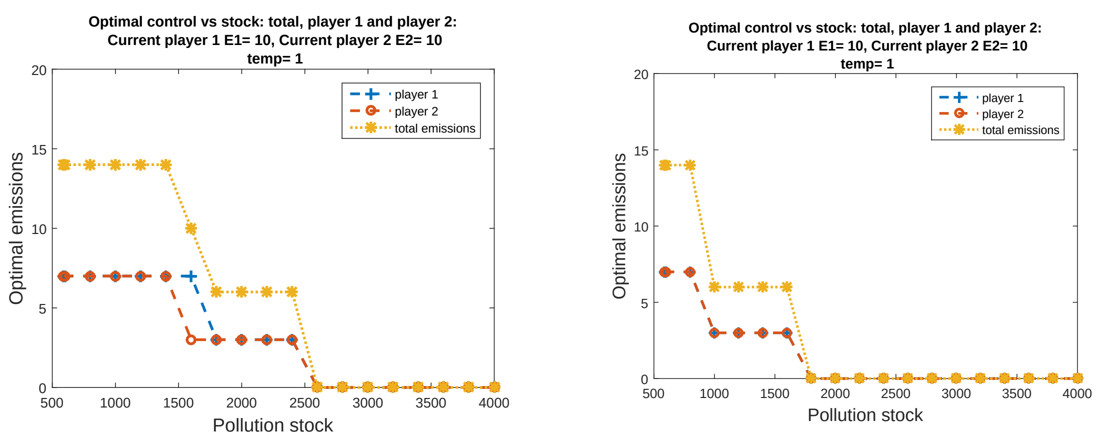
Figure 5 depicts the optimal controls for the game and the Social Planner versus the stock of carbon conditional on a temperature of 0.86 °C (the current value). For reference, recall that the current stock of carbon is about 870 GT. We see that optimal emissions fall as the stock of carbon is increased. In the game, the leader cuts back emissions when the stock reaches 1700 GT compared to 1500 GT for the follower. This reflects an advantage to the leader who can choose her emissions level, knowing that the follower will respond appropriately with emissions reductions which benefit both players. The Social Planner cuts back emissions more aggressively and at a lower carbon stock than the players in the game.



(a) Leader utility, $V_1(e_1, e_2, x, s, t)$ (b) Follower utility, $V_2(e_1, e_2, x, s, t)$

Figure 4: Values versus temperature for various carbon stock levels for the game, base case

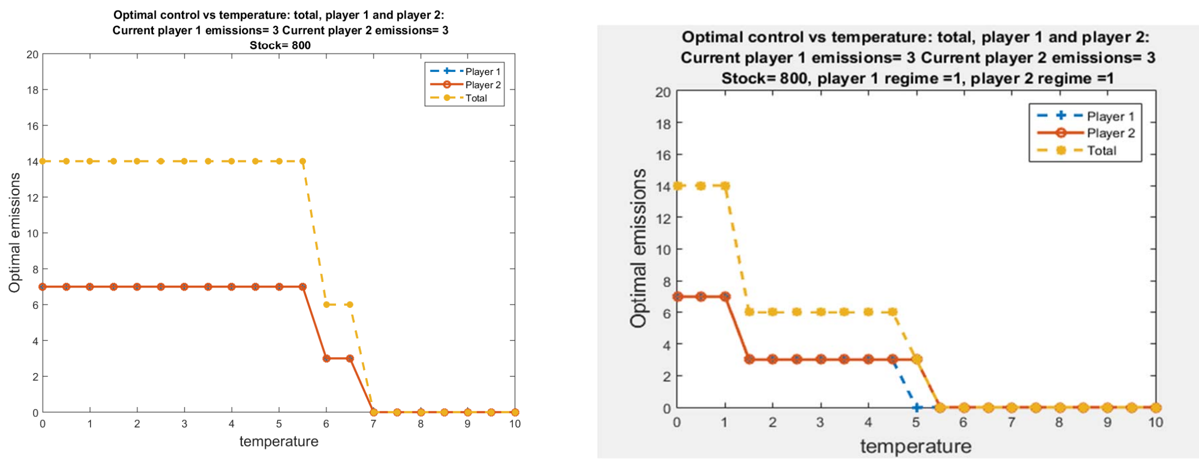
At a stock of 1000 GT of carbon, the Planner chooses total emissions of 7 compared to 14 for the total emissions under the game. The model is describing a clear tragedy of the commons where strategic interactions of the two decision makers leave both worse off than if decisions are made by a central planner.



(a) Game (b) Social planner

Figure 5: Optimal control versus pollution stock, contrasting game and Social Planner, base case, exponential damages, current temperature = 0.86 degrees C

Figure 6 contrasts the optimal controls for the game and the Social Planner versus temperature when the carbon stock is at 800 GT, which is close to the current value. Here the difference between the game and Social Planner is even more stark. Under the game the players both choose to emit 7 GT per year (compared to a maximum possible of 10) even when the temperature reaches a very high level. The Social Planner is, in contrast, much more responsive to temperature and reduces emissions when temperature exceeds 1 °C.



(a) Game

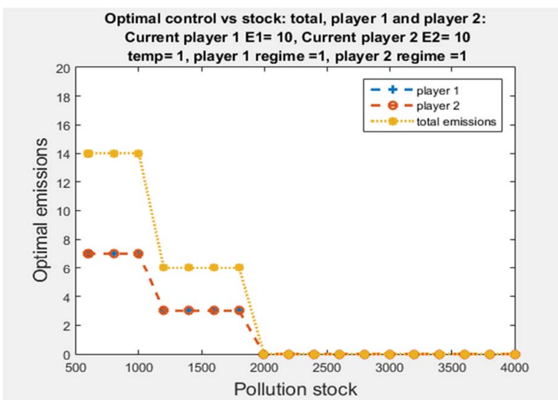
(b) Social planner

Figure 6: Optimal control versus temperature, game (symmetric players) and Social Planner, exponential damage, carbon stock at 800 GT

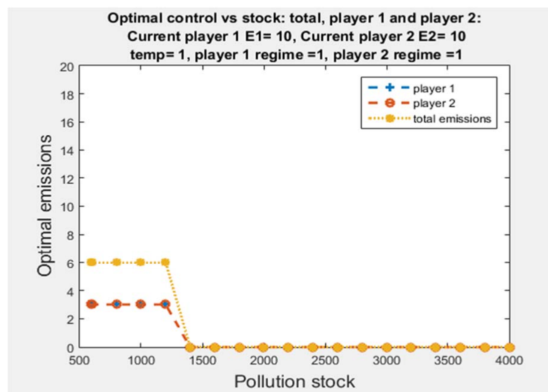
6.2 Importance of volatility

While the long run trend in temperature is the central concern of climate change, temperature volatility is also important. As noted previously, temperature is modelled as an Ornstein-Uhlenbeck process (Equation (3)) with base case volatility estimated at $\sigma = 0.1$. As time goes to infinity, the probability density of the Ornstein-Uhlenbeck process is Gaussian with a mean of $\bar{X}(t)$ and variance $\sigma^2/(2\eta)$. Based on 2015 data this implies $\bar{X}(t) = 1.93$ and a standard deviation of 0.44. Hence there is a 2.3% probability that temperature rises by 2 standard deviations or 0.88 °C due to randomness alone, regardless of carbon emissions.

Figure 7 compares the optimal controls of the game versus the Social Planner when volatility is tripled to 0.3. Higher volatility results in earlier emissions reduction for both the game and the planner but the impact on the planner is more marked. In the game, total emissions fall from 14 to 6 when the carbon stock reaches 1200 GT, compared to the base case when emissions remain at 14 until the carbon stock reaches about 1700 GT. For the Social Planner, optimal emissions are reduced compared to the low volatility case over all levels of atmospheric carbon stock, and fall to zero by the time the stock reaches 1400 GT.



(a) Game

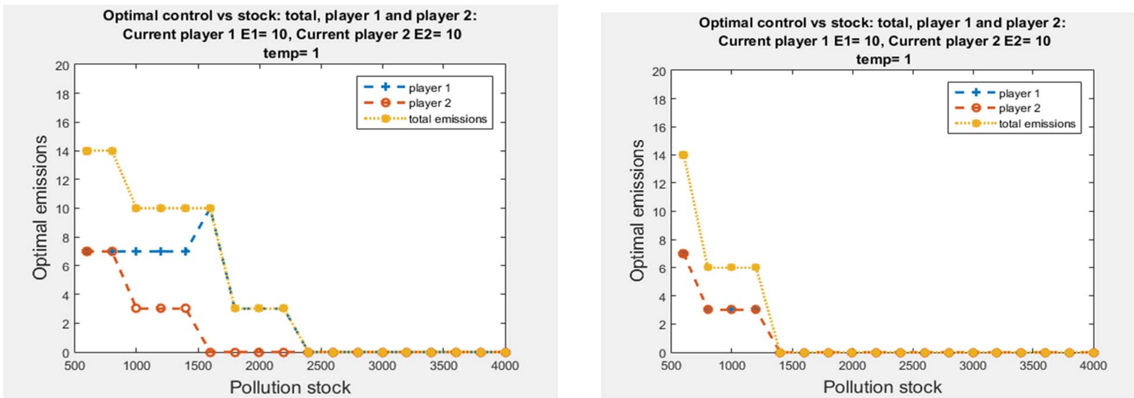


(b) Social planner

Figure 7: Optimal control versus carbon stock for high volatility case, game (symmetric players) and Social Planner, exponential damage, $\sigma = 0.3$

6.3 Asymmetric damages

We consider a case in which the follower has significantly higher damages than the leader. Specifically, κ_3 for player 2 in Equation (29) is increased from 1 to 1.1. The optimal controls in this case are shown in Figure 8. Comparing Figure 7(a) with 8(a) we observe that the advantage of the leader is increased when damages are asymmetric. The leader is able to increase emissions at a fairly high carbon stock level (above 1500 GT) as the leader knows that the follower will cut back emissions in response to high damages. Comparing Figure 7(b) and 8(b) we see that the Social Planner cuts back total emissions more aggressively when one player experiences greater harm from increased temperature compared to the base case. The results show that the tragedy of the commons is exacerbated by asymmetric damages.



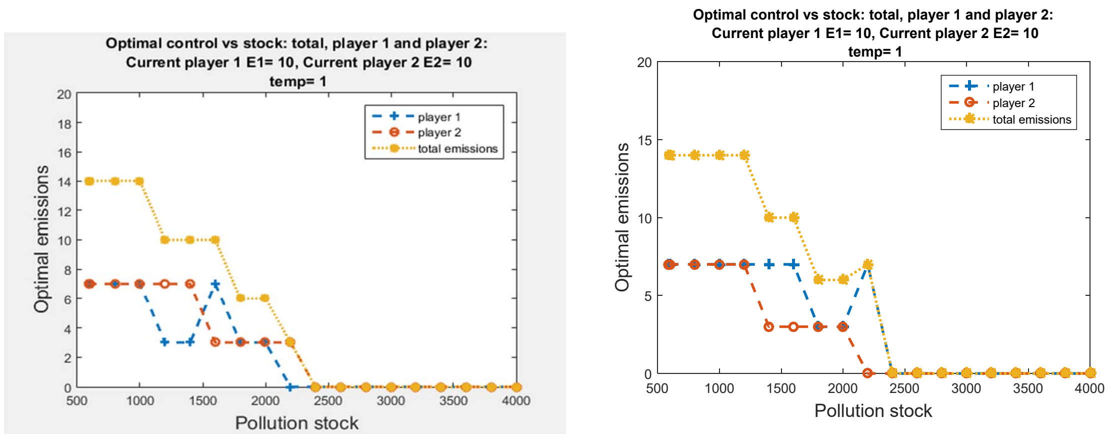
(a) Game (b) Social planner

Figure 8: Optimal control versus pollution stock, contrasting game and Social Planner, follower has higher damages, current temperature = 0.86 °C, $\kappa_3 = 1$ for player 1 and $\kappa_3 = 1.1$ for player 2

6.4 Asymmetric preferences

We consider a case in which one of the players is more environmentally aware than the other. We assume that the environmental friendly player is willing to pay 2 utility units, ($\theta_p = 2$) for reductions in emissions below the benchmark $\bar{E}$. The results are shown in Figure 9. We observe from Figure 9(a) that when the leader has green sentiments, she cuts back emissions sooner than in the base case. The follower does not respond to this change, and maintains the same emissions as in the base case. The leader increases emissions once the follower cuts back. When the follower has green sentiments (see Figure 9(b)), she cuts back emissions at a lower stock than in the base case. The leader does respond and increases emissions even though carbon stock is over 2000 GT. So in this case we see a type of green paradox mentioned by Wirl 2011 whereby increasing environmental preferences by one player is offset by the actions of the less environmentally concerned player.

The impact of green sentiments in one player requires further investigation. In future work we will explicitly model the preferences of players and consider the possibility of players switching from brown to green preferences.



(a) Green leader

(b) Green follower

Figure 9: Optimal control versus pollution stock when leader or follower has green environmental sentiments. current temperature = 0.86 °C.

6.5 Alternate damage functions

Sensitivities were conducted using the alternate damage function Equation (28). Using a quadratic function, ($\kappa_2 = 2$), the optimal choice of emissions is the maximum possible in both the game and the Social Planner. In contrast a cubic damage function, ($\kappa_2 = 3$) results in some curtailment of emissions, but still at higher levels than with exponential damages. We consider the exponential damage function to be the most reasonable as the damages quickly become very large at temperature above 3 °C.

7 Concluding comments

In this paper we have examined the strategic interactions of large regions making choices about greenhouse gas emissions in the face of rising global temperatures. We have modelled optimal decisions of players in a fully dynamic, closed loop Stackelberg game and have demonstrated its numerical solution. Our modelling of the evolution of carbon stock and temperature is based on Nordhaus's Integrated Assessment Model (Nordhaus 2013). Unlike some previous work, we do not assume that climate effects are zero if emissions are zero. Consequently, we inherently model disastrous long term effects of climate change. We also take into account the fundamental random nature of the temperature response to atmospheric carbon levels. In fact, our analysis shows that purely random effects are likely to cause global temperature changes of $\pm(0.5 - 1.0)$ degrees regardless of emission levels.

In our model, we find that the commonly used quadratic damage function results in very little emissions curtailment even at very high temperatures and carbon stock levels. We recommend an exponential damage function to account for disastrous effects for global average temperatures above 3 °C relative to pre-industrial levels.

Our results indicate that the leader in the pollution game has an advantage particularly in the case when the follower experiences higher damages from rising temperatures. By having knowledge of the follower's best response function, the leader is able to secure higher emissions (and hence utility) than the follower. In contrast with the game, a Social Planner cuts back on carbon emissions much more aggressively, indicating a classic tragedy of the commons. The tragedy of the commons is made worse under asymmetric damages.

We also examined a case where one of the players received a psychic benefit from emissions reductions compared to a benchmark, an effect we labelled the green reward. We observed that when the follower experienced a green reward, the leader was able to take advantage and increase her own emissions, thus counteracting the environmentally friendly sentiments of the follower.

Our results also highlight the importance of uncertain temperature. Increasing the volatility of temperature exacerbates the tragedy of the commons in that the Social Planner cuts back emissions by more than the players in the game, compared to the base case. Although the drift in long run temperature is key in climate change policy, the impact of volatility on strategic interactions of decision makers is significant.

Appendices

A Numerical methods

A.1 Advancing the solution from $t_{m+1}^- \rightarrow t_m^+$

Since we solve the PDEs backwards in time, it is convenient to define $\tau = T - t$ and use the definition

$$
\begin{aligned}
\hat{V}_p(e_1, e_2, x_i, s, \tau) &= V_p(e_1, e_2, x_i, s, T-\tau) \\
\hat{\pi}_p(e_1, e_2, x_i, s, \tau) &= \pi_p(e_1, e_2, x_i, s, T-\tau) \ . \qquad (32)
\end{aligned}
$$

We rewrite Equation (11) in terms of backwards time $\tau = T - t$

$$
\begin{aligned}
\frac{\partial \hat{V}_p}{\partial \tau} &= \hat{\mathcal{L}}\hat{V}_p + \pi_p + [(e_1+e_2) + \rho(\bar{S} - s)]\frac{\partial \hat{V}_p}{\partial s} \\
\hat{\mathcal{L}}\hat{V}_p &\equiv \frac{(\sigma)^2}{2}\frac{\partial^2 \hat{V}_p}{\partial x^2} + \eta(\bar{X} - x)\frac{\partial \hat{V}_p}{\partial x} - r\hat{V}_p \ . \qquad (33)
\end{aligned}
$$

Defining the Lagrangian derivative

$$
\frac{D\hat{V}_p}{D\tau} \equiv \frac{\partial \hat{V}_p}{\partial \tau} + \left(\frac{d\ s}{d\ \tau}\right)\frac{\partial \hat{V}_p}{\partial s} \ , \qquad (34)
$$

then Equation (33) becomes

$$
\frac{D\hat{V}_p}{D\tau} = \hat{\mathcal{L}}\hat{V}_p + \pi_p \qquad (35)
$$

$$
\frac{d\ s}{d\ \tau} = -[(e_1+e_2) + \rho(\bar{S} - s)] \ . \qquad (36)
$$

Integrating Equation (36) from τ to $\tau - \Delta\tau$ gives

$$
s_{\tau-\Delta\tau} = s_\tau \exp(-\rho\Delta\tau) + \bar{S}(1 - \exp(-\rho\Delta\tau)) + \big(\frac{e_1+e_2}{\rho}\big)(1 - \exp(-\rho\Delta\tau)) \qquad (37)
$$

We now use a semi-Lagrangian timestepping method to discretize Equation (33) in backwards time τ. We use a fully implicit method as described in Chen & Forsyth (2007).

$$\begin{aligned}\hat{V}_p(e_1,e_2,x,s_\tau,\tau) &= (\Delta\tau)\hat{\mathcal{L}}\hat{V}_p(e_1,e_2,x,s_\tau,\tau) \\ &\quad +(\Delta\tau)\pi_p(e_1,e_2,x,s_\tau,\tau)+\hat{V}_p(e_1,e_2,x,s_{\tau-\Delta\tau},\tau-\Delta\tau)\ . \end{aligned} \tag{38}$$

Equation (38) now represents a set of decoupled one-dimensional PDEs in the variable x, with (e_1, e_2, s) as parameters. We use a finite difference method with forward, backward, central differencing to discretize the $\hat{\mathcal{L}}$ operator, to ensure a positive coefficient method. (See Forsyth & Labahn (2007/2008) for details.) Linear interpolation is used to determine $\hat{V}_p(e_1, e_2, x, s_{\tau-\Delta\tau}, \tau - \Delta\tau)$. We discretize in the x direction using an unequally spaced grid with n_x nodes and in the S direction using n_s nodes. Between the time interval t^-_{m+1}, t^+_m we use n_τ equally spaced time steps. We use a coarse grid with $(n_\tau, n_x, n_s) = (2, 27, 21)$. We repeated the computations with a fine grid doubling the number of nodes in each direction to verify that the results are sufficiently accurate for our purposes.

A.2 Advancing the solution from $t^+_m \to t^-_m$

We model the possible emission levels as four discrete states for each of e_1, e_2, which gives 16 possible combinations of (e_1, e_2). We then determine the optimal controls using the methods described in Section 4.2.1. We use exhaustive search (among the finite number of possible states for (e_1, e_2)) to determine the optimal policies. This is, of coarse, guaranteed to obtain the optimal solution.

B Monotonicity of the Numerical Solution

Economic reasoning dictates that if the value function is decreasing as a function of temperature x at $t = t^-_{m+1}$, then this property should hold at t^+_m. In our numerical tests with extreme damage functions, which resulted in rapidly changing functions π_p, we sometimes observed numerical solutions which did not have this property. In order to ensure that this desirable property of the solution holds, we require a timestep restriction. To the best of our knowledge, this restriction has not been reported previously. In practice, this restriction is quite mild, but nevertheless important for extreme cases.

We remind the reader that since we solve the PDEs backwards in time, it is convenient to use the definitions

$$\begin{aligned}\hat{V}_p(e_1, e_2, x_i, s, \tau) &= V_p(e_1, e_2, x_i, s, T - \tau) \\ \hat{\pi}_p(e_1, e_2, x_i, s, \tau) &= \pi_p(e_1, e_2, x_i, s, T - \tau) \ .\end{aligned} \tag{39}$$

Assuming that we discretize Equation (38) on a finite difference grid $x_i, i = 1, \ldots, n_x$, we define

$$\begin{aligned}V_i^{n+1} &= \hat{V}_p(e_1, e_2, x_i, s_{\tau^{n+1}}, \tau^{n+1}) \\ c_i \equiv c(x_i) &= \hat{\pi}_p(e_1, e_2, x_i, s_{\tau^{n+1}}, \tau^{n+1})\Delta\tau + \hat{V}_p(e_1, e_2, x_i, s_{\tau^n}, \tau^n)\end{aligned} \tag{40}$$

Using the methods in Forsyth & Labahn (2007/2008), we discretize Equation (38) using the definitions (40) as follows

$$-\alpha_i \Delta\tau V_{i-1}^{n+1} + (1 + (\alpha_i + \beta_i + r)\Delta\tau)V_i^{n+1} - \beta_i \Delta\tau V_{i+1}^{n+1} = c_i \ , \tag{41}$$

for $i = 1, \ldots, n_x$. Note that the boundary conditions used (see Section 4.1) imply that $\alpha_1 = 0$ and that $\beta_{n_x} = 0$, so that Equation (41) is well defined for all $i = 1, \ldots, n_x$. See Forsyth & Labahn (2007/2008) for precise definitions of α_i and β_i.

Note that by construction α_i, β_i satisfy the positive coefficient condition

$$\alpha_i \geq 0 \quad ; \quad \beta_i \geq 0 \quad ; \quad i = 1, \ldots, n_x \ . \tag{42}$$

Assume that

$$
\begin{aligned}
\hat{V}_p(e_1, e_2, x_{i+1}, s_{\tau^n}, \tau^n) - \hat{V}_p(e_1, e_2, x_i, s_{\tau^n}, \tau^n) &\leq 0 \\
\hat{\pi}_p(e_1, e_2, x_{i+1}, s_{\tau^{n+1}}, \tau^{n+1}) - \hat{\pi}_p(e_1, e_2, x_i, s_{\tau^{n+1}}, \tau^{n+1}) &\leq 0 \ ,
\end{aligned} \tag{43}
$$

which then implies that

$$
c_{i+1} - c_i \leq 0 \ . \tag{44}
$$

If Equation (44) holds, then we should have that $V_{i+1}^{n+1} - V_i^{n+1} \leq 0$ (this is a property of the exact solution of Equation (38) if $c(y) - c(x) \leq 0$ if $y > x$).

Define $U_i = V_{i+1}^{n+1} - V_i^{n+1}, i = 1, \ldots, n_x - 1$. Writing Equation (41) at node i and node $i+1$ and subtracting, we obtain the following Equation satisfied by U_i,

$$
\begin{aligned}
[1 + \Delta\tau(r + \alpha_{i+1} + \beta_i)]U_i - \Delta\tau\alpha_i U_{i-1} - \Delta\tau\beta_{i+1}U_{i+1} &= \Delta\tau(c_{i+1} - c_i) \\
& \quad i = 1, \ldots, n_x - 1 \\
& \quad \alpha_1 = 0 \ ; \ \beta_{n_x} = 0 \ .
\end{aligned} \tag{45}
$$

Let $U = [U_1, U_2, \ldots, U_{nx-1}]'$, $B_i = \Delta\tau(c_{i+1} - c_i)$, $B = [B_1, B_2, \ldots, B_{nx-1}]'$. We can then write Equation (45) in matrix form as

$$
QU = B \ , \tag{46}
$$

where

$$
\big[QU\big]_i = [1 + \Delta\tau(r + \alpha_{i+1} + \beta_i)]U_i - \Delta\tau\alpha_i U_{i-1} - \Delta\tau\beta_{i+1}U_{i+1} \ . \tag{47}
$$

Recall the definition of an M matrix (Varga 2009),

Definition 5 (Non-singular M-matrix)**.** *A square matrix Q is a non-singular M matrix if (i) Q has non-positive off-diagonal elements (ii) Q is non-singular and (iii) $Q^{-1} \geq 0$.*

A useful result is the following (Varga 2009)

Theorem 1. *A sufficient condition for a square matrix Q to be a non-singular M matrix is that (i) Q has non-positive off-diagonal elements (ii) Q is strictly row diagonally dominant.*

From Theorem 1, and Equation (47), a sufficient condition for for Q to be an M matrix is that

$$1 + \Delta\tau[r + (\alpha_{i+1} - \alpha_i) + (\beta_i - \beta_{i+1})] > 0 \ , \ i = 1, \ ... \ n_{x-1} \tag{48}$$

which for a fixed temperature grid, can be satisfied for a sufficiently small $\Delta\tau$. In practice, for smoothly varying coefficients, $|\alpha_{i+1} - \alpha_i|$ and $|\beta_i - \beta_{i+1}|$ are normally small, so the timestep condition (48) is quite mild.

Proposition 2 (Monotonicity result)**.** *Suppose that (i) condition (48) is satisfied and (ii) $B_i = \Delta\tau(c_{i+1} - c_i) \le 0$, then $U_i = V_{i+1}^{n+1} - V_i^{n+1} \le 0$.*

Proof. From condition (48), Definition 5, and Theorem 1 we have that $Q^{-1} \ge 0$, hence from Equation (46)

$$U \ = \ Q^{-1} B \le 0 \ . \tag{49}$$

□

The practical implication of this result is that if conditions (43) hold at $\tau = T - t_{m+1}^-$, then $\hat{V}(\cdot, \tau = T - t_m^+)$ is a non increasing function of temperature. However, this property may be destroyed after application of the optimal control at $\tau = T - t_m^+ \to T - t_m^-$. In other words, if we observe that the solution is increasing in temperature, this can only be a result of applying the optimal control, and is not a numerical artifact.

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