# Asset Allocation During High Inflation Periods: A Stress Test

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## 1 Introduction

Inflation is now on everyone's mind. We have just been through a long period of benign inflation, and low (real) short term interest rates. Some would argue that this has led to a bubble in asset prices. Now, after the Covid crisis, we are seeing high inflation statistics in most of the world.

Going forward, we have to be cognizant of the risk of inflation. If we enter into a long period of even moderate inflation (i.e. something like the 5% per year that we observed (in Canada) during 1950-1983, see Hatch and White (1985)), will the traditional passive approach using a mix of capitalization weighted stock indexes and moderate term bonds still work?

Our objective in this white paper is to filter the historical time series (1926-2022) to uncover periods of high, sustained inflation. We concatenate these high inflation regimes, to produce a series of returns which were observed in inflationary times. We then examine the performance of various asset allocation strategies, by bootstrapping returns from the high inflation series. We consider a long term investor, with an investment horizon of 30 years. This would be typical of an investor saving for retirement. This is also relevant for a 65-year old retiree planning an investment strategy to age 95.

We can think of this concatenated series of high inflation regimes to be a stress test for a portfolio allocation strategy. We believe that the probability of a thirty year period of high inflation is low. However, it is instructive to see the effects of long term inflation on traditional allocation strategies.

Our main conclusion is that an investor should use a mix of short-term bonds and an equal-weighted stock index, to mitigate inflationary effects. In the event that inflation does not materialize, this portfolio should still do reasonably well.

## 2 Data

We use data from the Center for Research in Security Prices (CRSP) on a monthly basis over the 1926:1-2022:1 period.[1] [2] We also use the the U.S. CPI index, also supplied by CRSP.

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[1]The date convention is that, for example 1926:1 refers to January 1, 1926.

[2]More specifically, results presented here were calculated based on data from Historical Indexes, ©2022 Center for Research in Security Prices (CRSP), The University of Chicago Booth School of Business. Wharton Research Data Services (WRDS) was used in preparing this article. This service and the data available thereon constitute valuable intellectual property and trade secrets of WRDS and/or its third-party suppliers.

Our objective is to select high inflation periods as determined by the CPI. Monthly data is quite volatile, so we used the following filtering procedure. We use a moving window of $k$ months, and we determine the cumulative CPI index log return (annualized) in this window. If the cumulative annualized CPI is greater than a cutoff, then all the months in the window are flagged as part of a high inflation regime. Note that some months may appear in more than one moving window. Any months which do not meet this criteria are considered to be in low inflation regimes. See Algorithm A.1 for the filtering pseudo-code.

This approach requires specification of the cutoff, and the window size. The average annual inflation over the period 1926:1-2022:1 was 2.9%. In other words, inflation of about 3% was normal. After some experimentation, we used a cutoff of 5%. Figure 2.1 shows the filtering results for windows of size 12, 60 and 120 months. We can see that the five year window produces two obvious inflation regimes: 1940:8-1951:7 and 1968:9-1985:10, which correspond to well known market shocks (i.e. the second world war, and price controls; the oil price shocks and stagflation of the seventies). Increasing the window size to 10 years, resulted in similar looking plots as the five year window size, but the number of months in each window increased, and lowered the average inflation rate. Since our objective is to determine the effect of high inflation periods on allocation strategies, we decided to use the five year window results.

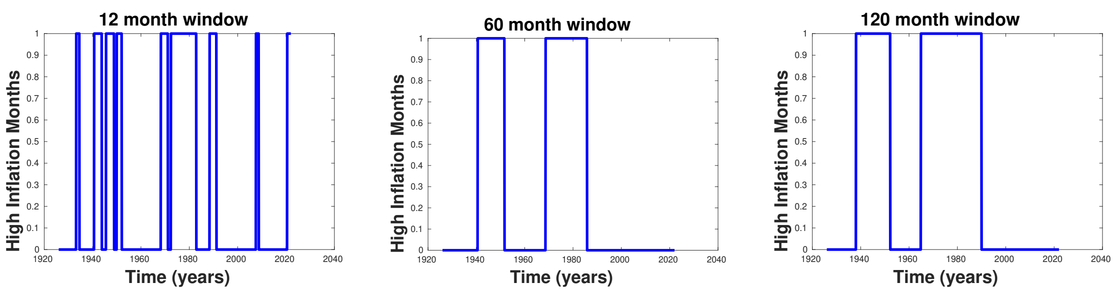



FIGURE 2.1: *High inflation regimes, using the moving window method, with the window size shown. The cutoff for* high inflation *regimes was* 0.05. *High inflation months have a label value of one, and low inflation months have a label value of zero. CPI data in the range 1926:1-2022:1.*

Table 2.1 shows the average annual inflation over the two regimes identified from the moving window filter.

| Time Period | Average Annualized Inflation |
|---|---|
| 1940:8-1951:7 | .0564 |
| 1968:9-1985:10 | .0661 |

TABLE 2.1: *Inflation regimes determined using a five year moving window with a cutoff of* 0.05.

For possible investment assets, we considered the 30-day T-bill index (CRSP designation "t30ind"), and we also constructed a constant maturity ten year US treasury index. [3] [4]

[3]The 10-year Treasury index was generated from monthly returns from CRSP back to 1941 (CRSP designation "b10ind"). The data for 1926-1941 were interpolated from annual returns in Homer and Sylla (2005).

[4]The 10-year treasury index is constructed by (a) buying a ten year treasury at the start of each month, (b) collecting interest during the month, and then (c) selling the treasury at the end of the month. We repeat the process again at the start of the next month. The gains in the index then reflect both interest and capital gains and losses.

In addition, we also studied the Capitalization weighted index (CapWt) and the equal weight index (EqWt), also from CRSP.[5] We remind the reader that the CRSP indexes are total return indexes, which include all distributions for all domestic stocks trading on major U.S. exchanges. All of these various indexes are in nominal terms, so we adjust them for inflation by using the U.S. CPI index, also supplied by CRSP.

As an initial filter for these assets, we assume that each real (deflated) index follows geometric Brownian motion (GBM). For example, given an index with value $S$, then

$$dS = \mu S \, dt + \sigma S \, dZ \tag{2.1}$$

where $dZ$ is the increment of a Wiener process. We use maximum likelihood to fit the drift rate $\mu$ (expected arithmetic return) and volatility $\sigma$ in each regime, for each index, as shown in Table 2.2. We also show a series constructed by: converting the indexes in each regime to returns, concatenating the two return series, and converting back to an index. This concatenated index does not, of course, correspond to an actual historical index, but is a pseudo index constructed from high-inflation regimes. This amounts to a worst case sequence of returns, that could plausibly be expected during a long period of high inflation.

It is striking that in each regime in Table 2.2, the drift rate $\mu$ for the equal weight index is much larger than the drift rate for the capitalization weighted index. Observe that the geometric return (i.e. the median return assuming GBM) for the capitalization weighted index, in the period 1968:9-1985:10, was only about one per cent per year.

It is also noticeable that bonds performed very poorly in the period 1940:8-1951:7. As well, during the period 1968:9-1985:10, there was essentially no term premium for 10-year treasuries, compared with 30-day T-bills. In addition, the 10-year treasury index had a much higher volatility compared to the 30-day T-bill index. Looking at the concatenated series, it appears that 30-day T-bills, are arguably the defensive asset here, since the volatility of this index is quite low (but with a negative (real) drift rate).

## 2.1 Bonds

It is instructive to examine the real returns of short and long term bonds over the entire 1926:1-2022:1 period. These indexes are shown in Figure 2.2.

We can observe the following from Figure 2.2.

- Over the long term, short term T-bills are, at best, stores of real value. You can't expect to generate real returns using short term bonds. But you won't lose much either.
- Long term bonds are sometimes very bad investments. For example, if you bought a 10 year treasury index in 1940, and held it until 1980, you would have lost about one-half of your real wealth.
- This contrasts with the last 40 years, where long term bonds have been fantastic investments. However, this was due to falling interest rates and low inflation. We can't expect this to continue.[6]

[5] The capitalization weighted total returns have the CRSP designation "vwretd", and the equal weighted total returns have the CRSP designation "ewretd".

[6] Consider the risk parity idea applied to a portfolio consisting of a stock index, a long term bond index, and cash. In this case, in an attempt to equal weight the risk, the final portfolio will be: long the stock index, and with a large long term bond component, financed by borrowing cash. In other words, this will be a highly leveraged long term bond and stock portfolio. We can see from Figure 2.2 that this was a great idea for the last 40 years. However, it is unlikely that we can extrapolate the long term bond index forward for the next 40 years, based on the last 40 years.

| Index | $\mu$ | $\sigma$ | $\mu - \sigma^2/2$ |
|---|---|---|---|
| | | | 1940:8-1951:7 |
| CapWt | 0.079 | 0.140 | .069 |
| EqWt | 0.145 | 0.190 | .127 |
| 10 Year Treasury | -0.035 | 0.036 | -.036 |
| 30-day T-bill | -0.050 | 0.029 | -.050 |
| | | | 1968:9-1985:10 |
| CapWt | 0.026 | 0.164 | .013 |
| EqWt | 0.065 | 0.220 | .041 |
| 10 Year Treasury | 0.011 | 0.093 | .007 |
| 30-day T-bill | 0.009 | 0.012 | .009 |
| | Concatenated: 1940:8-1951:7 and 1968:9 - 1985:10 | | |
| CapWt | 0.049 | 0.156 | .038 |
| EqWt | 0.098 | 0.209 | .076 |
| 10 Year Treasury | -0.008 | 0.076 | -.011 |
| 30-day T-bill | -0.014 | 0.022 | -.014 |

TABLE 2.2: *GBM parameters for the indexes shown. All indexes are real (deflated). $\mu$ is the expected annualized arithmetic return. $\sigma$ is the annualized volatility. $(\mu - \sigma^2/2)$ is the annualized geometric mean return (the median return assuming GBM).*

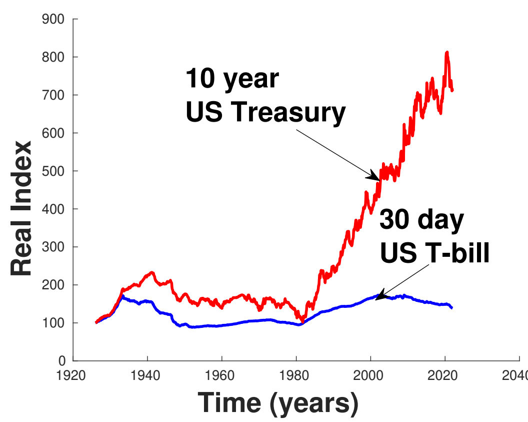



FIGURE 2.2: *Short and long term constant maturity bond indexes, data in the range 1926:1-2022:1. All indexes are deflated using the CPI.*

Consequently, in the following, we will focus attention on 30-day T-bills, the capitalization weighted index, and the equal weight index.

# 3 Bootstrap Resampling

We will be testing allocation strategies using stationary block bootstrap resampling (Politis and Romano, 1994; Politis and White, 2004; Patton et al., 2009; Dichtl et al., 2016; Forsyth and Vetzal, 2019; Ni et al., 2022). See Appendix B for detailed pseudo code for bootstrap resampling. Briefly, each bootstrap resample consists of (i) selecting a random starting date in the historical return series, (ii) then selecting a block (of random size) of consecutive returns from this start date, and

| Data series | Optimal expected block size $\hat{b}$ (months) |
|---|---|
| Real 30-day T-bill index | 26 |
| Real CRSP cap-weighted index (CapWt) | 2 |
| Real CRSP equal-weighted index (EqWt) | 4 |

TABLE 3.1: *Optimal expected blocksize* $\hat{b} = 1/v$ *when the blocksize follows a geometric distribution* $Pr(b = k) = (1 - v)^{k-1}v$. *The algorithm in Patton et al. (2009) is used to determine* $\hat{b}$. *Sample period: concatenated returns:1940:8-1951:7 and 1968:9-1985:10.*

(iii) repeating this process until a sample of the total desired length is obtained.

An important parameter is the expected blocksize, which, informally, is a measure of serial correlation in the return data. Table 3.1 shows estimates for the expected blocksize, for each return series, using the algorithm in Patton et al. (2009). It appears that the stock return data have very little serial correlation. However, the expected blocksize for the real 30-day T-bill index is about 2 years. This is not surprising, since short term rates are primarily driven by central banks, and hence are quite *sticky*. This poses a bit of a problem with our concatenated series, since there is a break in the data between the two historical regimes of high inflation. However, we will show results using a range of blocksizes, including i.i.d. assumptions (i.e. blocksize equal to one month). We will see that the results are relatively insensitive to blocksize.

## 3.1 Use of concatenated series

We view it as unlikely that a period of high inflation will persist over a 30 year period. However, the two periods of sustained, high inflation, during the last 100 years, total to about 28 years. By concatenating these periods of high inflation into a lengthy pseudo series of continuous high inflation, we view this pseudo series as a stress test for allocation strategies. Note that each of the data series in the two high inflation regimes are actual historical time series. The only liberty we have taken from the historical record is to concatenate the returns of these two disjoint (in time) high inflation regimes.

# 4 Investment Scenario

The details of the investment scenario are given in Table 4.1. Briefly, we begin with an initial wealth of 1000, with no further cash injections and withdrawals. The investment horizon is thirty years, with annual rebalancing to a weight of 60% in stocks and 40% in bonds. We evaluate the investment results by examining the distribution of the final wealth $W_T$ at $T = 30$ years.

# 5 More on Bootstrap Resampling

As discussed, we will use bootstrap resampling (Politis and Romano, 1994; Politis and White, 2004; Patton et al., 2009; Dichtl et al., 2016), to analyze the performance of using the equal weight index compared to the capitalization weighted index, during periods of high inflation (our concatenated series: 1940:8-1951:7, 1968:9-1985:10).

First, we examine the effect of the expected blocksize parameter in the bootstrap resampling algorithm. We will use a paired sampling approach, where we simultaneously draw returns from

| | |
|---|---|
| Investment horizon $T$ (years) | 30.0 |
| Equity market indexes | CRSP cap weighted index (real) |
| | CRSP equal weighted index (real) |
| Bond index | 30-day T-bill (US) (real) |
| Index Samples | Concatenated 1940:8-1951:7, 1968:9-1985:10 |
| Initial portfolio wealth $W_0$ | 1000 |
| Rebalancing times (years) | $t = 0{,}1.0, 2.0, \ldots, 29.0$ |
| Cash Injections/withdrawals | None |
| Equity fraction range | 0.60 at each rebalancing |
| Rebalancing interval (years) | 1.0 |

TABLE 4.1: *Investment scenario.*

the bond and stock indexes.[7] The algorithm in Politis and White (2004) was developed for single asset time series. It is therefore out of theory to apply the results in Table 3.1 to paired sampling. In Table 5.1, we examine the effect of different blocksizes on the statistics of stationary block bootstrap resampling. If we choose the blocksize based on the heuristic ($.60 \times (blocksize_{equity}) + .40 \times (blocksize_{bonds})$) then a blocksize of one year seems reasonable.

| Expected blocksize (months) | Median[$W_T$] | E[$W_T$] | std[$W_T$] | ES (5%) |
|---|---|---|---|---|
| 1 | 4161.0 | 5409.6 | 4506.6 | 994.5 |
| 3 | 4352.6 | 6207.1 | 6334.0 | 816.7 |
| 6 | 4423.2 | 6467.5 | 6881.4 | 752.3 |
| 12 | 4483.9 | 6478.7 | 6688.7 | 756.3 |
| 24 | 4547.7 | 6292.5 | 5969.0 | 820.5 |

TABLE 5.1: *Effect of expected blocksize, on the statistics of the final wealth $W_T$ at $T = 30$ years. Constant weight, scenario in Table 4.1. Equity weight: 0.60, rebalanced annually. Bond index: 30-day T-bill. Equity index: equal weight. Concatenated series: 1940:8-1951:7, 1968:9-1985:10 (high inflation regimes). All quantities are real (inflation adjusted). Initial wealth 1000. Bootstrap resampling, $10^6$ resamples (Appendix A.1).*

Perhaps a more visual way of analyzing the effect of the expected blocksize is shown in Figure 5.1, where we show the cumulative distribution function (CDF) of the final wealth after 30 years, for different blocksizes. We show the CDF since this gives us a visualization of the entire final wealth distribution, not just a few summary statistics.

Since the data frequency is at one month intervals, specifying a geometric mean expected blocksize of one month means that the blocksize is always a constant one month. This effectively means that we are assuming that the data is i.i.d. However, the one-month results are an outlier, compared to the other choices of expected blocksize. There is hardly any difference between the CDFs for any choice of expected blocksize in the range 3-24 months. From this point on, we will use an expected blocksize of 12 months (one year).

[7] This preserves correlation effects.

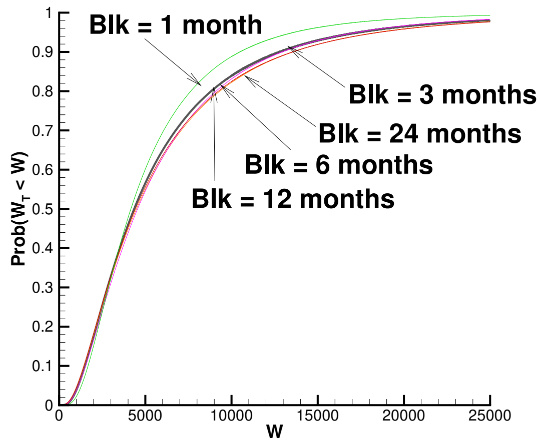



FIGURE 5.1: *Cumulative distribution function (CDF), final wealth $W_T$ at $T = 30$ years, effect of expected blocksize. Constant weight, scenario in Table 4.1. Equity weight: 0.60, rebalanced annually. Bond index: 30-day T-bill. Equity index: equal weight. Concatenated series: 1940:8-1951:7, 1968:9-1985:10 (high inflation regimes). All quantities are real (inflation adjusted). Initial wealth 1000. Bootstrap resampling, expected blocksize one year, $10^6$ resamples (Appendix A.1).*

# 6 Stochastic Dominance

We remind the reader of the concept of first order stochastic dominance. Suppose have two investment strategies, A and B. Consider the CDFs (cumulative distribution functions) of both strategies, as a function of the terminal wealth $W$. We denote the CDF of strategy A by $F_A(W)$ and that of strategy B by $F_B(W)$. If $W_T$ is a possible value of wealth at time $T$, then we can interpret the CDF $F_A(W)$ as

$$Prob(W_T < W) \quad = \quad F_A(W) \; . \tag{6.1}$$

Strategy $A$ stochastically dominates (in the first order sense) strategy B if

$$F_A(W) \quad \leq \quad F_B(W) \; , \tag{6.2}$$

and there exists at least one point $\hat{W}$ such that $F_A(\hat{W}) < F_B(\hat{W})$. This means that strategy $A$ never gives a lower level of terminal wealth at every level of probability, compared to strategy $B$. And there is at least one value of wealth such that strategy A achieves this wealth at a higher probability than strategy B. Any investor who has a preference for more final wealth rather than less will prefer strategy A.

It is in fact rare to find that one strategy strictly dominates another strategy, so we have the concept of partial stochastic dominance (van Staden et al., 2021). Strategy A dominates B in a partial sense if

$$F_A(W) \quad \leq \quad F_B(W) \; ; \; W_{\min} \leq W \leq W_{\max} \; . \tag{6.3}$$

This is obviously a practical criteria. If $W_{\max}$ is very large (i.e. we would be worth billions), then we don't care about stochastic dominance at extreme large wealth values (we won't be able to spend all our wealth anyway). On the other hand, if $W_{\min}$ is very small, then, under both strategy A and B, we are bankrupt, and so it doesn't really matter if we have one cent in our pocket compared to two cents. Or perhaps $F_A(W_{\min})$ is so small,[8] that these events have a tiny probability, so we don't care about what happens in these cases either. For more discussion of this, see Forsyth (2022).

# 7 Bootstrap Tests: equal weight vs. capitalization weight

We consider the investment scenario described in Table 4.1. We used block bootstrap resampling of the concatenated CRSP data 1940:8-1951:7, 1968:9-1985:10. An expected blocksize of one year was specified, with $10^6$ resamples. Figure 7.1 compares the use of the equal weight index and the capitalization weighted index for the stock component of the strategy. Remarkably, the strategy which uses the equal weight index appears to stochastically dominate the strategy which uses the capitalization index.

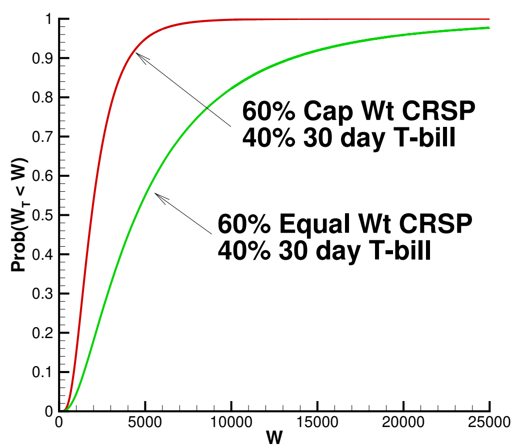



FIGURE 7.1: *Cumulative distribution function of final real wealth $W$ at $T = 30$ years, bootstrap resampling expected blocksize one year, $10^6$ resamples (Appendix A.1). $T = 30$ years. Data: concatenated returns, 1940:8-1951:7, 1968:9-1985:10. Scenario in Table 4.1.*

This is also reflected in the summary statistics in Table 7.1, which shows that the use of the equal weight index gives greatly increased values of $E[W_T]$, Median$[W_T]$. We also show the expected shortfall at the 5% level ($ES(5\%)$) which is simply the mean of the worst 5% of the outcomes. This is a measure of left tail risk. By this measure, the equal weight strategy produces a larger (better) result than the capitalization weighted index.

However, the standard deviation (std$[W_T]$) of the equal weighted index is much larger that the standard deviation of the capitalization weighted index. This would normally be considered a red flag, and would likely generate a bad Sharpe ratio. However, this is an example of a case where the standard deviation is a poor measure of risk, since both upside and downside are penalized. If we look at the CDF plot Figure 7.1, bearing in mind Table 7.1, we can see that the large standard deviation of the equal weight strategy is due to the large right skew of the distribution, i.e. higher probabilities of obtaining very large wealth values.[9]

[8] Recall the definition of $F_A(W)$ in equation (6.1).

[9] Many years ago, I was sitting next to a banker on a flight to New York. I was amused by his comment: *"Actually, we like volatility when stocks go up."*

| Equity Index | Median[$W_T$] | E[$W_T$] | std[$W_T$] | ES (5%) |
|---|---|---|---|---|
| Capitalization weight | 1890.8 | 2253.2 | 1474.2 | 534.2 |
| Equal weight | 4483.9 | 6478.7 | 6688.8 | 756.3 |

TABLE 7.1: *Constant weight, scenario in Table 4.1. Equity weight: 0.60, rebalanced annually. Bond index: 30-day T-bill. Concatenated series: 1940:8-1951:7, 1968:9-1985:10 (high inflation regimes). All quantities are real (inflation adjusted). ES(5%) is the mean of the worst five per cent of the outcomes. Bootstrap resampling, expected blocksize one year, $10^6$ resamples (Appendix A.1). $T = 30$ years.*

## 7.1 A Closer Look at the Left Tail

But, perhaps we should examine the left tail in a bit more detail. Figure 7.2 shows a zoomed in portion of Figure 7.1. We can see that the equal weight strategy does not strictly dominate the cap weighted strategy, since the CDFs cross at a wealth of around 230, at a probability of about $5 \times 10^{-4}$, so we have only partial stochastic dominance. However, bear in mind that $Median[W_T]$ for the equal weight strategy is about 4484, and that we have an initial wealth of 1000. If we end up with wealth below 230 after 30 years of investing, then this is a very bad result. As a concrete example, the cap weighted strategy has $Prob[W_T < 150] = 3.4 \times 10^{-4}$ while the equal weighted strategy has $Prob[W_T < 150] = 6.7 \times 10^{-4}$. Does this really matter? In some sense, at this wealth level, the cap weighted strategy is *twice as good* as the equal weight strategy, but both results are very bad, with extremely low probabilities.

In fact, to get this in perspective, it is useful to look at Figure 7.1. Strictly speaking, we have only partial stochastic dominance of equal weight over cap weighted stock indexes, with $W_{\min} \simeq 230$ (see equation (6.3) ). However, if we look at Figure 7.1, we can see that $W = 230$ is very small on the scale of the x-axis values, and that $Prob[W_T < 230]$ $(5 \times 10^{-4})$ is also very small, on the scale of the y-axis, for both strategies. This would suggest that any reasonable investor, on the basis of Figure 7.1, would choose the equal weight index.

So, with some loss of rigor, we will refer to anything like Figure 7.1 as showing stochastic dominance, even though this is not strictly true. However, we have stochastic dominance for any practical purpose.

# 8 CDFs for entire period 1926:1-2022:1

Our results for inflationary times seem to suggest that an equal-weight stock index is the way to go. However, let's review some CDF plots from a previous white paper (Forsyth, 2022). Figure 8.1(a) shows the bootstrapped CDFs for the equal weight and cap weighted CRSP index for the entire period 1926:1-2022:1. We can see that, for the entire historical period, the equal weight index dominates the cap weighted index.

However, Figure 8.1(b) compares the bootstrapped equal and cap weighted CRSP indexes, but this time only using the data in the range 1980:1-2022:1. The equal weight dominance has almost disappeared.

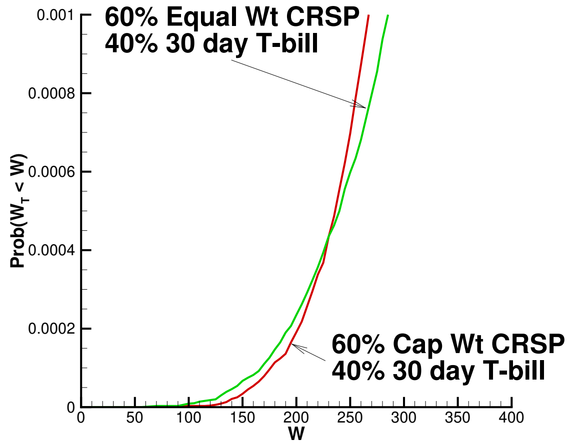
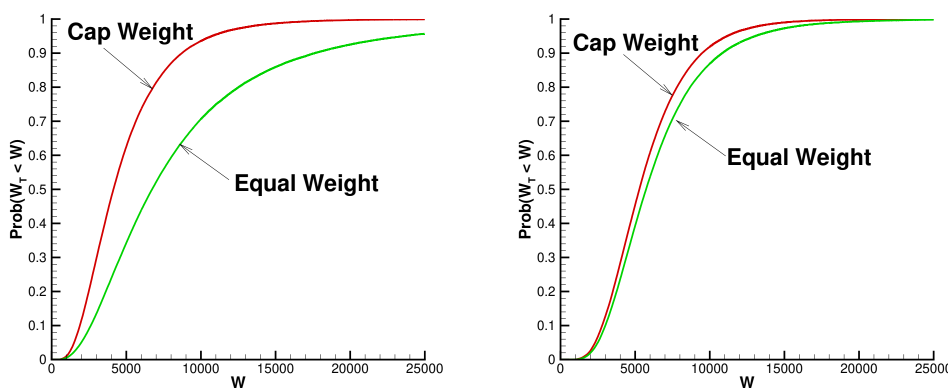



FIGURE 7.2: *Cumulative distribution function of final real wealth $W$ at $T = 30$ years, bootstrap resampling expected blocksize one year, $10^6$ resamples (Appendix A.1). Concatenated series: 1940:8-1951:7, 1968:9-1985:10 (high inflation regimes). $T = 30$ years. Scenario in Table 4.1. Zoom of Figure 7.1.*




(a) *1926:1-2022:1*

(b) *1980:1-2022:1*

FIGURE 8.1: *Cumulative distribution functions (CDFs) for cap weighted and equal weighted indexes, as a function of final real wealth $W$ at $T = 30$ years. Initial stake $W_0 = 1000$, no cash injections or withdrawals. Block bootstrap resampling, expected blocksize 2.0 years. 60% stocks, 40% bonds, rebalanced annually. Bond index: 30 day US T-bills. Stock index: CRSP capitalization weighted or CRSP equal weighted index. Data range shown. All indexes are deflated by the CPI. $10^6$ resamples.*

# 9 For and Against Equal Weighting

## 9.1 Against Equal Weighting

Clearly, an equal weighted portfolio will give a greater weight to small cap stocks than a capitalization weighted portfolio. The fact that, for many years, small cap stocks outperformed large cap stocks was first noted in Banz (1981). However, the small cap effect seems to have largely disappeared (Ahn et al., 2019). This, of course, would be consistent with basic financial reasoning: once everyone knows about a market anomaly, then everyone will trade to exploit this, and the effect will disappear.

So, the argument here would be that the equal weight outperformance observed during periods of high inflation is simply due to the (at the time) unknown small cap effect. This is also consistent with our bootstrap CDFs, which show very little improvement of the equal weight portfolio vs. the cap weighted portfolio for the last 40 years.

## 9.2 For Equal Weighting

See Plyakha et al. (2014); Tljaard and Mare (2021) and the references cited therein for a summary of many studies which are consistent with Figure 8.1(a): over the long term, equal weight indexes are superior to cap weighted indexes. Plyakha et al. (2014) argue that the outperformance of the equal weight index is partially due to the small cap factor, but there is also a significant effect due to rebalancing. In other words, the equal weight strategy is fundamentally contrarian: sell winners and buy losers. Observe that the multi-period optimal mean-variance strategy has this property: buy stocks and sell bonds when stocks lose; buy bonds and sell stocks when stocks gain (Forsyth and Vetzal, 2019; van Staden et al., 2021). Hence, the equal weight portfolio simply applies this idea to the stock basket. In Tljaard and Mare (2021), this diversification idea is discussed in detail. See also Edwards et al. (2018).

Note that Tljaard and Mare (2021) agree that equal weight portfolios outperform in the long term, but also have underperformed for significant periods, in particular the last 10 years. They offer various reasons for this. In particular, the authors argue that the relative performance of the equal weight vs. the cap weight index will suffer in periods where the cap weighted portfolio becomes highly concentrated. The intuition behind this is clear: if a small number of companies become very successful over long periods, and dominate the cap weighted index, then an equal weighted portfolio will surely suffer. However, in this case, the cap weighted portfolio amounts to highly concentrated bets on a small number of stocks, which, historically, has been a bad idea.

Oderda (2015) shows that, under certain assumptions, rule based portfolios (equal weight, minimum variance) outperform capitalization weighted indexes. The determination of the optimal weights for these portfolios is independent of estimates of the expected returns of individual stocks. Hence this outperformance portfolio is robust to uncertainty in the expected return parameters. Coqueret and Andre (2022) use reinforcement learning to attempt to determined optimal factor portfolios. The end result of this learning exercise is essentially a $1/n$ portfolio, i.e. equal weighted in the factors.[10] The authors provide the intuition that since financial data are dominated by noise, the best strategy is to be agnostic about factor return characteristics, and simply weight all factors equally.

In Table 9.1, we show the historical annualized compound return and volatility for the CapWt and EqWt indexes, for the periods 2002:1-2012:1 and 2012:1-2022:1. We can see that the last decade was something of an anomaly, with the CapWt index outperforming the EqWt index by 300 bps. However, during the turbulent period 2002:1-2012:1, the EqWt index outperformed the CapWt index by 480 bps. The small cap effect was well known by the decade 2002:1-2012:1, so it is doubtful that the small cap effect can explain this result.

All this work leads to the conclusion that the historical outperformance of an equal weight portfolio is not simply due to the small cap effect.

[10] In this case, there are $n$ factors, so the $1/n$ rule simply allocates $1/n$ of the total wealth to each factor.

| Index | Annualized log return | $\sigma$ |
|---|---|---|
| | 2002:1-2012:1 | |
| CapWt | .016 | .17 |
| EqWt | .064 | .200 |
| | 2012:1-2022:1 | |
| CapWt | .118 | .132 |
| EqWt | .088 | .158 |

TABLE 9.1: *Real (deflated) return, single historical path, dates shown.*

# 10 Conclusion

The historical evidence, based on bootstrapping returns during inflationary times, suggests that equal weight indexes significantly outperform capitalization weighed indexes. In addition, bootstrapped returns for the entire historical period of 1926:1-2022:1, show once again that equal weight indexes outperform. Even during the last 40 years (an unprecedented period of falling real interest rates, low inflation, and high performing FAANG stocks), equal weight indexes basically perform similarly to cap weighted indexes.

The real question is whether the equal weight outperformance during historical periods is solely due to the small cap effect. If this is the case, then probably we can't expect the equal weight index to be much protection during inflationary times. On the other hand, looking at the last 40 years only, it seems that you are not hurt by using an equal weight index.

However, there is good evidence to suggest that a large portion of the equal weight outperformance is due to the contrarian aspect of equal weighting. This also explains the lackluster performance of equal weighting during the last decade, where we have seen the cap weighted index become highly concentrated with tech stocks.[11]

Consequently, if you think that inflation will be an issue going forward, an equal weight stock index is a good bet. If you are wrong, and inflation turns out not to be an issue, than your equal weighted index will probably at least keep up with a cap weighted index.[12] In fact, Tljaard and Mare (2021) suggests a truly optimal strategy is a dynamic mix of equal and cap weighted portfolios. However, we now have to pick the optimal weight.

In the absence of any other information, perhaps we should use a portfolio with (i) 40% short term bonds (ii) 30% cap weighted stock index and (iii) 30% equal weighted index, and rebalance annually.

[11] As of June 2022, the top five companies in the S&P 500 were: Apple, Microsoft, Amazon, Alphabet (class A and C) and Tesla. In total, these companies accounted for over 20% of the market capitalization of the S&P 500.

[12] With the caveat that the equal weight strategy will do poorly if FAANG stocks rally and dominate the cap weighted index for the next 30 years.

# Appendix

## A Windowed Inflation Filter

**Algorithm A.1:** Pseudocode window inflation filter

```
Data:
   CPI[i]; i = 1,...,N /* CPI Index */
   Cutoff /* High inflation cutoff:  annualized */
   Δt /* CPI index time interval */
   K /* smoothing window size */
Result: Flag[i]; i = 1,...,N /* = 1 high inflation month; = 0 otherwise */
/* initialization */
Flag[i] =0; i = 1,...,N;
for i = 1,...,N − K do
   if log(CPI[i + K]/CPI[i])/(K ∗ Δt) > Cutoff then
      for j = 0,...,K do
         Flag[i+j] = 1 ;
      end
   end
end
```

## B Bootstrap Algorithm

Algorithm B.1 presents pseudocode for the stationary block bootstrap. See Ni et al. (2022) for more discussion concerning this algorithm. Note that the index must be converted to a series of returns before applying the bootstrap.

**Algorithm B.1:** Pseudocode for stationary block bootstrap

```
/* initialization                                                         */
bootstrap_samples = [ ];
/* loop until the total number of required samples are reached            */
while True do
    /* choose random starting index in [1,...,N], N is the index of the last
       historical sample                                                  */
    index = UniformRandom( 1, N );
    /* actual blocksize follows a shifted geometric distribution with expected
       value of exp_block_size                                            */
    blocksize = GeometricRandom( 1 / exp_block_size );
    for i = 0; i < blocksize; i = i + 1 do
        /* if the chosen block exceeds the range of the historical data array,
           do a circular bootstrap                                        */
        if index + i > N then
            bootstrap_samples.append( historical_data[ index + i - N ] );
        else
            bootstrap_samples.append( historical_data[ index + i ] );
        end
        if bootstrap_samples.len() == number_required then
            return bootstrap_samples;
        end
    end
end
```

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