Equal Weight vs Capitalization Weight Indexes

Peter A. Forsyth*

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1 Introduction

Nowadays, many retail customers (e.g. yours truly) buy index exchange traded funds (ETFs). A good portfolio for a Canadian investor, for example, could be constructed using

- A domestic index portfolio (TSX 60)
- A US stock index (S&P 500)
- An *ex-North America* index
- A domestic bond index

You can do just fine with these basic building blocks. You don't need exposure to cryptocurrencies, NFTs or alternative assets. Bear in mind the wise words of Walter Bagehot, Editor of the *Economist*, 1861-1877.

> *"The history of the trade cycle had taught me that a period of a low rate of return on investments inexorably leads toward irresponsible investment...People won't take 2% and cannot bear a loss of income. Instead, they invest their careful savings in something impossible - a canal to Kamchatka, a railway to Watchet, a plan for animating the Dead Sea."*

Of course, the trick is to try to optimally allocate wealth between these four indexes. A simple strategy is to choose constant weights, and rebalance to those weights at yearly or quarterly intervals.

Most standard ETFs use a capitalization (cap) weighted index. In other words, the proportion of each stock in the index is just the market capitalization of that stock divided by the total stock market capitalization. If there are no complications such as companies going bankrupt, or new companies being added to the index, then an advantage of a cap weighted index is that once it is set up, very little trading is required (e.g. reinvestment of dividends).

But there are many other possible ways to construct an index. We are going to look at an equal weighted index, compared to the usual cap weighted index. An equal weighted index simply invests a equal amount in each stock in the index. Of course, as stock prices change, the weighting rapidly becomes unbalanced. The index must be rebalanced periodically, usually at monthly intervals. This involves selling the winners and buying the losers, so this a fundamentally contrarian strategy, and is the exact opposite of a momentum strategy.

There are two interesting papers on this topic:

1. *"Why does an equal-weighted portfolio outperform value-and price-weighted portfolios?"* Y. Plyakha, R. Uppal, G. Vilkov, (2012) SSRN 2724535.

*Cheriton School of Computer Science, University of Waterloo, Waterloo, Ontario, Canada, N2L 3G1, paforsyt@uwaterloo.ca

2. *"Why has the equal weight portfolio underperformed and what can we do about it?* B. H. Tljaard and E. Mare, *Quantitative Finance* 21:11 (2021)

Paper [1] (Plyakha et al, 2012) suggests that equal weight indexes outperform cap weighted indexes over the long term. However, paper [2] (Tljaard et al, 2021) does not disagree, but suggests that the equal weight index may underperform for short periods, in particular, the last decade.

1.1 Stochastic Dominance

If you are already familiar with the concept of stochastic dominance, skip to the next section.

How do we compare the performance of various investment strategies, or indexes? Simplistic measures are things like mean, variance and Sharpe Ratio. However, these are just a few summary statistics of investment performance. We would like to look at the entire probability distribution function for each strategy.

Suppose we have an investment strategy, which, starting at time zero, generates wealth W at time T. Of course, in the real world, W is a random variable, with probability density $p(W)$. The cumulative distribution function $F(W)$ is given by

$$F(W) \quad = \quad \int_{-\infty}^{W} p(W')\ dW' \ . \tag{1.1}$$

If W_T is a possible value of wealth at time T, then we can interpret the CDF as

$$Prob(W_T < W) \quad = \quad F(W), \tag{1.2}$$

that is, $F(W)$ is the probability that we end up with less than W dollars. Suppose we would like to obtain a final wealth of W^* dollars. Then, we would like $F(W^*)$ to be as small as possible, i.e. the probability of ending up with less than W^* is very small, which is what we want.

Figure 1.1 shows the cumulative distribution functions of the final wealth W, for two investment strategies, A and B, which we denote by $F_A(W)$ and $F_B(W)$. Consider the point on the x-axis $W = 10,000$. We can see that for strategy A, the probability of obtaining less than $10,000$ is 0.6, while for strategy B, the probability of obtaining less than $10,000$ is 0.86. Hence, if we want to have a strategy which minimizes the probability of obtaining less than $10,000$, we would prefer strategy A. However, note that for Figure 1.1 we have that

$$F_A(W) \leq F_B(W) \ ; \ -\infty \leq W \leq \infty \tag{1.3}$$

and there is at least one point $\widetilde{W}$ such that $F_A(\widetilde{W}) < F_B(\widetilde{W})$, i.e. a point where equation (1.3) holds with strict inequality. In fact, in Figure 1.1, the strict inequality holds for many points. So, we can repeat the argument we went through for $W = 10,000$ for every value of W along the x-axis. In other words, for every value of W, strategy A has a smaller probability of ending up with less than W compared to strategy B.

In this case, we say that strategy A stochastically dominates (in the first order sense) strategy B. Any reasonable investor (i.e. with any reasonable utility function) would always prefer strategy A to strategy B. Note that this criteria is based on the entire distribution function, not just a few summary statistics. First order stochastic dominance is easy to spot from the CDFs. If the CDF of strategy A always plots at or below the CDF of strategy B, then A dominates B.

However, in practice, given two reasonable strategies, it is rare to find that one strategy dominates another. Often the CDFs cross at various points. For example, Figure 1.2 shows CDFs for various strategies. Each of these strategies is *reasonable*, yet no strategy strictly stochastically dominates another strategy, which would be typical.

This leads us to the definition of partial stochastic dominance.[1] We say that strategy A partially dominates strategy B if

$$F_A(W) \quad \leq \quad F_B(W) \ ; \ -\infty \leq W \leq W^* \ . \tag{1.4}$$

[1]see *On the Distribution of Terminal Wealth under Dynamic Mean-Variance Optimal Investment Strategies*, P. M. van Staden, D-M. Dang, P. A. Forsyth, SIAM Journal on Financial Mathematics 12:2 (2021) 566–603.

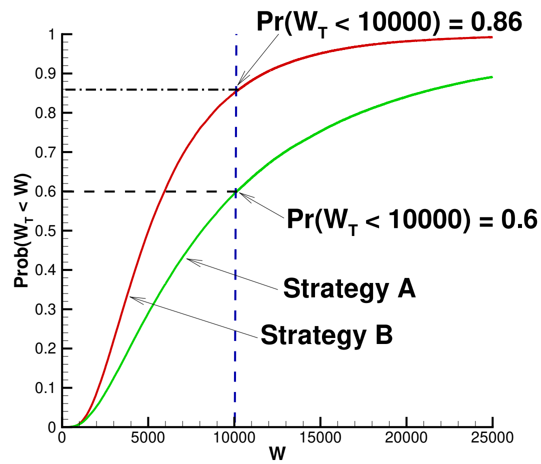


Figure 1.1: Illustration of first order stochastic dominance. Cumulative distribution functions for two strategies, in terms of final wealth W. Strategy A stochastically dominates strategy B (in the first order sense). Any reasonable investor would always prefer strategy A.

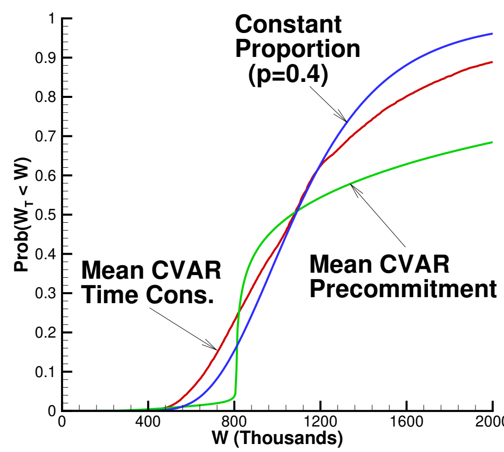


Figure 1.2: Comparison of CDFs for various strategies. Note that in general, the CDFs cross, and we do not observe strict stochastic dominance. See P. A. Forsyth, "Multi-period mean-CVAR asset allocation: is it advantageous to be time consistent?," SIAM Journal on Financial Mathematics 11:2 (2020) 358-384.

This a practical criteria: if W^* is quite large (i.e. we would be fabulously wealthy if $W_T > W^*$), then we really don't care if strategy A underperforms B for these large wealth values, as long as A outperforms B for all values of $W_T \leq W^*$. We are very happy with any amount larger than W^*.

We can generalize this a bit more. We can say that A partially dominates B if

$$F_A(W) \quad \leq \quad F_B(W) \ ; \ \hat{W} \leq W \leq W^* \ . \tag{1.5}$$

We have just explained why the upper bound can be a reasonable criteria for partial stochastic dominance. However, at first sight it seems foolhardy to also apply a lower bound criteria $\hat{W}$. This means that we allow A to have a worse performance than B in the left tail, where results are bad.

However, sometimes this can be reasonable. Suppose we start off with an initial wealth of 1000, and that strategy A has $Med[W_T] = 10,000$, which looks quite good. Suppose condition (1.5) is satisfied with $\hat{W} = 10$ and $W^* = 50,000$. We don't care what happens if we start off with 1000 and end up with more than $50,000$.

However, A underperforms B in the left tail where $W_T < 10$. These are the scenarios where essentially everything has turned bad. We started with 1000, and after years of investing, we are left with only 10. Basically, we are bankrupt. Under strategy A, perhaps our probability of having, say, five dollars or less, is twice the probability of strategy B having five dollars or less. So, strategy A has twice the probability of being in this extreme left tail compared to strategy B. This sounds bad. But this is all peanuts compared to our original stake of 1000. So, perhaps in this case, we don't care about the extreme left tail either. The fact that in these bad cases, we are more likely to end up with two cents in our pocket from strategy B compared to one cent from strategy A is cold comfort.

1.2 Expected Shortfall

If $F(W)$ is the cumulative distribution function of W_T, with density $p(W)$, then suppose we select a value $\mathbb{W}$ such that

$$F(\mathbb{W}) = \int_{-\infty}^{\mathbb{W}} p(W')\, dW' = \alpha \ ; \ 0 < \alpha < 1 \tag{1.6}$$

We define the expected shortfall at level α ($ES(\alpha)$) as

$$ES(\alpha) = \frac{\int_{-\infty}^{\mathbb{W}} W'\, p(W')\, dW'}{\alpha} \ , \tag{1.7}$$

which is just the mean of the worst α fraction of outcomes. Note that this is essentially the negative of the Conditional Value at Risk (CVAR). Since W is the final wealth, a larger value of $ES(\alpha)$ is better. This is measure of left tail risk.

1.3 Data

We are going to compare an equal weight index with a capitalization based index. We will use the Center for Research in Securities (CRSP) capitalization weighted total return index (includes all dividends and distributions).[2] Similarly, we will also use the CRSP equal weighted index. The equal weighted index has an equal amount invested in all stocks in the index. This index is rebalanced back to equal weights monthly. The CRSP data covers the range 1926.00 - 2022.00. We use the monthly data for both series. For the bond indexes, we consider two cases: a constant maturity 10-year US treasury index,[3] and a short term 30 day US T-bill index.[4] These indexes are also for monthly data. We adjust all the indexes for inflation, by dividing by the CPI (also from CRSP). In other words, all investments are in real dollars.

Note that the 10-year Treasury index was constructed by (i) buying a 10 year treasury at the start of every month, (ii) selling the 10-year treasury at the start of the next month, and then (iii) immediately buying a fresh 10 year treasury. The return over the month includes interest and capital gains/losses, all in constant dollars.

It is interesting to observe that the number of stocks in the equal weighted CRSP index was 496 in 1926, while as of the end of 2021, the number of stocks in the equal weighted index is 8598.

[2] More specifically, results presented here were calculated based on data from Historical Indexes, ©2022 Center for Research in Security Prices (CRSP), The University of Chicago Booth School of Business. Wharton Research Data Services (WRDS) was used in preparing this article. This service and the data available thereon constitute valuable intellectual property and trade secrets of WRDS and/or its third-party suppliers.

[3] The 10-year Treasury index was constructed from monthly returns from CRSP back to 1941. The data for 1926-1941 were interpolated from annual returns in Homer and Sylla, "A history of interest rates," (2005).

[4] The 30 day T-bill index was obtained from CRSP

1.4 Scenario

In order to model a realistic scenario, we consider an investor who has a portfolio of 60% in the equity index, and 40% in a bond index. The investor rebalances the investments in the stock and bond index, back to the 60 : 40 ratio once a year. The investor starts with 1000, with no injection or withdrawals of cash over the investment horizon, and we examine the statistics of the terminal wealth W_T at $T = 30$ years. We will consider a long term 30 year investment strategy, since this would be typical of an investor saving for retirement.

1.5 Bootstrap results

We will compare the equal weighted index to the capitalization weighted index, using stationary block bootstrap resampling of the data for the period 1926.00-2022.00. [5] This approach for evaluating investment policies is entirely data driven [6]. We use the standard algorithm[7] for estimating the optimal expected blocksize for each series. The bootstrap procedure concatenates randomly selected blocks of data to account for possible serial correlation.[8] We use the average of the optimal expected blocksize for each individual time series, and then simultaneously draw samples from the stock and bond indexes.

Figure 1.3 compares using the cap weighted CRSP index compared to the equal weighted index, in terms of the CDFs for both strategies. The bond index is a constant maturity 10 year US treasury index. Figure 1.3 shows that the portfolio with the equal weighted stock index clearly stochastically dominates the portfolio with the capitalization weighted stock index. This is a somewhat surprising result, but is consistent with some previous work.

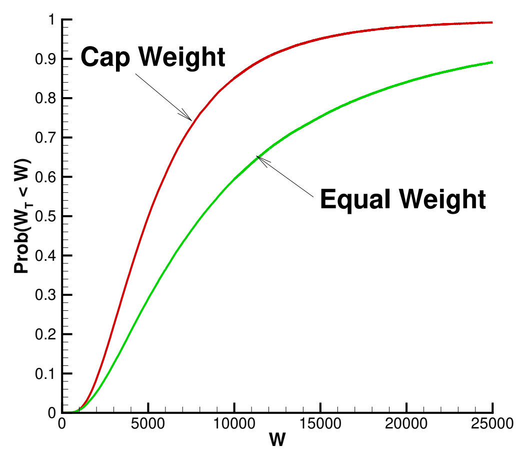


Figure 1.3: Cumulative distribution functions (CDFs) for cap weighted and equal weighted indexes. Initial stake $W_0 = 1000$, no cash injections for withdrawals, $T = 30$ years. Block bootstrap resampling, expected blocksize 0.5 years. 60% stocks, 40% bonds, rebalanced annually. Bond index: constant maturity 10 year US treasuries. Stock index: CRSP capitalization weighted or CRSP equal weighted index. Data range 1926.00 - 2022.00. All indexes are deflated by the CPI. 10^6 resamples.

[5] D. Politis and J. Romano, "The Stationary Bootstrap", Journal of the American Statistical Association, 89 (1994) 1303-1313.

[6] H. Dichtl, W. Drobetz, M. Wambach, "Testing rebalancing strategies for stock-bond portfolios across different asset allocations," Applied Economics 48 (2016) 772-788.

[7] A. Patton, D. Politis, H. White, "Correction to: automatic block-length selection for the dependent bootstrap," Econometric Reviews 28 (2009) 372-375.

[8] For detailed pseudocode for block bootstrap resampling, see P. Forsyth, K. Vetzal, "Optimal Asset Allocation for Retirement Saving: Deterministic vs. Time Consistent Adaptive Strategies," Applied Mathematical Finance 26:1 (2019) 1-37.

	Equal Weight	Cap Weight
$E[W_T]$	12546	6237
$std[W_T]$	15664	4639
$Med[W_T]$	8122	5016
ES(5%)	1431	1280
$\mathcal{S}$	0.73	1.13

Table 1.1: Initial stake $W_0 = 1000$, no cash injections for withdrawals, $T = 30$ years. Block bootstrap resampling, expected blocksize 0.5 years. 60% stocks, 40% bonds, rebalanced annually. Bond index: constant maturity 10 year US treasuries. Stock index: CRSP capitalization weighted or CRSP equal weighted index. Data range 1926.00 - 2022.00. All indexes are deflated by the CPI. ES(5%) is the mean of the worst 5% of the outcomes. $\mathcal{S}$ is the continuously compounded Sharpe ratio, as defined in equation (1.8).

	Equal Weight	Cap Weight
$E[W_T]$	9058	4877
$std[W_T]$	8653	3091
$Med[W_T]$	6726	4135
ES(5%)	1484	1184
$\mathcal{S}$	.93	1.25

Table 1.2: Initial stake $W_0 = 1000$, no cash injections for withdrawals, $T = 30$ years. Block bootstrap resampling, expected blocksize 2.0 years. 60% stocks, 40% bonds, rebalanced annually. Bond index: 30 day US T-bills. Stock index: CRSP capitalization weighted or CRSP equal weighted index. Data range 1926.00 - 2022.00. All indexes are deflated by the CPI. ES(5%) is the mean of the worst 5% of the outcomes. $\mathcal{S}$ is the continuously compounded Sharpe ratio, as defined in equation (1.8).

The cumulative, continuously compounded Sharpe ratio is defined as

$$\mathcal{S} = \frac{E[W_T] - W_0 e^{rT}}{std[W_T]} \ , \tag{1.8}$$

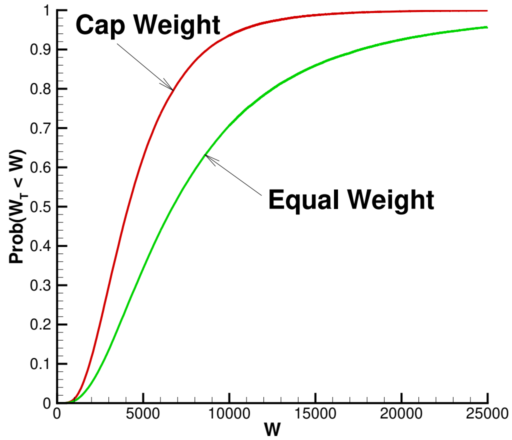
where r is the risk-free rate, and in our case $T = 30$. Note that equation (1.8) should not be confused with the usual *instantaneous* Sharpe ratio, which is estimated using average short term arithmetic returns. In our case, since we carry out annual rebalancing, the equity fraction between rebalancing dates is not constant, so the usual procedure does not make sense.

Since we are examining real (adjusted for inflation) quantities, there is no real risk-free asset. However, the annualized real return of a 30-day T-bill over the entire historical period is approximately zero. Using this value of $r = 0$ in equation (1.8) we obtain the results shown in Table 1.1. Even though we have seen that the equal weight portfolio dominates the cap weighted portfolio, the continuously compounded Sharpe ratio for the equal weight 60:40 portfolio is actually worse (smaller) than for the 60:40 cap weighted portfolio.

This is essentially because standard deviation is a poor measure of risk, since it penalizes upside as well as downside. The equal weighted index generates a much bigger right skew compared with using the cap weighted index. Most investors would prefer a large right skew, which is penalized by the standard deviation. The left tail risk, as measured by ES(5%) is larger (better) for the equal weighted portfolio. This is consistent with our intuition from Figure 1.3.

Figure 1.4 and Table 1.2 show similar results for 60:40 portfolios, this time the bond index is based on 30-day T-bills. The results are qualitatively similar to the 10 year treasury case.

However, more recent papers suggest that equal weighted portfolios have underperformed. Figure 1.5 shows the bootstrap results, for the 60:40 equal and cap weighted portfolios, using data in the range 1980.00-



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2022.00. The bond index is based on 30 day T-Bills. In this case, the use of the equal weighted portfolio still stochastically dominates the cap weighted portfolio, but the effect is very small. Table 1.3 gives some additional statistics for the past two decades. It appears that the equal weighted portfolio underperforms for the past decade.

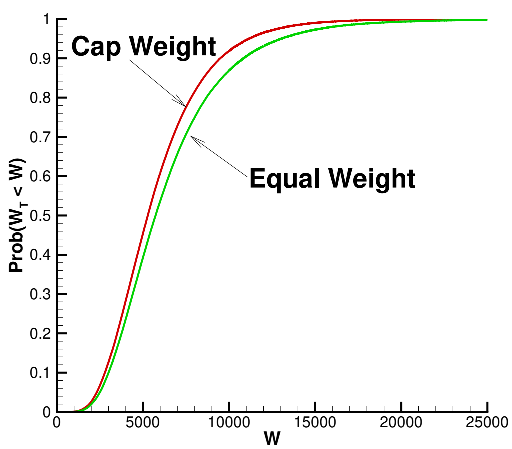

Figure 1.5: Cumulative distribution functions (CDFs) for cap weighted and equal weighted indexes. Initial stake $W_0 = 1000$, no cash injections for withdrawals, $T = 30$ years. Block bootstrap resampling, expected blocksize 2.0 years. 60% stocks, 40% bonds, rebalanced annually. Bond index: 30 day US T-bills. Stock index: CRSP capitalization weighted or CRSP equal weighted index. Data range 1980.00 - 2022.00. All indexes are deflated by the CPI. 10^6 resamples.

Data series	Annualized Return 2012.00-2022.00	Annualized Return 2002.00-2022.00
Real CRSP cap weighted index	11.79%	6.72%
Real CRSP equal-weighted index	8.78%	7.58%
Real 10 year US treasury index	1.61%	2.33%
Real 30-day US T-bill index	-1.58%	-1.33%

Table 1.3: Recent historical real returns of CRSP equal and cap weighted stock indexes (100% stocks, no bonds). Also, the return of the 10 year treasury index and the 30-day T-bill index (0% stocks, 100% bonds).

How can we explain this? The Tljaard et al (2021) paper suggests that this is a short term effect. However, an equal weighted index will obviously put a lot more weight on small cap stocks, compared to a capitalization weighted index. Years ago, it was noted that small cap stocks seemed to perform better than you would expect. In fact, small cap portfolios were one of the original factors in the Fama-French three factor model of stock returns.[9]

A reasonable explanation for the small cap effect would be that analysts ignored small cap stocks, so nobody was really following them. This allowed skilled stock pickers to do better than you would expect (a market imperfection).[10]. The usual assumption in academic finance is the *no arbitrage* principle, i.e. no free lunch. If the equal weight index outperformance is simply due to the small cap effect, then, once everyone knows about it, the effect will disappear (i.e. arbitraged away). So, if we look at the bootstrap results from 1980.00, we see a much smaller effect than on the entire data set from 1926.00. The classic small cap effect paper was published in 1981. Is this just a coincidence?

1.6 What about taxes and distributions?

Note that the above indexes assumed that any distributions (e.g. dividends) were immediately reinvested. In addition, the equal weighted portfolios are re-balanced monthly (back to equal weights). The dividends and capital gains are usually subject to taxation. This is clearly not a problem if the portfolio is held in a tax advantaged account, such as an RRSP or TFSA in Canada, or a 401(k) in the US.

However, in a taxable account, how realistic is the assumption that all taxes on gains are deferred? In Canada, there are swap based index ETFs, which do not distribute any dividends. Effectively, all dividends are reinvested and tax deferred until the ETF is sold.

It is also interesting to note that in the US, it is perfectly legal for ETFs to use *heartbeat transactions*, which essentially defer taxes on any gains from rebalancing.[11].

1.7 Summary: equal weight vs. cap weight

Bootstrap resampling using long term data 1926.00 - 2022.00, shows that equal weighted indexes stochastically dominate cap weighted indexes. However, if we repeat the experiment, this time using data in the range 1980:00 - 2022:00, the effect almost disappears. Is this a permanent effect, or are the last 40 years anomalous? Certainly, the last 40 years have had very low inflation, and declining interest rates, which is historically unusual, and perhaps provided a tailwind for large cap tech stocks. On the other hand, maybe the small cap effect has actually disappeared, now that there is a large literature on this.

It is, of course, tempting to ignore long term historical data, and assume that only more recent data is relevant. This can be dangerous

[9] E. F. Fama, K. R. French, "The cross section of expected stock returns," Journal of Finance 47:2 (1992) 427-465.

[10] see *"The relationship between return and market value of common stocks"*, R. Banz, Journal of Financial Economics 9:1 (1981) 3-18.

[11] `https://www.bloomberg.com/graphics/2019-etf-tax-dodge-lets-investors-save-big/`

> *The four most expensive words in the English language are: “This time it’s different.”* (Sir John Templeton)

Maybe the answer is to equal weight the cap weight and equal weighted indexes?