Bankers, Bonuses and Busts

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Market-Meltdown 2008-9

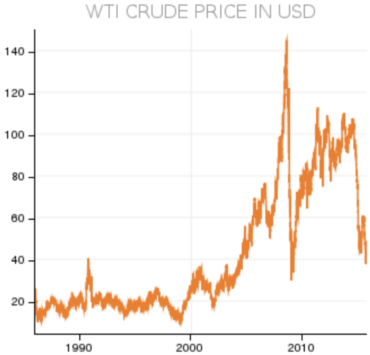
It is commonly believed that poor modelling of *financial derivatives* caused the market meltdown of 2008-9.

- Should we ban complex financial instruments?
- What caused the problems?

Motiviation: Why do we need financial derivatives?

Suppose that you want to build an oil sands plant in Alberta

- Recent data suggests that you need to get \$75/bbl to break even on a new oil sands plant
- You are concerned that oil prices could fall below \$75/bbl
- As insurance, you buy put options on oil



A put option gives you the right (but not the obligation) to sell oil at $K = 75$, at time T in the future.

- Terminology: K is the strike price, T is the expiry time.

Put Options: Financial Insurance

Let $S(t)$ be the spot price of crude at time t

- If $S(T) < K$, you exercise the option, and effectively get paid \$75/bbl for the oil you produce
- If $S(T) > K$, then you do not exercise the option, and you simply sell oil on the open market

This contract pays off at $t = T$

$$\begin{aligned} \text{Payoff} &= \max(K - S, 0) \\ T &= \text{ Expiry time} \\ K &= \text{ strike price} \end{aligned}$$

By buying a put option, you have locked in the minimum price you will receive for your oil.

Derivative Contract

This is a simple example of a derivative contract, (a put option), also known as a contingent claim.

- In this example, the underlying asset S is the price of crude oil
- Other possibilities: oil, electricity, TSX index, defaults of subprime loans, etc.
- Used by firms and individuals to *hedge* risk (financial insurance)

$$\text{Call Payoff } = \max(S-K,0) \;\; ; \quad \text{Put Payoff } = \max(K-S,0)$$
$$\text{Straddle Payoff } = \max(S-K,0)+\max(K-S,0) \;\; ; \quad \text{Etc } \ldots$$

What is it worth?

So, how much is a bank going to charge me for a put option?

- The bank is not in the business of taking risk.
- The bank would like to make money, regardless of what happens to the price of the underlying asset S
- The bank sells me the option, at some price today, and hedges its exposure to me

Price Process

Let S be the price of an underlying asset (i.e. TSX index).

- A standard model for the evolution of S through time is Geometric Brownian Motion (GBM) (Black-Scholes-Merton 1973)

$$
\begin{aligned}
\frac{dS}{S} &= \mu\ dt + \sigma\phi\sqrt{dt} \\
& \mu = \text{ drift rate,} \\
& \sigma = \text{ volatility,} \\
& \phi = \text{ random draw from a} \\
& \qquad \text{standard normal distribution}
\end{aligned}
$$

Monte Carlo Paths: GBM

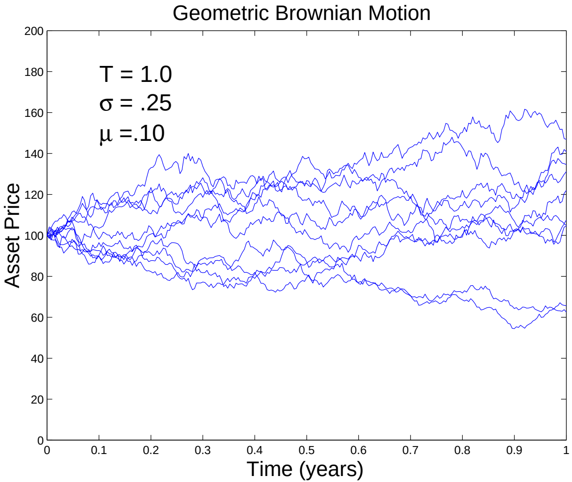


Figure : Ten realizations of possible random paths.

Efficient Markets

Note: we have not assumed that the prices in the market are rational

- We are assuming that the prices are stochastic, i.e. unpredictable

Why is this reasonable?

- If the price process was predictable, I (and many others) would eventually be able to determine any patterns in the data
- In particular, I would be rich, and I would not be speaking to you today

Sadly, you will observe that I am speaking to you today.

Hedging

Let $V(S,t)$ be the value at any time of the option.

• The bank will sell the option to me for $V(S,t=0)$ today, and construct the following portfolio Π ($-tive \ \rightarrow$ short)

$$
\begin{aligned}
\Pi &= -V + eS + B \\
& \quad V = \text{ value of option} \\
& \quad S = \text{ price of underlying} \\
& \quad B = \text{ cash in risk free money market account} \\
& \quad e = \text{ units of underlying}
\end{aligned}
$$

Hedging (Black-Scholes-Merton (1973); Scholes-Merton; Nobel Prize (1997))

Hedging objective, determine e (units of underlying), so that

$$\begin{aligned} d\Pi &= \Pi(t+dt) - \Pi(t) = 0 \\ &\Pi = -V + eS + B \end{aligned}$$

Stochastic calculus, etc.: value of option $V(S,t)$ given by the solution to the Black-Scholes Partial Differential Equation (PDE).

$$V_t + \frac{\sigma^2 S^2}{2} V_{SS} + SV_S - rV = 0$$
$$r = \text{ risk free interest rate}$$

The optimal number of units in the underlying asset turns out to be $e = \frac{\partial V}{\partial S}$. This choice for e minimizes the hedge error.

Hedging

So, what does the bank do?

- Solve the BS equation, sell me the option today for V(S, 0).
- Construct the portfolio Π, by buying $e(S, t = 0)$ units at price S, and depositing B in the money market account
- As $t \to t + dt$, $S \to S + dS$ (randomly)
- At $t + dt$ bank rebalances the hedge, by buying/selling underlying so that $e(S + dS, t + dt) = V_S$
- Hedging portfolio is *Delta Neutral*

Delta Hedging

This strategy is called *Delta Hedging*.

- Note that this is a dynamic strategy (rebalanced at infinitesimal intervals)
- It is self-financing, i.e. once the bank collects cash from selling option, no further injection of cash into Π is required.
- At time T in the future, the bank liquidates Π, pays off short option position, at zero gain/loss, *regardless of random path followed by* S.

No-arbitrage Price

The value of the option $V(S,t)$ is the *no-arbitrage* value

- $V(S, t = 0)$ is the cost of setting up the portfolio Π at $t = 0$
- The value of the option is *not* the discounted expected payoff

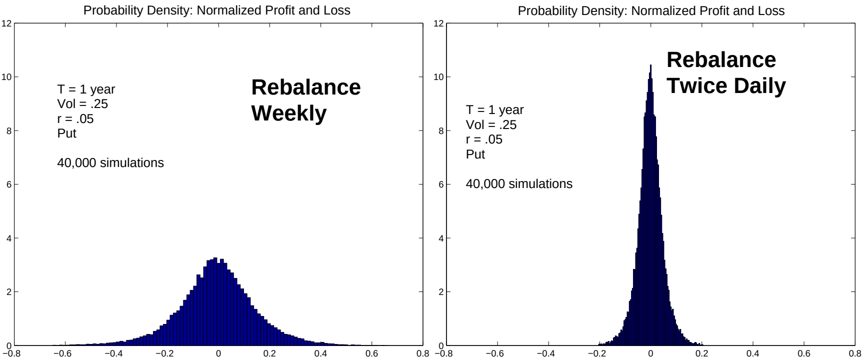
Does this actually work? Can we construct a hedge so we can't lose, regardless of the random path followed by S?

- Simulate a random price path, along path, carry out delta hedge at finite rebalancing times (not a perfect hedge)
- Liquidate portfolio at expiry, pay off option holder, record normalized profit and loss ($P\&L$)

$$\text{Normalized } P\&L \;=\; \frac{P\&L}{\text{Initial Option Premium}} \tag{1}$$

- Repeat this many times

Monte Carlo Delta Hedge Simulation: Normalized Profit and Loss



As rebalancing interval $\to 0$, standard deviation of relative $P\&L \to 0$.

Hedging: One stochastic path

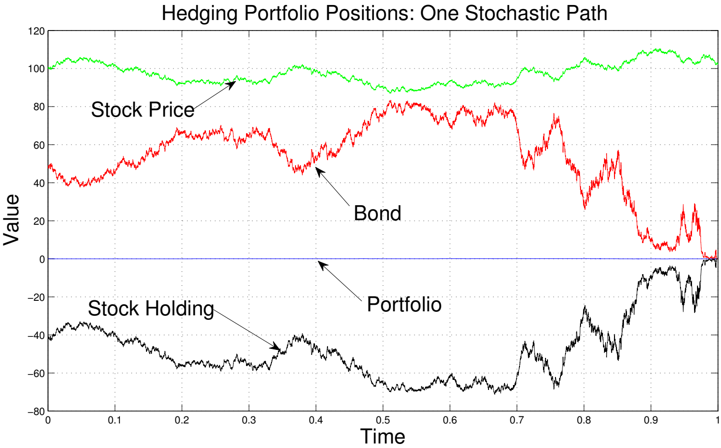


Figure : One year put, $K = 100$, $r = .02$, $\sigma = .20$, $\mu = .10$, $S_0 = 100$, rebalanced 10000 times.

Reality

- Nobody hedges at infinitesimal intervals, volatility $\neq$ *const*., GBM not a perfect model
- Bank wants to make a profit

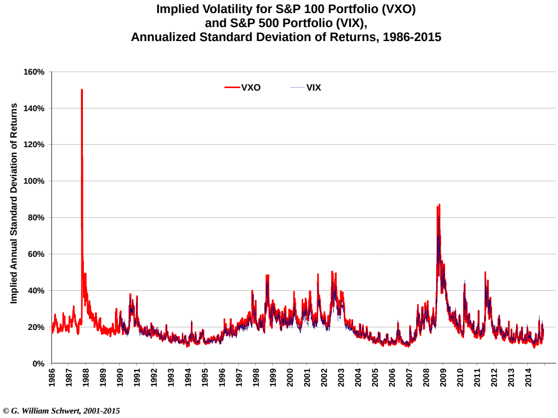
$$
\begin{aligned}
V_{buy}^{market} &= V(S,t)^{model} + \epsilon_1 + \epsilon_2 \\
V_{sell}^{market} &= V(S,t)^{model} - \epsilon_1 - \epsilon_2
\end{aligned}
$$

$$\epsilon_1 = \text{ profit}$$

$$\epsilon_2 = \text{ compensation for imperfect hedge}$$

$$V_{buy}^{market} - V_{sell}^{market} = \text{ bid-ask spread}$$

What's Wrong with GBM? Volatility is *not* constant!



So, volatility is actually stochastic. This can be modelled and hedged fairly easily.

$\hookrightarrow$ But there is a worse problem.

What's More Wrong with GBM?

- Equity return data suggests market has *jumps* in addition to GBM
  - Sudden discontinuous changes in price
- Risk management: if we don't hedge the jumps
  - We are exposed to sudden, large losses
- Conventional Wisdom
  - Jumps are too expensive to hedge
- So, most people ignore the jumps, i.e. market crashes
- But, it seems that we get a financial crisis occurring about once every ten years
- Does it make sense to ignore these events?
- Jumps are also known as **Black Swans** (see the book with the same title by Nassim Taleb)

Example: A Drug Company

- If this were Geometric Brownian Motion, this pattern would occur only once in a time long compared to the age of the Universe

“ Sigh: No Forest”

Figure : SinoForest

- June 2, 2011 *Muddy Waters Research* releases report claiming accounting irregularities at SinoForest.

US CRSP Index monthly log returns 1926-2015

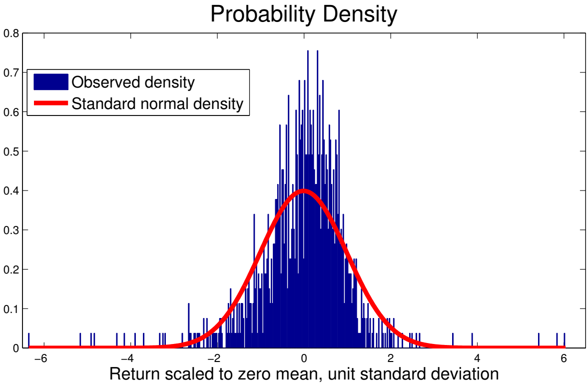
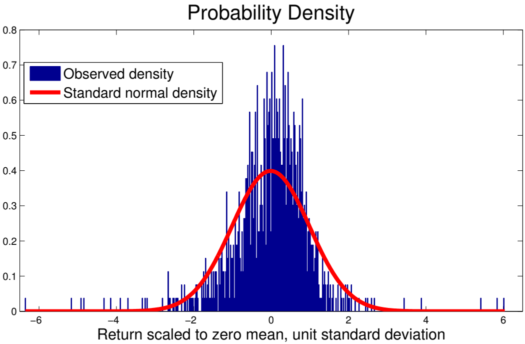


Figure : Scaled to zero mean and unit standard deviation, standard normal distribution also shown

US CRSP Index monthly log returns 1926-2015

- Extreme events more likely than simple GBM
- Higher peak, fatter tail than normal distribution
- Plots of EuroStoxx, TSX, etc. all look similar



A Better Model: Jump Diffusion (1976)

$$\frac{dS}{S} = \overbrace{\mu \; dt + \sigma\phi\sqrt{dt}}^{GBM} + \overbrace{(J-1)dq}^{Jumps}$$

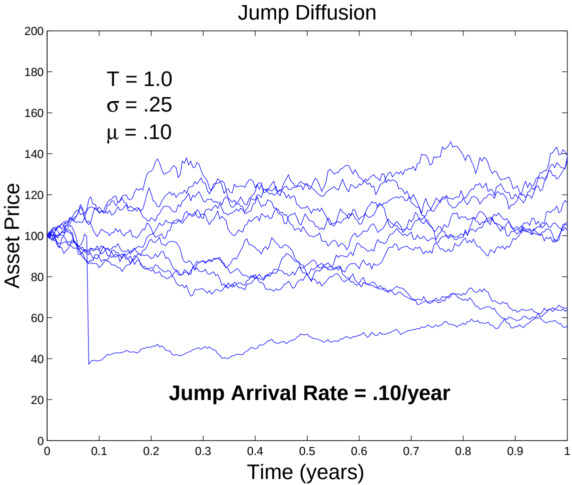
$$dq = \begin{cases} 0 & \text{with probability } 1 - \lambda dt \\ 1 & \text{with probability } \lambda dt, \end{cases}$$

$\lambda =$ mean arrival rate of Poisson jumps; $S \to JS$

$J =$ Random jump size.

- GBM plus jumps (jump diffusion)
- When a jump occurs, $S \to JS$, where J is also random
- This simulates a sudden market crash

Monte Carlo Paths



The arrival rate of the Poisson jump process is .1 per year. Most of the time, the asset follows GBM. In only one of ten stochastic paths, in any given year, can we expect a crash.

Option Price $V = V(S, t)$ Given by PIDE/Variational Inequality

$$
\begin{aligned}
\min(V_\tau - \mathcal{L}V - \lambda\mathcal{I}V, V - V^*) &= 0 \qquad \text{American} \\
V_\tau = \mathcal{L}V - \lambda\mathcal{I}V & \qquad \text{European}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}V &\equiv \frac{\sigma^2}{2} S^2 V_{SS} + (r - \lambda\kappa) S V_S - (r + \lambda) V \\
\mathcal{I}V &\equiv \int_0^\infty V(SJ) g^{\mathbb{Q}}(J)\, dJ \\
T &= \text{maturity date}, \quad \kappa = E^{\mathbb{Q}}[J - 1], \quad V^* = \text{ payoff },\\
r &= \text{risk free rate }, \quad \tau = T - t, \\
g^{\mathbb{Q}}(J) &= \text{probability density function of the jump amplitude } J
\end{aligned}
$$

- An American option can be exercised at any time in $[0, T]$
- American price given by solution of a Hamilton Jacobi Bellman (HJB) PIDE

Technical Note: Option Price, Jump Diffusion

Option price: Hamilton Jacobi Bellman (HJB) Variational Inequality (VI)

- HJB VI is non-linear
- In general, solutions to the HJB VI are non-smooth
- What does it mean to solve a differential equation where *solution* is not differentiable?
- Need to consider the idea of *viscosity solution* [1]
- Need to construct numerical algorithms which guarantee convergence to the viscosity solution
- Easy to construct examples where seemingly reasonable numerical methods converge to the wrong (i.e. non-viscosity) solution

[1] Pierre-Louis Lions was awarded the Fields medal in 1994 for his work on viscosity solutions

The Trader Strategy

- Suppose the real world follows jump diffusion
- You work at a bank, and convince your boss that you don't have to worry about jumps (e.g. sudden increase in default rates).
- You sell put options, and use the simple delta hedge.
- You use the constant volatility Black Scholes equation to compute the delta hedge (e.g. back out implied volatility from traded options)
- What happens? (Note: this is the wrong model)

First: recall $P\&L$ for delta hedging, real process GBM

Recall Delta Hedging: Real Process GBM

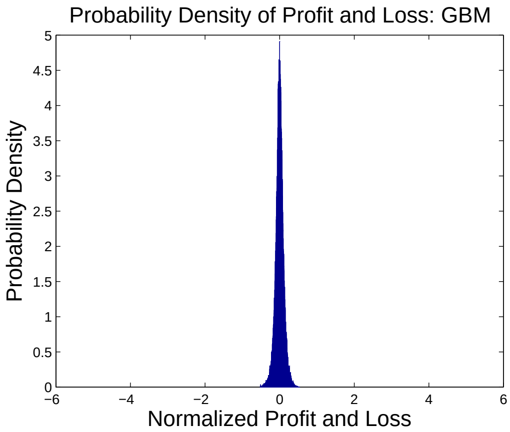
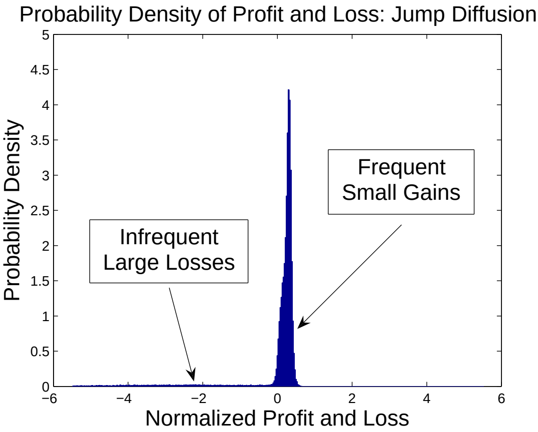


Figure : Normalized profit and loss, daily rebalancing

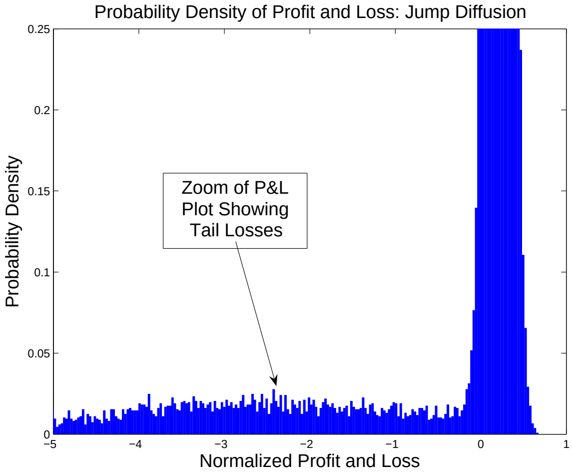
- I have rescaled the previous plot (we will see why later).

Delta Hedging: Real Process Jump Diffusion (daily rebalancing)



- The peak of the probability density is shifted to the right.
- There is a big *tail* of enormous losses to the left.

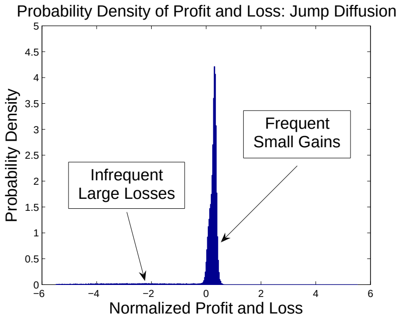
But look at the tail!



- The 95% Conditional Value at Risk (CVAR) is about -4.0.
- In other words, the average of the worst 5% of the tail losses is about 400% of the option premium.

What should happen?

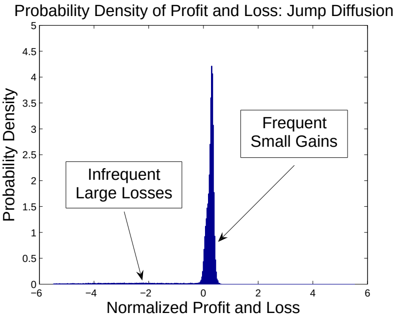
- Most of the time, if you are only going to delta hedge, you make small gains
- You should put these gains in a reserve fund, so that you will be able to use the reserve when the jump (i.e. a crash) occurs



Current data suggests that crashes occur about once every ten years.

What actually happens?

- If you ignore the tail risk in our toy example
- The mean of the (tail-less) distribution is $+.20$
- Most of the time you make a profit.

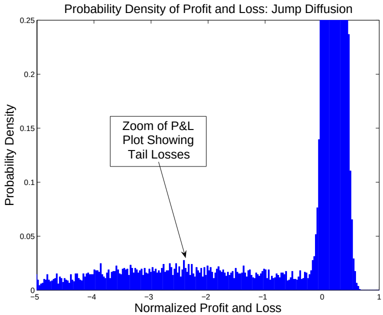


You use leverage (i.e. borrowing) to magnify these small gains.

Your boss is happy, your shareholders are happy, you get paid a large bonus.

Everybody is happy: until . . .

- This goes on for some time (on average, ten years in the above example)
- Then a crash occurs
- You retire rich, shareholders and taxpayers pay for the losses
- Sounds like a perfect strategy!



Of course, you claim that the jump was a totally unforeseen event, should happen only once every 10^6 years, etc. (assuming GBM)

Technical Note: How can we hedge jumps?

In fact, a trading strategy can be devised which hedges against crashes.

Problems

1. We don't know when a jump will occur
2. We don't know how big the jump will be

Solution:

- Design a hedging strategy which does not require good estimates of jump frequency or jump size
- Involves solving the HJB VI numerically, and solving an optimization problem

Hedging the Jumps

- See *"Dynamic Hedging Under Jump Diffusion with Transaction Costs,"* Kennedy, Forsyth, Vetzal, **Operations Research** Vol. 57 (2009) 541-559; *"Calibration and hedging under jump diffusion,"* He, Kennedy, Coleman, Forsyth, Li, Vetzal, **Review of Derivatives Research** Vol 9 (2006) 1-35.
- This strategy is robust to bad estimates of jump frequency and jump size distribution
- But this would cost a bit more (price of options would be about 10% higher).
- This would cut into the (apparent) short term profits.
- So, nobody does this

The current problems

The recent large scale bailouts of financial firms were caused by

- Poor hedging of credit derivatives
- No allowance made for sudden changes in model parameters
    - $\rightarrow$ i.e. no hedging of jumps!

Charles Prince, ex CEO of Citigroup[2], explaining why Citigroup was aggressively involved in credit derivatives (2007)

> "As long as the music is playing, you've got to get up and dance."

---

[2]Prince received a $105 Million exit payment. Citi shares have lost 80% of their value

Warning bells have been sounding for years

From a paper we wrote in 2002, about complex products sold by insurance companies

> *"If one adopts the no-arbitrage perspective...in many cases these contracts appear to be significantly underpriced, in the sense that the current deferred fees being charged are insufficient to establish a dynamic hedge for providing the guarantee...This finding might raise concerns at institutions writing such contracts."*

Windcliff, Forsyth, LeRoux, Vetzal, North American Actuarial J., 6 (2002) 107-125

What Happened?

As described in a Globe and Mail article (Report on Business, December 2, 2008, "Manulife, in red, raises new equity,"), one of the large Canadian insurance companies, Manulife, posted a large mark-to-market writedown to account for losses associated with these segregated fund guarantees. From the Globe and Mail Streetwise Blog, November 7, 2008

> *"Concerns that the market selloff will translate into massive future losses at Canada's largest insurer sent Manulife shares reeling last month. Those concerns were a result of Manulife's strategy of not fully hedging products[3] such as annuities and segregated funds, which promise investors income no matter what markets do."*

[3] In 2004, CEO Dominic D'Alessandro decided not to hedge these products. The board of Manulife awarded him $25 Million in 2009 for *extraordinary performance.*

And the pain continues...

Financial Post, August 6, 2010

> *"Two years after the market meltdown exposed a critical weakness at Manulife Financial Corp., the life insurance giant continues to be plagued by market gyrations that contributed to a record loss of $2.4-billion in the second quarter...*
>
> *Manulife is also increasing prices where possible, hedging a greater proportion of the variable annuity businesses as markets rise, and contemplating hedging interest rates, executives said yesterday."*

Globe and Mail, November 8, 2012

> *"...insurer took a $1-billion charge that stemmed largely from a change in behaviour by its customers ...variable annuities..."*

Are Mathematicians/Computer Scientists to blame for the meltdown of 2008?

- Black Scholes model, constant volatility (1973)
- Jump diffusion (i.e. Black Swan) (1976)
- Local volatility $\sigma = \sigma(S, t)$ (1990)
- Stochastic volatility plus jumps (1995)

Most common models used by banks: local volatility, maybe stochastic volatility, **no jumps**

Why no jumps?

> "Jump models are too hard to calibrate"
> "Jump models take too long to compute."
> "Jump models give prices which are too high."
> "It is too hard to hedge with jump models."

Reality: ignoring jumps $\Rightarrow$ bonus generating machine (disguises risk)

What is going to stop another meltdown from happening again?

- It is in the interests of bank executives and traders to underestimate long term risks.
- They get paid big dollars to be optimistic.
- Did you know that Citigroup (under various names) has been bailed out by the US government four times in the last eighty years (New York Times, November 1, 2009).
- It is easy to devise trading strategies which give distributions similar to that shown in our toy example.
- One could argue that bankers are simply maximizing their own welfare by taking advantage of the *Yellen Put*.
- We are likely to see another banking sector meltdown in $5 - 10$ years.