Banks, Bonuses and Busts

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Introduction

Derivative contracts are used by firms and individuals as protection against risk—they act as financial insurance.

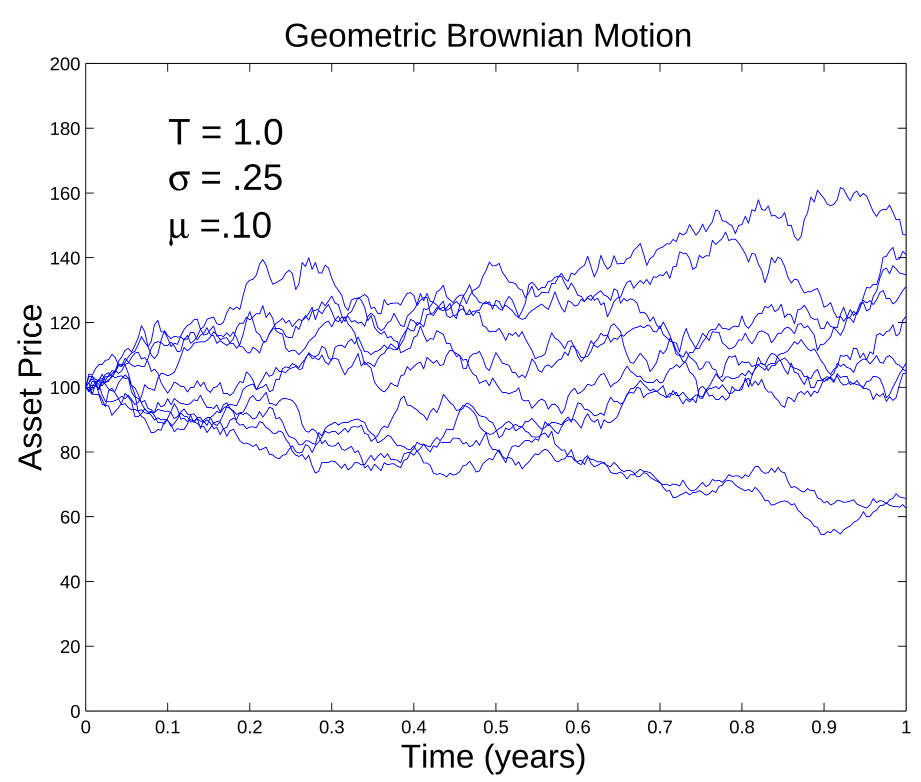
For example, a put option gives its holder the right, but not the obligation, to sell some underlying asset at a future time T for a fixed price K, regardless of its price S_T at that time. Payoff: $\max(K - S_T, 0)$. A put option can be viewed as protection against a market collapse.

The issuer (e.g. a bank) needs (1) to determine the price at which to sell the option; and (2) a way to *hedge* their risk associated with the contract.

Geometric Brownian Motion (GBM)

Many studies have shown that financial markets are efficient, in the sense that financial assets follow a random process. Note that an efficient market does not mean that prices are rational!

A basic model for the evolution of the asset price S through time is geometric Brownian motion (GBM). The plot below shows ten possible realizations for an asset following GBM. Nobody can say which of these possible stochastic paths will actually occur.

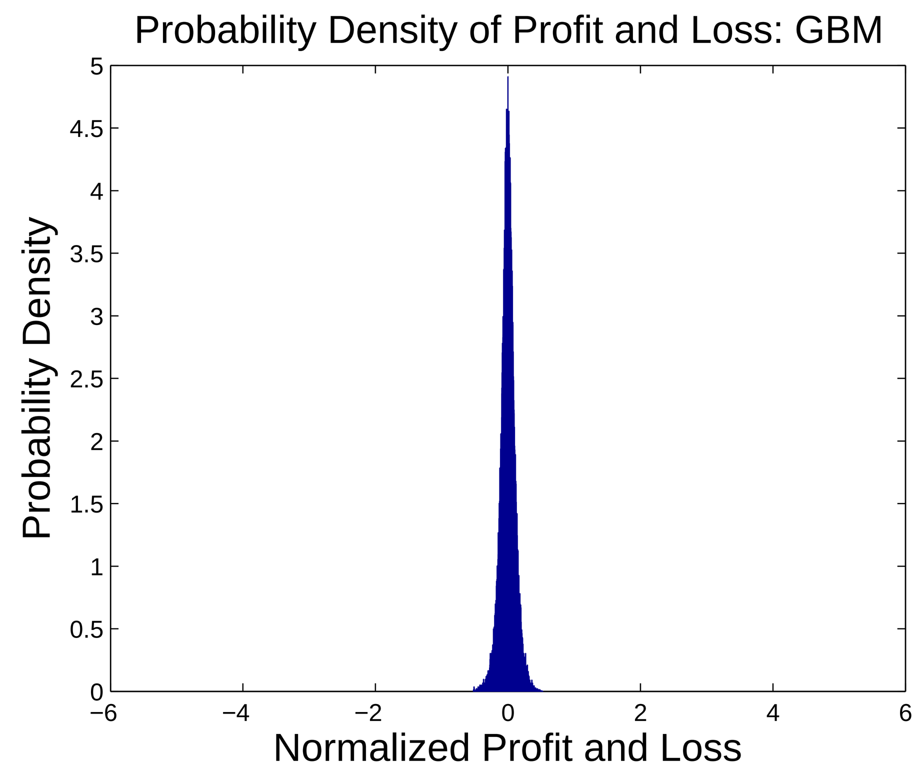


Delta Hedging

If the real world assets follow GBM, then the bank can sell the option for the Black-Scholes price, and hedge their position through a simple strategy known as *delta hedging*.

The plot below shows the results of a simulation of delta hedging of a put option, with daily rebalancing. The y-axis represents the probability density and the x-axis shows the corresponding normalized profit and loss (actual $P\&L$ divided by the initial option premium).

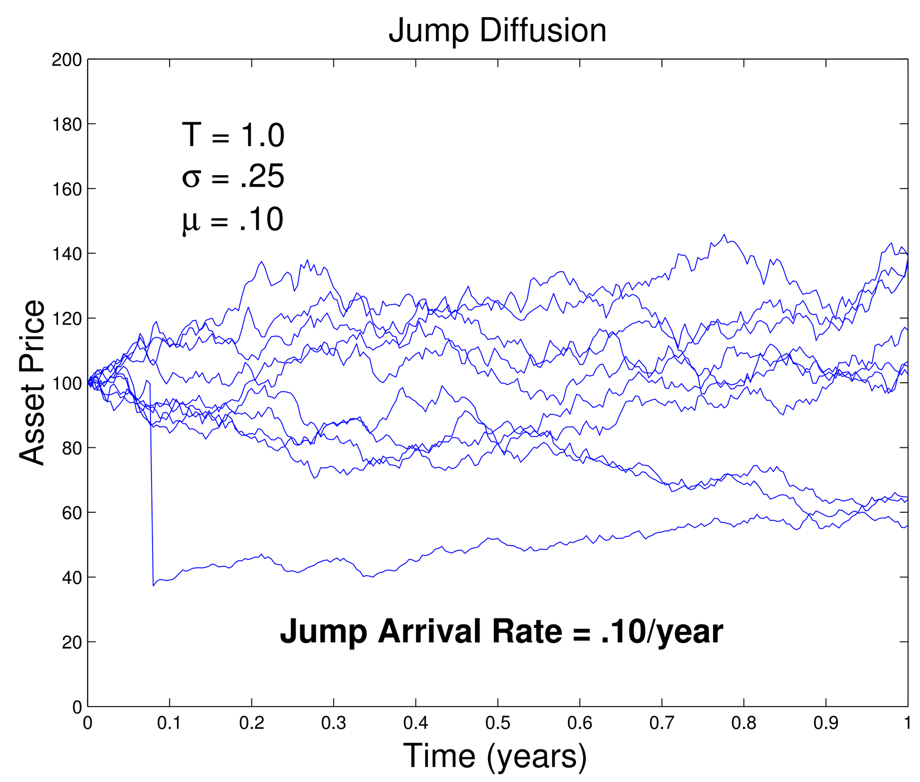
The mean of this distribution is zero. The bank charges a small mark-up over the Black-Scholes price, in order to make a small profit at low-risk.



The Jump-Diffusion Model

But GBM is not realistic. A better model would be to assume GBM is punctuated by infrequent market crashes. Based on recent evidence, such market crashes seem to occur about once every ten years.

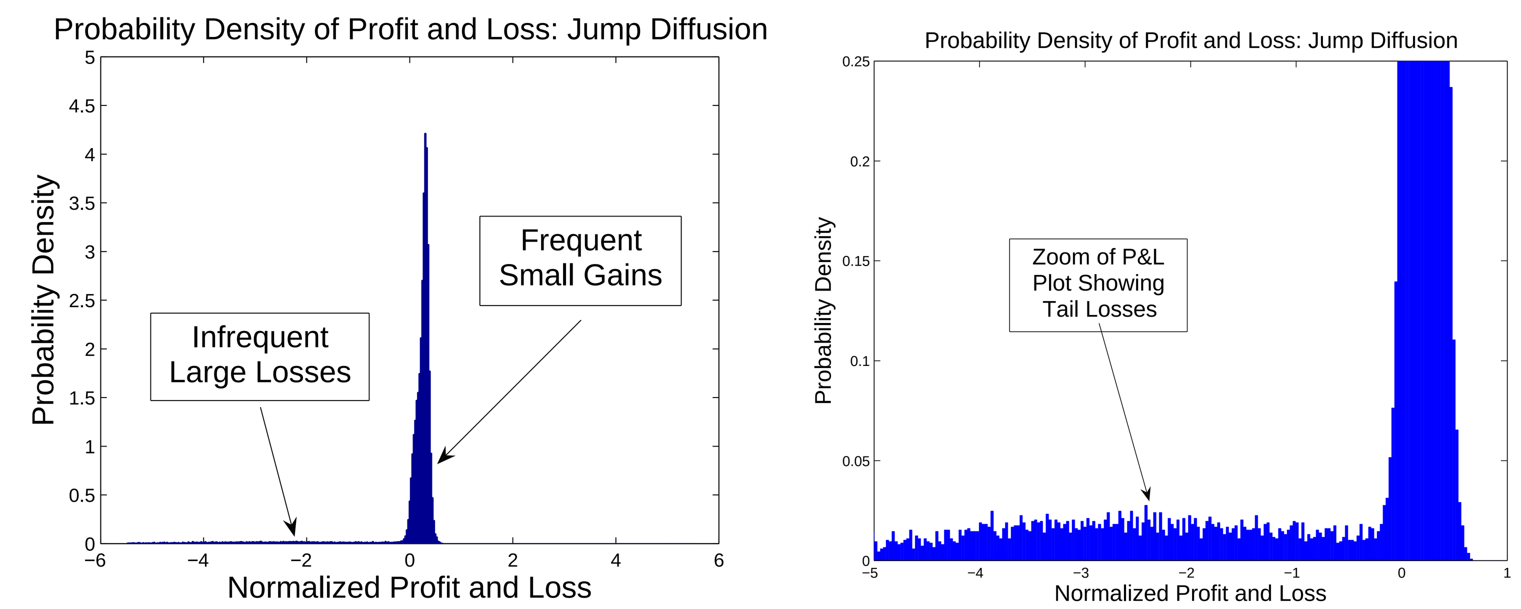
The plot below shows ten possible realizations of an asset which follows a jump diffusion.



The arrival rate of the Poisson jump process is $.1$ per year. Most the time, the asset follows GBM. In only one of ten stochastic paths, in any given year, can we expect a crash.

The Trader Strategy

Suppose the real world follows jump diffusion, but you work at a bank, and convince your boss that you don't have to worry about jumps. You sell put options, and use the simple delta hedge. What happens?



The peak of the probability density is shifted to the right. But there is a big *tail* of enormous losses to the left. The 95% Conditional Value at Risk (CVAR) is about -4.0. In other words, 5% of the time, the average of the tail losses is about 400% of the option premium.

However, if you ignore the tail risk, the mean of this distribution is $+.20$. Most of the time you make a profit. Your boss is happy, your shareholders are happy, you get paid a large bonus.

This goes on for some time (on average, ten years in the above example) then a crash occurs, you retire rich, the shareholders and the taxpayers pay for the losses. Sounds like a good strategy!

In fact, a trading strategy can be devised which hedges against crashes. See *"Dynamic Hedging Under Jump Diffusion with Transaction Costs,"* Kennedy, Forsyth, Vetzal, **Operations Research** Vol. 57 (2009) 541-559. But this would cost more, and cut into the (apparent) short term profits.

What is going to stop this from happening again?

It is in the interests of bank executives and traders to underestimate long term risks. They get paid big dollars to be optimistic. Did you know that Citigroup (under various names) has beeen bailed out by the US government four times in the last eighty years (New York Times, November 1, 2009).

It is easy to devise trading strategies which give distributions similar to that shown above. Unfortunately, we are likely to see another banking sector meltdown in another few years.