You can't go faster than the speed of light and the no-arbitrage condition

Peter A. Forsyth*

May 7, 2018

About 25 years ago, I received an email from a prospective graduate student. He informed me that he had an undergraduate degree in Computer Science, an MBA, and was currently working in the financial sector. He wanted me to be his supervisor and write a thesis on *Computational Finance*. I had never heard of this topic.

However, being a professor, I was not phased by my lack of knowledge, and agreed to meet with the prospective student. To research the topic, I walked to the library (I physically went to the library and searched through the card catalogue, nobody does that anymore). I found two books on option pricing and hedging, which I proceeded to skim through over a few days.

Even though I had no background in finance, I found that the math concepts were very familiar. There were two basic financial concepts

Concept 1: Stock prices follow a random process. Over the long run (i.e. more than ten years), stocks generally go up in price. However, over the short run (five years or less), stock returns are simply noise.

Concept 2: The no-arbitrage condition applies.

First a word about concept (1). As a first approximation, stocks were assumed to follow a constant parameter Geometric Brownian Motion (GBM). This is really just the same thing as assuming that stock returns follow the familiar bell curve, also known as a normal distribution.[1] However, the authors of these books were quick to point out that in practice, nobody used GBM, and you had to adjust for changing volatility and deviations from the GBM model.

The last few chapters of these books described how to use more sophisticated return models (i.e. different random processes). This was 25 years ago. Remember this when anyone tells you that academic finance is all wrong since normal returns are assumed.

By the way, concept (1) does not require an assumption that investors behave rationally. In fact, one can even use random processes which have bubbles built-in (e.g. regime switching or jump diffusion models). Concept (1) is simply based on statistical observations of market prices.

The really interesting concept is *no-arbitrage.* An example will illustrate. Suppose you have two investment choices

A An investment in a CDIC insured bank account with interest rate r. Since this is sure fire, we will call this a *risk-free* investment, and r the risk-free rate of return.

*David R. Cheriton School of Computer Science, University of Waterloo, Waterloo ON, Canada N2L 3G1, paforsyt@uwaterloo.ca, +1 519 888 4567 ext. 34415.

[1] "Options, futures and other derivatives," John Hull, Pearson, New York (10th edition); "Option pricing: mathematical models and computation," P. Wilmott, J. Dewynne, S. Howison, Oxford Financial Press, Oxford. The student who emailed me ended up doing a PhD. Jeff Dewynne was the external examiner.

B An investment portfolio which is risk-free, and which has a return greater than the risk-free rate of return r.

Where will you put your money? Well, of course you will invest in strategy B. So will everyone else. Since this portfolio is in high demand, the cost of the portfolio will rise, and the return will fall very quickly to the risk-free rate r.

This is the basis of the principle of no-arbitrage. There are several ways to state this

- There is no free lunch.
- It is not possible that a riskless investment yields a return higher than the risk-free rate.
- Without risk there is no abnormal reward.
- There is no trading strategy, such that an investor, starting with zero initial wealth[2], has a non-zero probability of gain, with zero probability of loss.

We have seen with our example why this property holds. Arbitrage opportunities may exist, but they are rapidly exploited, and they disappear very quickly, so that for typical investors, arbitrage opportunities are not available.

For many years, I taught a course in computational finance at the University of Waterloo. I would tell students in Computer Science that no knowledge of finance was required: I would teach them all they needed to know about finance in the first week of lectures (professors are not known for their humility). After all, it is not rocket science. The most enjoyable lecture was on no-arbitrage. I would go over a simple example, which had an arbitrage opportunity, and I could see the lights go on all over the class. I would always end the class with my well worn joke.

> A professor and a student are walking down the street. There is a \$50 dollar bill on the sidewalk. The student turns to the professor, and asks *"Aren't you going to pick that up?"* The professor responds *"By no-arbitrage, there is no point. By the time I bend over to pick this up, somebody else will have got it before me."* The student then reaches over and picks up the \$50 dollar bill. The professor then exclaims *"See, I told you so. The no-arbitrage condition is true."*

So, the principle of no-arbitrage holds because, arbitrage opportunities do exist, but only for tiny periods of time, and then they are gone.

Going back to our two basic concepts: stocks follow a random process and no-arbitrage. From these two ideas, an amazing amount of useful results follow. In essence, these principles are as basic to finance as the *"you can't go faster than the speed of light"* principle is in physics.

Over the next few months, I plan on writing a number of blogs on investing and finance. I have had an inside look at what goes on in the financial sector, mainly through my former Masters and PhD students who are working in finance. Now that I am retired, and not chasing after grants and contracts, I am free to write with candour about the financial sector.

[2] It is assumed that the investor can borrow at the risk-free rate, which is close to true for a large bank