

Some Thoughts on Current Issues in Wealth Management and Retirement Planning

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Who am I?

I graduated with a PhD in Applied Mathematics in 1979

- Worked in the oil patch in Calgary, optimal depletion of oil reservoirs
- Founder of a software company
- After enduring two oil price crashes, I gave up
- 1987-2016: professor of Computer Science, Waterloo

For the last 30 years

- Research in Computational Finance
 - Pricing, hedging exotic options, variable annuities
 - Optimal trade execution
 - Optimal accumulation of retirement savings
 - After I retired in 2016 → optimal decumulation
- Over 20 years, taught $\simeq 1000$ 4th year students, Computer Science Computational Finance course

About Me: II

- Member, UofW DB pension plan investment committee
- Supervised over 60 MMath, PhD students
 - Quants in New York, London, Toronto
- Editor-in-chief, Journal of Computational Finance (2008-2013)
- Research grants/contracts: Neuberger Berman, Royal Bank of Canada, Scotiabank, Credit Suisse, SunLife, Tata Consultancy Services, Morgan Stanley, ITO33, Bell Canada, Global Risk Institute
- Board member of Aquanty (local tech company, models risk of climate change, industrial activity on water resources)

Target Date Funds: a triumph of marketing over math

Accumulation phase of saving for retirement

Standard suggestion: glide path

- Take risk when you are young, less risk near retirement
- Example

$$\text{Percentage in equities} = 110 - (\text{your age})$$

- Many target date ETFs with variants on this rule

However: many empirical papers cast doubt on the glide path idea

A theoretical result

Theorem 1 (Ineffectiveness of glide path strategies)

Assuming bla, bla, given any deterministic glide path, there exists a constant weight strategy which

(i) *has the same expected terminal wealth;*

(i)

$$\overbrace{\text{Variance}[\text{terminal wealth}]}^{\text{constant weight}} \leq \overbrace{\text{Variance}[\text{terminal wealth}]}^{\text{glide path}} \quad (1)$$

Proof.

See Forsyth, Vetzal, Appl Math Fin (2019)



Of course, many assumptions are made about stock market returns, etc.

What about real life?

Bootstrap simulations

Assume

- Investor injects cash yearly into a Vanguard 40 year target date fund¹
- Investor injects the same yearly cash flows into a retirement account, rebalanced yearly to constant weight in stocks
- Bootstrap resample² historical data to generate many paths
 - No assumptions made about market returns
 - Real historical data used

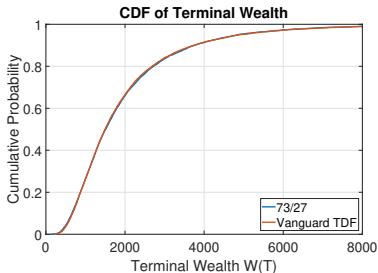
¹Starts at 90% equities, reduces to 50% at year 40

²We use block bootstrap resampling, which takes into account serial correlation in returns, i.e. not i.i.d.

Cumulative Distribution Function (CDF)

The CDF gives all the information about the entire probability distribution

- Not just mean, variance, etc.³



The two curves basically overlap

- This means that mean, variance, skewness, CVAR, VAR, etc are all identical

³Constant weight 73% equities, \simeq time average of glide path allocation

Glide Path ETFs: why do they exist?

Intuition (from theoretical result)

- When you are young, you have a small amount in your account
 - Glidepath: large returns on small amount
- When you are near retirement you expect to have a large amount in your account
 - Glidepath: small returns on large amount
- This all just averages out
 - ⇒ There is a constant weight strategy with same final wealth distribution

So, we have

- Theoretical analysis, backed up by empirical tests⁴

Why are target date ETFs still being sold?

⁴Not just us, other authors have reproduced these results.

DC plan decumulation: “Nastiest, hardest problem in finance”

Defined Benefit (DB) pension plans are disappearing

→ Replaced by Defined Contribution (DC) plans

Upon retirement, the DC plan holder has to

- Select an investment strategy
- Determine how much to withdraw each year

The DC plan holder is now exposed to

- Longevity risk
- Market risk

The Four per Cent Rule

Based on rolling 30-year historical periods, Bengen (1994) showed:

A retiree who

- Invested in a portfolio of 50% bonds, 50% stocks (US), rebalanced annually
- Withdrew 4% of initial capital (adjusted for inflation) annually
 - Would never have run out of cash, over any rolling 30-year period (from 1926)

Criticism

- Simplistic asset allocation strategy
- Simplistic withdrawal strategy
- Rolling 30 year periods contain large overlaps
 - Underestimates risk of portfolio depletion
 - Bootstrap simulations 10-15% risk of depleting savings

Modern Tontines (Individual Tontine Account)

DC members make irrevocable investment in pooled fund

- If the member dies during a year, their assets distributed to the other members as mortality credits
- The sharing rule is actuarially fair, i.e. expected gain from participating is zero
 - If you are older or have more assets
 - You get a larger share of mortality credits

Advantage:

- Transparent, peer-to-peer risk sharing
- Can decide your own investment strategy

Mortality Credits: Example

CPM2014 Life table: theoretical mortality credit

- Yearly credit for 76-year old male: 2%
- Yearly credit for 86-year old male: 8%
- Yearly credit for 96-year old male: 33%

Example:

- 86 year-old: total wealth W in account (December 31, 2023)
- If he is still alive on January 1, 2024
 - He will earn mortality credit of $0.08W$

Theoretical credit depends only on

- Your age
- Your account balance

Does not depend on how anyone else invests, their age, or their account balances!⁵.

⁵This is a counterintuitive result, see R. Fullmer, Tontines: a practitioner's guide, CFA Institute (2019), and references therein

How does this work in practice?

Define for each year:

$$\left\{ \begin{array}{l} \text{Group} \\ \text{Gain} \end{array} \right\} = G = \frac{\text{Total actual assets forfeited due to deaths in pool}}{\text{Total expected mortality credits for survivors}}$$

$$\begin{aligned} &\text{Actual mortality credit for each investor} \\ &= \text{Theoretical Credit} \times G \end{aligned}$$

This ensures that total mortality credits handed out

→ Equals total assets forfeited

Can show that $E[G] = 1$, $\text{Var}[G] \rightarrow 0$ if

- Pool is sufficiently large
- Diversity condition holds ⁶

⁶The expected total mortality credits must be large compared to any members expected credit. Simulations: perpetual pool size $\simeq 5,000$ -15,000.

Multi-period optimal stochastic control

Investor has access to two funds

- A broad stock market index fund
- A constant maturity bond index fund

The investor has two controls at each rebalancing time (yearly)

- The amount to withdraw
- The fraction of wealth in stocks

Constraints

- Minimum and maximum withdrawals specified
- No leverage, no shorting allowed

Objective Function

Find Pareto points which

- Maximize the total expected withdrawals over 30 year investment horizon (as in Bengen scenario)
- Minimize risk of running out of cash
 - Risk measure: Expected Shortfall (ES)
 - Mean of the worst 5% of outcomes
- Expected Shortfall is dollar number which can be directly compared to other assets.
 - E.g. real estate
- Expected Shortfall is a left tail measure, unlike, e.g. standard deviation

Scenario: all amounts indexed to inflation

- DC account at $t = 0$ (age 65) \$1,000K (one million)
 - Minimum withdrawal from DC account \$40K per year
 - Maximum withdrawal \$80K per year⁷
- No shorting, no leverage, annual rebalancing
- Retiree owns mortgage-free real estate
 - Hedge of last resort (reverse mortgage)⁸
- Fees: 50bps per year

Investment Horizon

- $T = 30$ years, i.e. from age 65 to 95

⁷Assume gov't benefits of 22K/year. Minimum income $\simeq 22K + 40K = 62K$ /year. Max income: $22K + 80K = 102K$ /year.

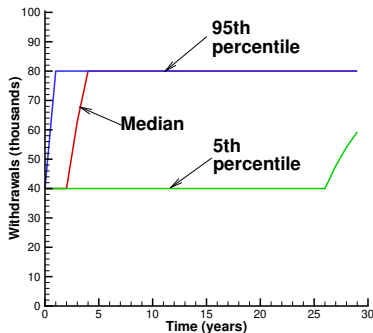
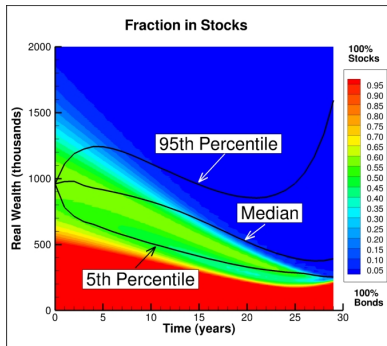
⁸I observe mental bucketing of real-estate by my fellow retirees.

Center for Research in Security Prices (CRSP) US

- Cap weighted index, all stocks on all major US exchanges
1926:1-2020:12
- US 30-day T-bill⁹
- Monthly data, inflation adjusted by CPI

⁹Also used 10-year treasuries, similar results

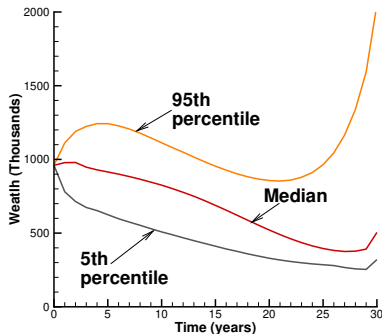
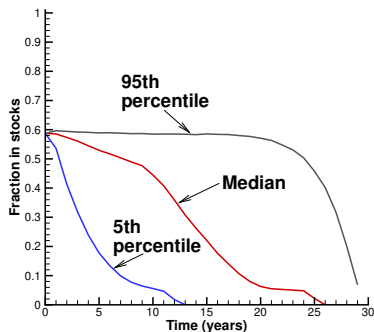
Optimal allocation controls (out-of-sample bootstrap simulations)



Median withdrawal control

- Withdraw at minimum (40K per year) for first few years
- Then, rapidly increase to 80K/year
- Minimizes sequence of return risk

Out of sample bootstrap tests



5th percentile wealth at $t = 30$ (age 95) $\simeq 300K$.

- Mortality credit earned at this age 33% per year (100K yearly income)

ES (mean of worst 5% of outcomes)

→ $ES = +200K$ at age 95

Tontine Summary

Pros:

- Average withdrawal $\simeq 7\%$ real of initial wealth
- Almost no probability of running out of cash
- Never withdraws less than Bengen rule
- Less risk than Bengen rule¹⁰
- Provider of tontine pool bears no risk \rightarrow minimal fees

Cons:

- Withdrawals can vary (but at least 4% per year)
- All wealth forfeited on death
 - "If you want more money when you are alive, you have to give up some when you are dead." (Moshe Milevsky)*
- Investor bears systematic mortality risk, market risk (in return for higher expected withdrawals)

¹⁰Bootstrap tests: Bengen 10-15% probability of negative wealth.


Conclusion

- Compared to option pricing, hedging, risk management
 - It appears that retirement planning is in the dark ages
 - No use of modern tools
- Retirement planning mantra “*Focus on long-term*”
 - But use of single period optimal policy (i.e. Sharpe ratio)
 - No use of multi-period optimal control
- It is time to use ideas such as optimal control, machine learning,¹¹ modern tontines
 - Optimal strategies for DC plan decumulation
 - Possible to include taxes, various income sources, personalized plans
 - Completely data-driven optimal control
- But, you have to do some work

¹¹Note that most work on machine learning in finance is nonsense, i.e. predicting the stock market

RESEARCH ARTICLE

Optimal performance of a tontine overlay subject to withdrawal constraints

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Abstract

We consider the holder of an individual tontine retirement account, with maximum and minimum withdrawal amounts (per year) specified. The tontine account holder initiates the account at age 65 and earns mortality credits while alive, but forfeits all wealth in the account upon death. The holder wants to maximize total withdrawals and minimize expected shortfall at the end of the retirement horizon of 30 years (i.e., it is assumed that the holder survives to age 95). The holder controls the amount withdrawn each year and the fraction of the retirement portfolio invested in stocks and bonds. The optimal controls are determined based on a parametric model fitted to almost a century of market data. The optimal control algorithm is based on dynamic programming and the solution of a partial integro differential equation (PIDE) using Fourier methods. The optimal strategy (based on the parametric model) is tested out of sample using stationary block bootstrap resampling of the historical data. In terms of an