Is Rebalancing Bad for Your Wealth?

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Motivation

Axiomatic in the wealth management industry: default strategy

- Retail investors should hold stock and bond index ETFs
- Rebalance to a constant weight in stocks (yearly or quarterly)
- For many years: 60% stocks, 40% bonds

However, I hope to demonstrate to you that this advice is dubious.

- In fact, it is difficult to argue that rebalancing is superior to *buy and hold*

I posted an op-ed paper on linkedin about this.

> *"It is good that Professor Forsyth has retired. He should not be in the classroom, telling his students that rebalancing is a bad idea."*

More precisely

Case 1

- A portfolio of 100 stocks
- Low pairwise correlation
- High volatility
- Investment horizon: 30 years

Case 2

- Portfolio: one stock index ETF and one bond index ETF
- Properties: historical US stock and bond indexes
- Investment horizon: 10 years

$\Rightarrow$ Case 2: is a typical retail investor

We will show that for a Case 2 investor

- *buy and hold* is arguably the best strategy

Buy and Hold: better than rebalancing?

This is not a new result

- Has been rediscovered many times

Small sample of literature:

- Perold and Sharpe (1988) FAJ
- Wise (1996) BAJ
- Dayanandan and Lam (2015)
- Hilliard and Hilliard (2018) RQFA
- Horn and Oehler (2020) JAM
- Bertrand and Prigent (2022) SSRN

So, I am not alone ☺

Simple Example: GBM

Stock index S:

$$\frac{dS}{S} = \mu\, dt + \sigma\, dZ$$

$\mu =$ arithmetic return

$\sigma =$ volatility

$dZ =$ increment of a Wiener process .

Bond index B:

$$dB = r\, B\, dt$$

Total wealth $W(t)$

$$W = S + B$$

Continuously rebalanced portfolio: constant fraction of wealth β in stocks

$$\frac{dW}{W} = \beta\left(\frac{dS}{S}\right) + (1-\beta)\left(\frac{dB}{B}\right)$$

GBM: exact solutions

The exact solution for $W(t)$

$$\frac{W(t)}{W(0)} = e^{(1-\beta)rt-\beta^2\sigma^2 t/2} e^{\beta\mu t+\beta\sigma(Z(t)-Z(0))}$$

$$Z(t)\text{: Brownian motion} \tag{1}$$

Exact solution for stock index is

$$\frac{S(t)}{S(0)} = e^{-\sigma^2 t/2} e^{\mu t+\sigma(Z(t)-Z(0))} \tag{2}$$

Using equations (1-2): eliminate $\left(e^{\mu t+\sigma(Z(t)-Z(0))}\right)^{\beta}$

$$W^{rebal}(t) = \text{Rebalanced portfolio}$$

$$\frac{W^{rebal}(t)}{W(0)} = e^{(1-\beta)rt+\beta(1-\beta)\sigma^2 t/2}\left(\frac{S(t)}{S(0)}\right)^{\beta} \tag{3}$$

GBM: exact solutions II

Expression for $W^{rebal}(t)$

$$\frac{W^{rebal}(t)}{W(0)} = e^{(1-\beta)rt} \exp(\overbrace{\beta(1-\beta)\sigma^2 t/2}^{rebalancing\ \ premium}) \left(\frac{S(t)}{S(0)}\right)^{\beta}$$

$$\text{if } 0 < \beta < 1 \Rightarrow (rebalancing\ \ premium) > 0$$

Alternative: buy and hold portfolio

- $\beta W^{bh}(0)$ in stocks $(t = 0)$
- $(1-\beta)W^{bh}(0)$ in bonds $(t = 0)$
- Never rebalanced

Then

$$W^{bh}(t) = \text{Buy and hold portfolio}$$

$$\frac{W^{bh}(t)}{W(0)} = \beta \left(\frac{S(t)}{S(0)}\right) + (1-\beta)e^{rt}$$

Rebalancing: a nonlinear strategy

$$\frac{W^{rebal}(t)}{W(0)} = e^{(1-\beta)rt+\beta(1-\beta)\sigma^2 t/2} \underbrace{\left(\frac{S(t)}{S(0)}\right)^{\beta}}_{\textit{nonlinear payoff}} \tag{4}$$

Derivative payoff

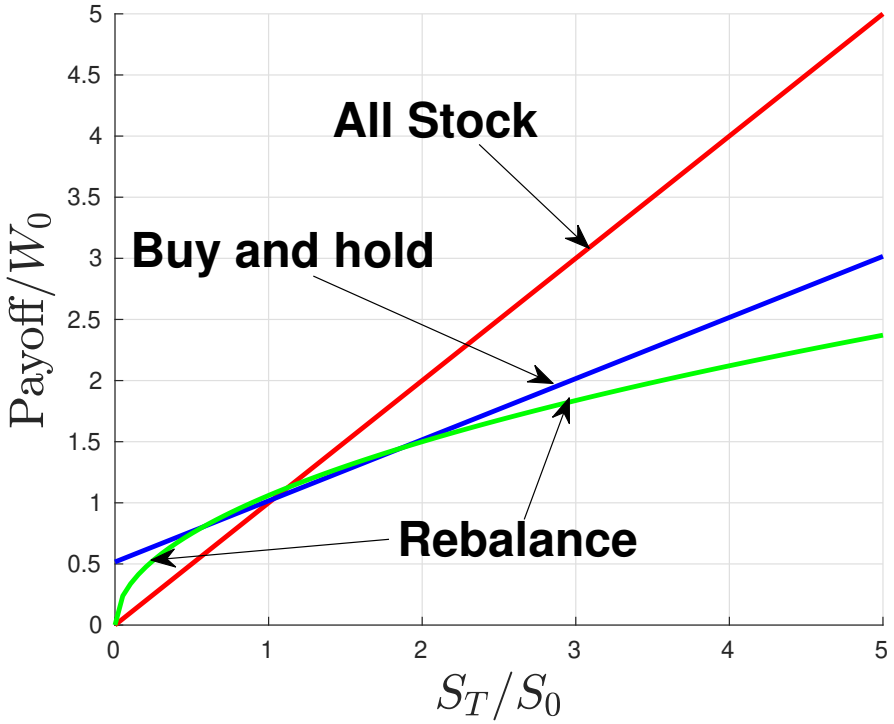
- Rebalancing is a dynamic trading strategy
- Produces a nonlinear payoff
  - $\beta = 0.5$ maximizes rebalancing premium
  - square root payoff
- Note equation (4) independent of drift μ

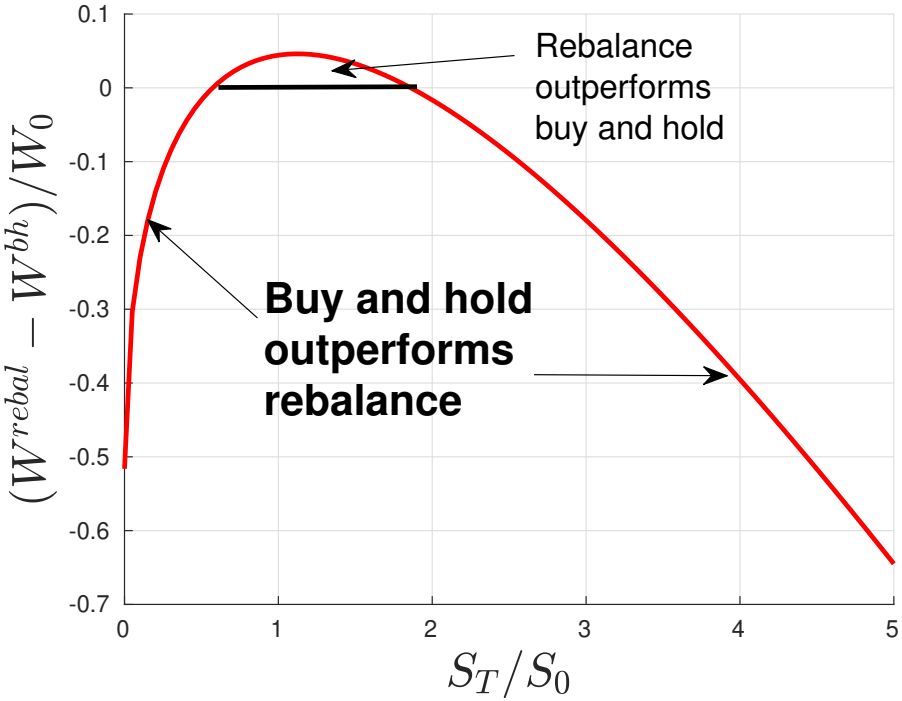
Example data

- CRSP[1] cap-weighted index 1926:1-2023:12 (inflation adjusted)
- r average real return of US 30-day T-bills, 1926:1-2023:12

[1] Center for Research in Securities Prices

Payoff Diagrams: T=10 years, $\beta = 0.5$

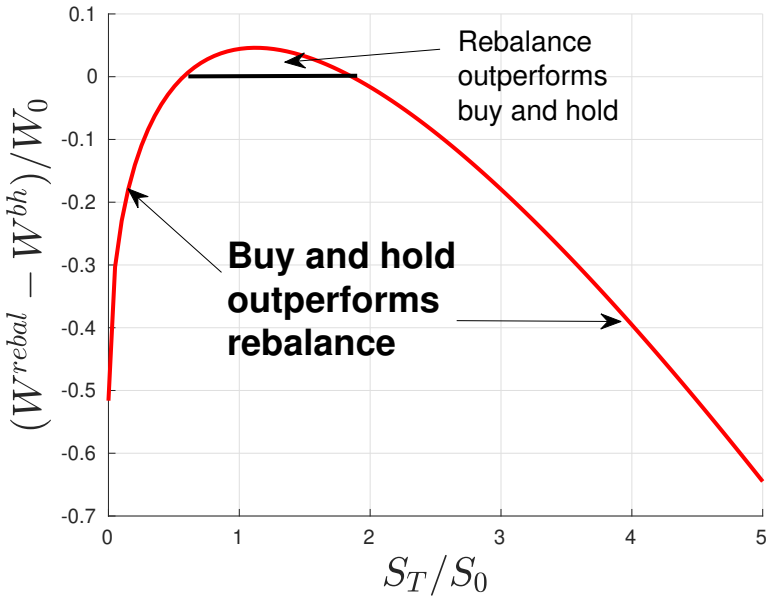




Why is buy and hold better in the wings?

- If the market trends down
  - Rebalancing keeps buying losing stocks
- If the market trends up
  - Rebalancing keeps buying losing bonds

Examine the entire distribution



Textbook analysis: examine single point on S_T/S_0 axis

- If this point is in the outperformance region
  $\rightarrow$ you conclude that rebalancing is good

But: more relevant
$\hookrightarrow$ how much of the time do you end up in the outperformance region?

More realistic: stock index: jump diffusion; discrete rebalancing

Inflation Adjusted	
Investment Horizon	$T = 10$ years
Initial Investment	$W_0 = 1000$
Rebalancing frequency	1 month
Rebalancing fraction	$\beta = 0.5$
T-bill return (r)	CRSP
Jump diffusion parameters	CRSP

	Median$[W_T]$	$W_T : 5^{th}$ percentile	$W_T : 95^{th}$ percentile	ES(5%)
Rebalance	1481	870	2563	754
Buy and Hold	1484	827	3302	746

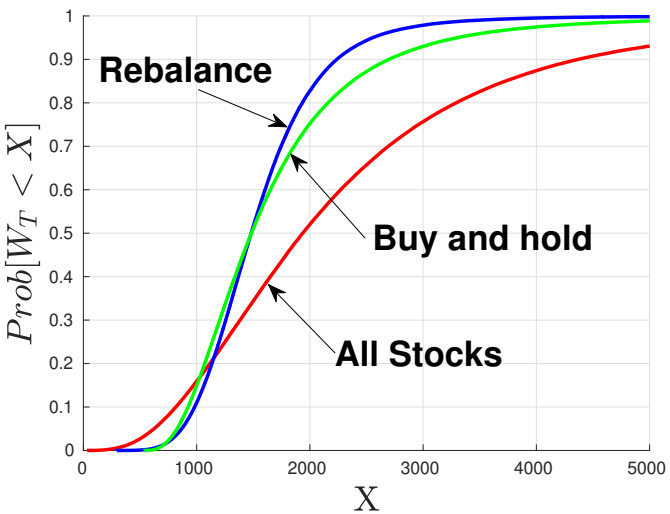
ES(5%): mean of worst 5% of the outcomes (larger is better).

Cumulative Distribution Functions (CDF)

Recall concept of 1st order stochastic dominance

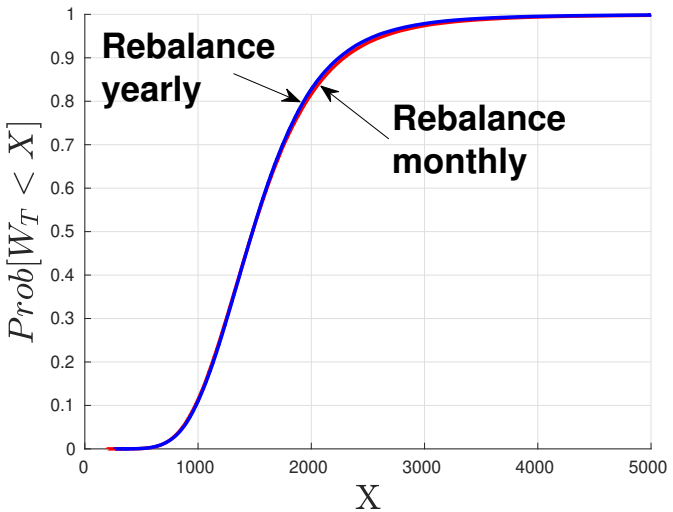
- If CDF of portfolio A plots below CDF of portfolio B
  ↪ Any rational investor will choose A

CDF of W_T



Buy and Hold
↪ Outperforms rebalancing right of median
↪ Underperforms (slightly) left of median

CDF of W_T



Monthly vs. yearly rebalancing
⇒ Almost no difference

Data Driven Test

We will use stationary block bootstrap resampling (Politis (1994))

- No assumptions about stochastic processes
- Popular with practitioners

Data:
↪ Monthly CRSP stock and bond returns (real) 1926:1-2023:12

For each stochastic path of T years

- Pick starting month at random (with replacement)
  - Generate a block of returns (pairwise sample of indexes)
- Pick another month, generate another block of returns
- Continue until we have hit T years

Block size follows geometric distribution

- Algo for expected block size
- Block resampling $\rightarrow$ serial correlation

Bootstrap Tests: Data 1926:1-2023:12

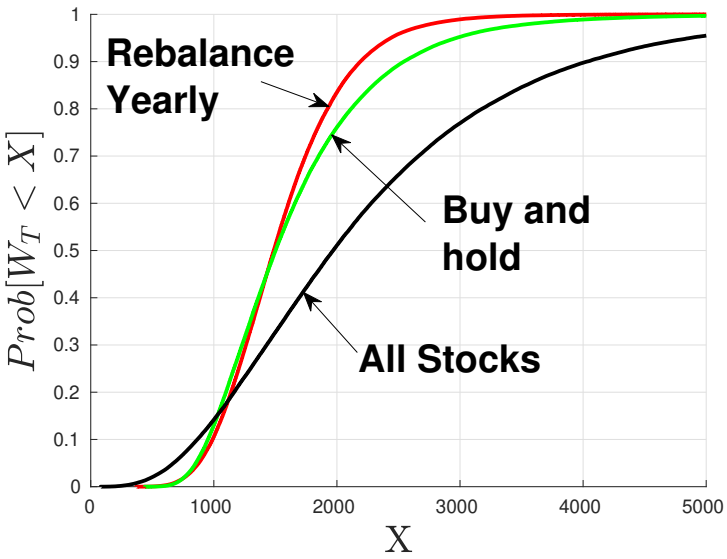
Inflation Adjusted	
T	10 years
Initial Investment	1000
Rebalancing frequency	1 year
Rebalancing fraction	$\beta = 0.5$
T-bill returns	CRSP
Stock returns	CRSP

	Median[W_T]	$W_T : 5^{th}$ percentile	$W_T : 95^{th}$ percentile	ES(5%)
Rebalance	1487	878	2438	768
Buy and Hold	1501	854	2970	764

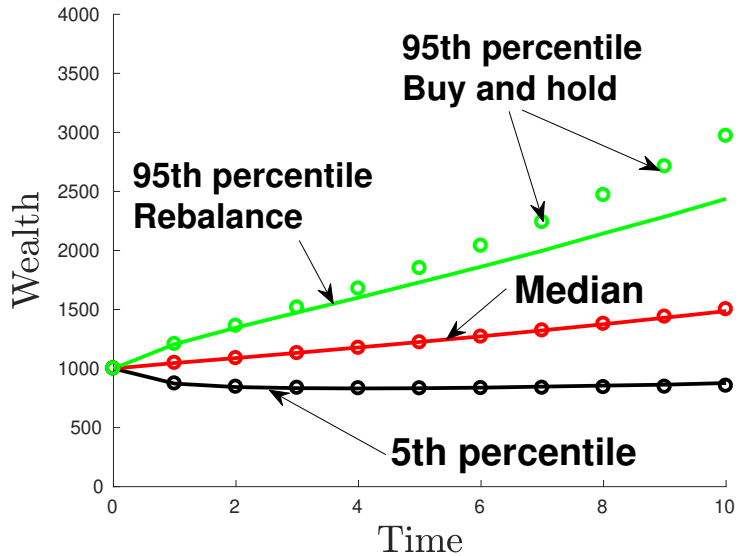
ES(5%): mean of worst 5% of the outcomes.

Bootstrap: T=10 years, $\beta = 0.5$

CDF of W_T



Percentiles $W(t)$



Lines: rebalance. Symbols: buy and hold

Left of median

↪ Slight underperformance of *buy and hold*.

Right of median

↪ Significent outperformance of *buy and hold*.

Bootstrap: pathwise analysis

Examine statistics of

$$R_t = \frac{W_t^{bh}}{W_t^{rebal}} \tag{5}$$

$R_t > 1$ indicates *buy and hold* outperforms

- Distribution of R_t gives us the pathwise probability of outperformance

Statistics for R_T

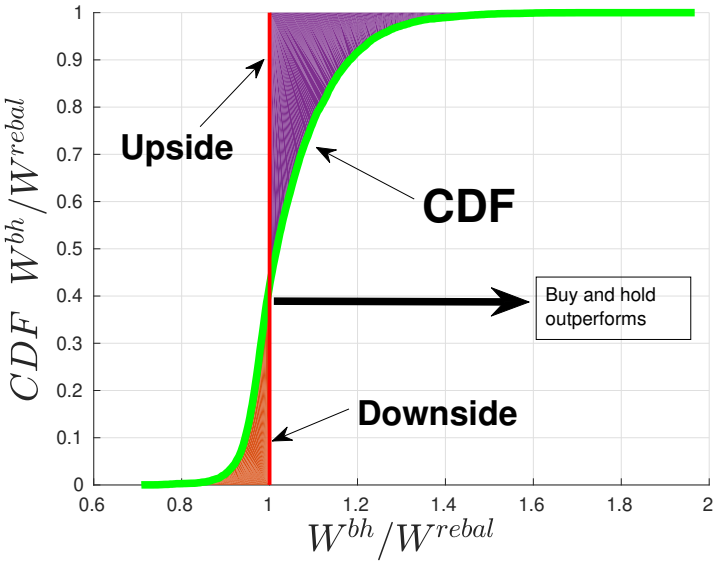
(W_T^{bh}/W_T^{rebal}) Median	(W_T^{bh}/W_T^{rebal}) 5^{th} percentile	(W_T^{bh}/W_T^{rebal}) 95^{th} percentile	ES(5%)
1.02	.924	1.25	.889

Extreme right skew

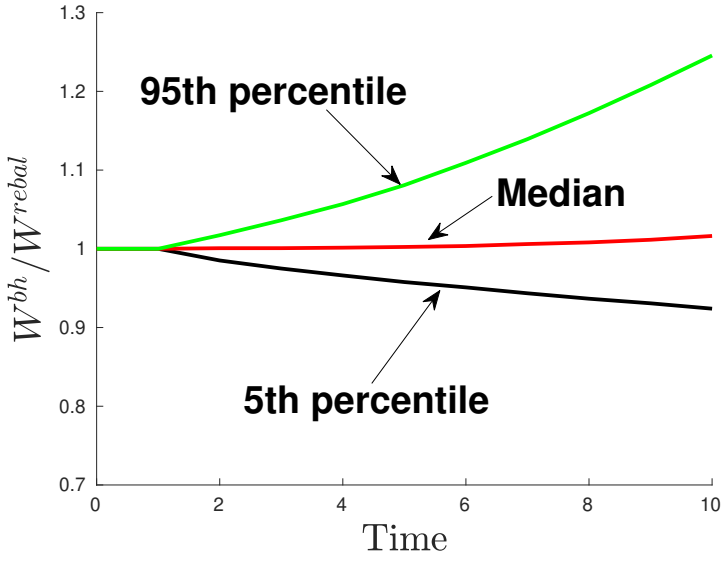
- Sharpe ratio worse for *buy and hold*
  $\rightarrow$ Sharpe ratio not useful for nonlinear strategies

More Bootstrap Plots: (W_t^{bh}/W_t^{rebal})

CDF of (W_T^{bh}/W_T^{rebal})



Percentiles (W_t^{bh}/W_t^{rebal})



$$\frac{\text{Purple Area}}{\text{Gold Area}} = \text{Omega Ratio} = 3.5$$

Along each path

$$Prob[W_T^{bh} > W_T^{rebal}] \simeq 0.60$$

$$5th \text{ percentile } (W_T^{bh}) \simeq 0.92 \times W_T^{rebal}$$

$$95th \text{ percentile } (W_T^{bh}) = 1.25 \times W_T^{rebal}$$

Other Issues

Instantaneous volatility: buy and hold

- Higher than W^{rebal}: path where stocks go up
- Lower than W^{rebal}: path where stocks go down
- If you are rich $\Rightarrow$ higher vol
- If your are poor $\Rightarrow$ lower vol

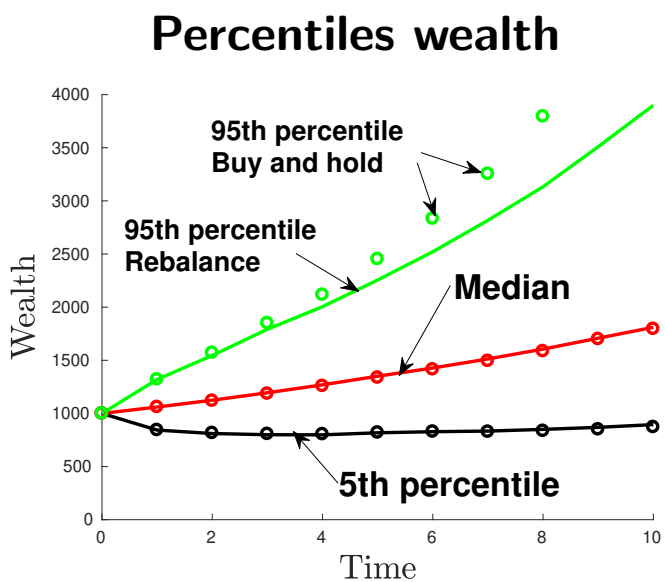
  $\hookrightarrow$ Is this bad?

Alternative data set:

- Stock index: CRSP Equal Weight
- Bond index: CRSP 10 year US Treasury

- Bootstrap tests

$\hookrightarrow$ Buy and hold outperforms rebalancing



Lines: rebal. Symbols: buy & hold

Conclusions

Retail investor

↪ Portfolio: stock index ETF, bond index ETF

↪ 10 year horizon

Default Advice for Retail Investors: Rebalance

- Rebalancing is hard to justify vs. buy and hold
  - → In a taxable account, you should not rebalance
- Rebalancing is a nonlinear strategy
- Textbook analysis
  - → Based on examining single points on CDF
  - → Use of linear criteria (e.g. Sharpe ratio)
- Need to examine entire CDF
  - → Different conclusion from textbook examples
- In order to beat buy and hold
  - → More sophisticated strategies, i.e. based on optimal control req'd