On the timing of non-renewable resource extraction with regime switching prices: A stochastic optimal control approach

Margaret Insley
Department of Economics, University of Waterloo

September 2015

Presentation at the University of A Coruña

Optimal decisions for a firm managing a natural resource asset

- This paper uses a "real options" paradign to examine a firm's optimal decisions about extracting a non-renewable resource over time and final abandonment of the project.
- An oil sands project is used as an example.
- Real options paradign uses concepts from finance for valuing financial options, and applies these to other types of investment decisions where irreversibility and uncertainty are key.

Applying option theory to other types of investment decisions

1980s - a surge of interest in applying option theory to the firm's decision about investments in real assets:

- Dixit (Quarterly Journal of Economics,1989) , "Hysteresis, import penetration, and exchange rate pass-through"
- Brennan and Schwartz (J. of Business, 1985): an early paper using a no-arbitrage approach and stochastic control theory to value a prototype mining project - the real options approach

- Paddock, Siegel and Smith (1988, Quarterly Journal of Economics) , "Option valuation of claims of real assets: the case of offshore petroleum leases"
- Morck, Schwartz and Strangeland (1989, Journal of Financial and Quantitative Analysis), "The Valuation of Forest Resources under Stochastice Prices and Inventories"

More recent literature

A huge literature in economics and business using real options.

- Mason (JEEM, 2001) extended Brennan and Schwartz by examining a firm's decision to commence or suspend extraction of a non-renewable resource
- Chen and Insley (JECD,2012) examine optimal forest harvesting with regime switching stochastic lumber prices
- Slade (JEEM, 2001) - optimal extractions from copper mines - option theory compared to actual firm decisions
- Conrad and Kotani (REE, 2005) - considered whether to allow drilling in wildlife refuge in the Arctic

Future development of the literature

- In economics the focus has been on problems with analytical solutions.
- Development of computational approaches to solving HJB equations allows us to analyze more complex decision problems.
- Modelling approach is now much less constrained by our ability to find closed form analytic solutions.
- Theory of viscosity solutions has put the solution of HJB equations on a firm mathematical footing. No need to use Markov chains and other probabilistic approaches

Future development of the literature

- Better models of stochastic prices or costs - regime switching, jumps, stochastic volatility
- Comparing actual firm decisions to optimal action
- Implications of the real options paradigm for public policy decisions when there is significant uncertainty - i.e. climate change
- Real options and game theory to analzye firms' strategic decisions under threat of preemption

Issues that motivate this paper

- Pace of natural resource extraction depends on volatile commodity prices - boom and bust cycles
- Serious environmental consequences of many resource extraction projects
- Environmental regulations may not be adequate for a sudden ramp up in operations
- Environmental damages may change through the life of the project

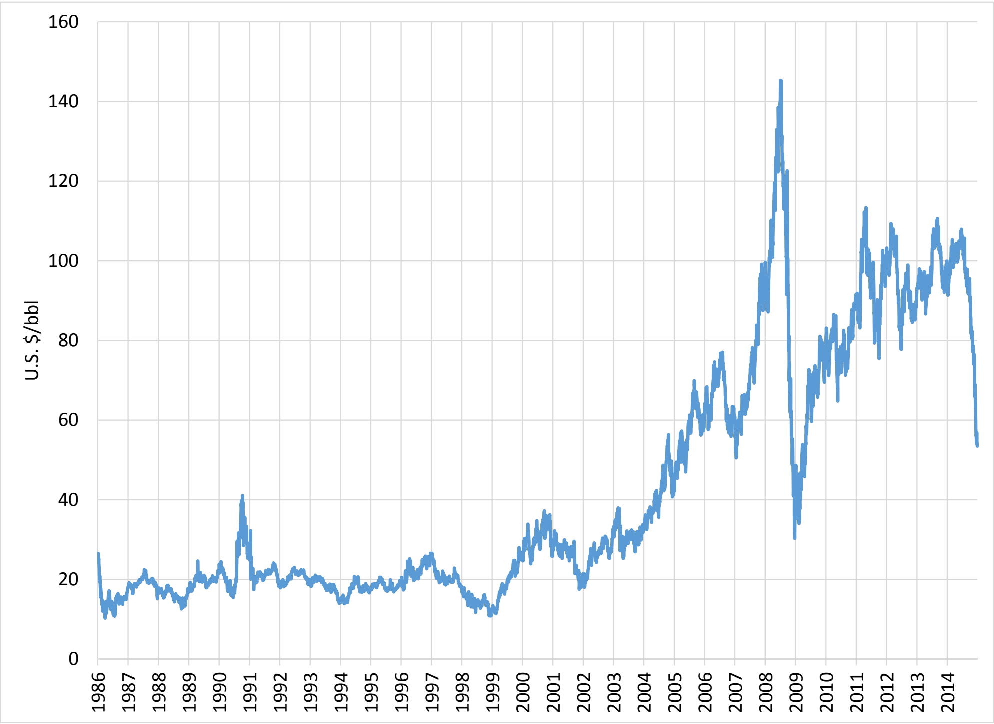


Figure 1: West Texas Intermediate Crude Oil Futures Price with one month expiry, U.S. $/barrel, Monthly data

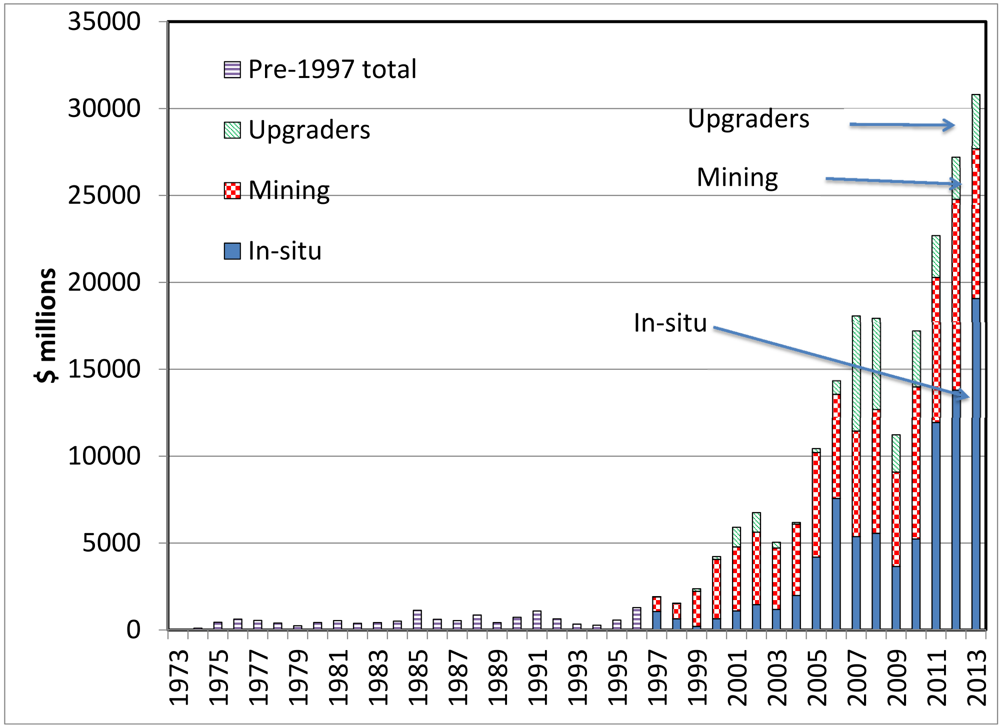


Figure 2: Alberta Oil Sands Capital Expenditures. Data Source: Canadian Association of Petroleum Producers

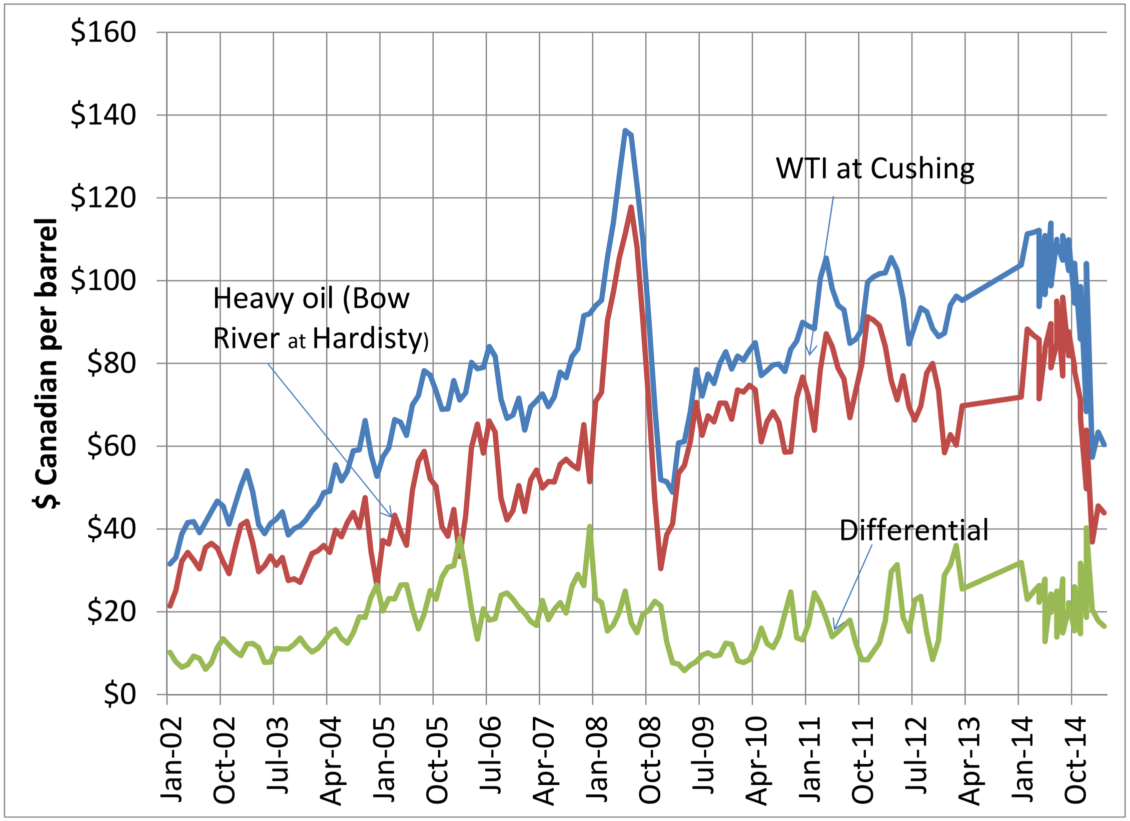


Figure 3: Heavy oil differential: WTI at Cushing in \$C/bbl, Heavy oil price at Hardisty, Alberta, Data Source: CAPP

Objectives of this paper

- To examine the impact of volatile prices and boom/bust cycles on the optimal decisions of non-renewable resource producer
- Use a regime switching model to capture oil price dynamics
- Use a switching model of resource investment - construction and operations can be paused and restarted
- Consider implications for environmental regulation

Model of a firm's optimal decisions

- Specify a Hamilton-Jacob-Bellman partial differential equation to model the decision to construct a resource extraction project - oil sands $in\ situ$ project
- Construction happens over a period of several years
- Once operational the project can be mothballed temporarily at a cost and reactivated at a further cost
- Can also be abandoned at a cost

Models of resource price

A general Ito process

$$dP = a(P,t)dt + b(P,t)dz$$

$$\begin{aligned} a(P,t), b(P,t) &= \text{known functions} \\ dz &= \text{increment of a Wiener process} \\ dz &= \epsilon\sqrt{dt}, \epsilon \sim N(0,1) \end{aligned}$$

Common models of commodity prices

- Geometric Brownian Motion

$$dP = \alpha P dt + \sigma P dz$$

- Processes with mean reversion in the drift

$$dP = \eta(\bar{P} - P)dt + \sigma P dz$$

$$dP = \eta(\mu - log(P))Pdt + \sigma P dz$$

Looking for better models

- Various researchers have sought improvements to these simple models.
- Criteria:
  - Ability to match the term structure of futures contracts
  - Simple enough to be useful in pricing options
- Schwartz (J. of Finance, 1997) compared three different models
  - One factor mean reverting
  - Two factor with stochastic convenience yield
  - Three factor adding in a stochastic interest rate

Looking for better models

- Stochastic volatility models - allows the variance of the process generating the time series to change at discrete points or continuously.
- Larsson and Nossman (Energy Economics, 2011) use stochastic volatility with jumps to model oil prices.
- Used WTI spot prices to estimate the parameters of their model.
- To price assets, parameters of the price model should be estimated under the Q-measure, risk adjusted process.

An alternative - a regime switching model

- Empirical analysis indicates that drift and volatility parameters are not constant
- A regime switching model accommodates changes in drift and volatility by defining different regimes and specifying probabilities of switching between regimes
- Some empirical studies find strong evidence of regime switching for crude oil price volatility (eg. Zou and Chen, 2013, Canadian Journal of Statistics)

Specification of regime switching model

- Two regimes:

$$\begin{aligned} dP &= \eta^j(\bar{P}^j - P)dt + \sigma^j P dz \qquad (1) \\ j &= 1, 2; \end{aligned}$$

- η^j is the speed of mean reversion in regime j
- $\bar{P}^j$ is the long run price level in regime j
- σ^j is the volatility in regime j
- dz = increment of a Wiener process

Probability of switching regimes

- The term dX_{jl} governs the transition between j and l:

$$dX_{jl} = \begin{cases} 1 & \text{with probability } \lambda_{jl}dt \\ 0 & \text{with probability } 1 - \lambda_{jl}dt \end{cases}$$

- There can only be one transition over dt

Futures Prices

- In order to estimate risk-adjusted parameters, the parameters in the above equation are calibrated using market natural gas futures prices and options on futures.
- Let $F^j(P,t,T)$ denote the futures price in regime j at time t with delivery at T while the spot price resides at P

Futures Prices

- The futures price equals the expected value of the spot price in the risk neutral world:

$$\begin{aligned} F^j(p,t,T) &= E^Q[P(T)|P(t)=p, J_t=j] \\ j &= 1,2. \end{aligned}$$

where E^Q refers to the expectation in the risk neutral world and J_t refers to the regime in period t.

Futures Prices

- Applying Ito's lemma results in two coupled pde's for the futures price, one for each regime, $j = 1, 2$:

$$(F^j)_t + \eta^j(\bar{P}^j - P)(F^j)_P + \frac{1}{2}(\sigma^j)^2 P^2 (F^j)_{PP} + \lambda_{jl}(F^l - F^j) = 0.$$

- Boundary condition: $F^j(P, T, T) = P$, $j = 1, 2$.
- Substituting a solution of the form

$$F^j(P, t, T) = a^j(t, T) + b^j(t, T)P$$

into the pde and boundary condition results in an ode system which can be solved.

Calibration Procedure

- This ode system can be used to find the model implied futures price for different parameter values
- A suite of parameters must be estimated such as $\theta = \{\eta^j, \mu^j, \sigma^j, \lambda^{jl} \mid j, l \in \{0, 1\}\}$
- In addition the current regime, $J(t)$ must be estimated.
- On each observation day, t, there are futures contracts with a variety of different maturity dates, T

Calibration

- The parameter values minimize the sum of squared differences between model-implied futures prices and actual futures prices.

$$min_{\theta,j(t)} \sum_{t} \sum_{T} (\hat{F}(J(t), P(t), t, T; \theta) - F(t, T))^2$$

where $F(t,T)$: market futures price on observation day t with maturity T and $\hat{F}(J(t), P(t), t, T; \theta)$ is the corresponding model implied futures prices.

Calibration

- A difficult optimization problem, with no unique solution
- Bounds are placed on the parameter estimates to achieve reasonable results
- Calibration is done using monthly data for futures prices of various maturities, 1995 - 2014.
- The speed of mean reversion η, long run equilibrium price $\bar{P}$, and probability of switching regimes λ^{jl} are calibrated independently of volatility, σ

Calibration

- For the assumed Ito process volatilities are the same in the P-measure and Q-measure
- Volatilities are estimated separately using the spot price.
- Use Matlab code written by Perlin (2012) for P-measure estimation of Markov state switching models.

Base Case Parameter Estimates

	Regime 1	Regime 2	lower bound	upper bound
η^j	0.29	0.49	.01	1
$\bar{P}^j$,	50	98	0	200
λ^{jl}	0.45	0.47	0.02	0.98
σ	0.28	0.34		

Table 1: $dP = \eta^j(\bar{P}^j - P)dt + \sigma^j P dz, j = 1, 2.$

- Risk adjusted parameter estimates
- Probability of switching regimes is $\lambda^{jl}dt$
- The average error is \$8.85.

Simulation of the price process

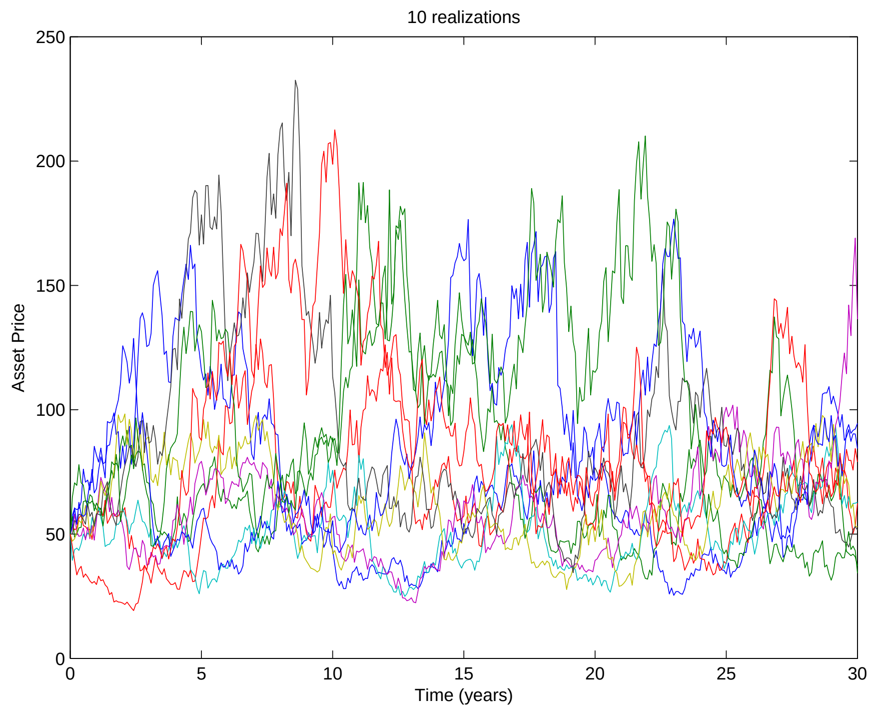


Figure 4: Simulation of base case regime switching price process, U.S. $/barrel, 10 realizations

Resource Valuation Model

- $V(P, S, \delta)$ - value of the resource asset; P is resource price, S is the size of the resource stock, and δ is the plant stage.
- M possible plant stages, δ_m such as: 0 percent complete, partially complete, fully operational, mothballed, abandoned.
- The firm chooses the timing of extraction as well as the plant stage to maximize V.
- Denote annual extraction by R. Then $dS = -Rdt$; A path dependent variable

Objective Function

The value of the project in regime j and stage m is $V_m^j(p, s, t)$.

$$V_m^j(p, s, t) = \max_{R, \delta_m} \mathrm{E}^Q \left\{ \int_{t_0}^{T} e^{-rt'} \left[\pi_m^j \right] dt \mid P(t) = p, S(t) = s \right\},$$

$$m = 1, ..., M; \; j = 1, ..., J$$

subject to $\int_{t_0}^{T} R(:, t) dt \leq S_0$.

V between decision dates

Standard contingent claims arguements derive a system of pde's which describe V between decision dates.

$$\frac{\partial V_m^j}{\partial t} = \max_{R \in Z(S)} \left\{ -\frac{1}{2} b^j(p,t)^2 \frac{\partial^2 V_m^j}{\partial p^2} - a^j(p,t)\frac{\partial V_m^j}{\partial p} + R_m^j \frac{\partial V_m^j}{\partial s} - \pi_m^j(t) + \sum_{l=1, l\neq j}^{J} \lambda^{jl}(V_m^l - V_m^j) - rV_m^j \right\}$$

$$j = 1, 2;\ m = 1, ..., M$$

where $a^j(p,t)$ is the risk adjusted drift rate conditional on $P(t) = p$ and λ^{jl} is the risk adjusted transition j to regime l from regime .

Decision dates for switching plant stages

Each year the firm checks to see if it is optimal to switch to a different stage of operations. Switching stages incurs a cost, but so does staying in the current stage.

- Stage 1: Before construction begins
- Stage 2: Project 1/3 complete
- Stage 3: Project 2/3 complete
- Stage 4: Project 100 % complete and in full operation
- Stage 5: Project is temporarily mothballed
- Stage 6: Project abandoned

Choosing the optimal plant stage

The optimal switching decision is given by:

$$V(t^-, \delta_{\bar{m}}) = \max\left\{V(t^+, \delta_1) - C_{\bar{m}1},\ \dots, V(t^+, \delta_{\bar{m}}) - C_{\bar{m}\bar{m}},\ \dots, V(t^+, \delta_M) - C_{\bar{m}M}\right\}$$

Solution Approach

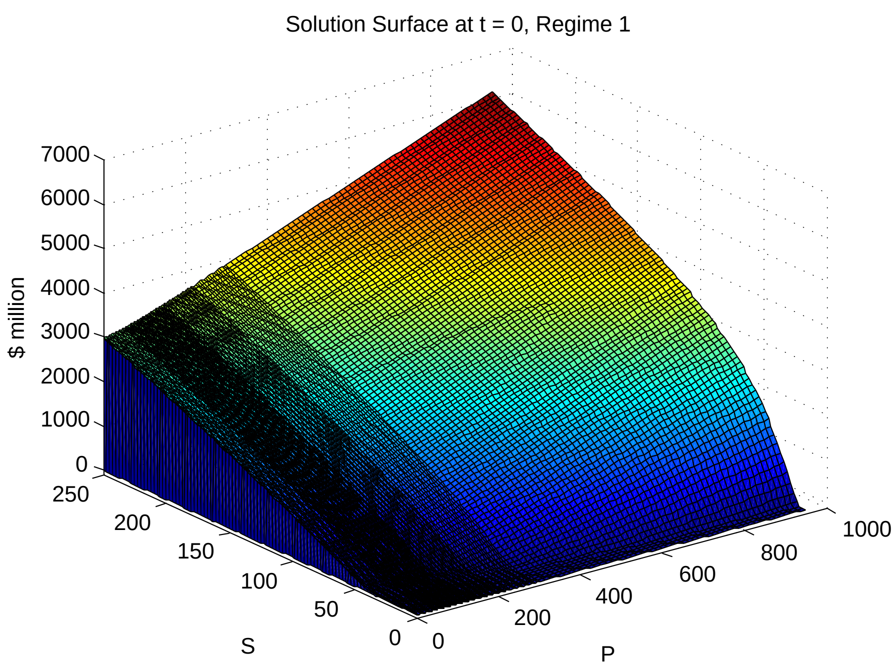
- A stochastic optimal control problem requiring a numerical solution
- A standard finite difference approach plus a semi-Lagrangian scheme

Production*	30,000 bbl/day, in situ, SAGD
Reserves*	250 million barrels
Lease length	30 years
Variable costs (energy):*	5.28% of WTI price
Variable costs (non-energy):*	$5.06/bbl
Fixed cost (operating)*	$34 million
Fixed cost (mothballed)	$21.9 million
Cost to mothball and reactivate	$ 5 million
Construction costs*	$960 million over three years
Corporate tax: Federal/Prov	15% / 10%
Carbon tax	$40 per tonne

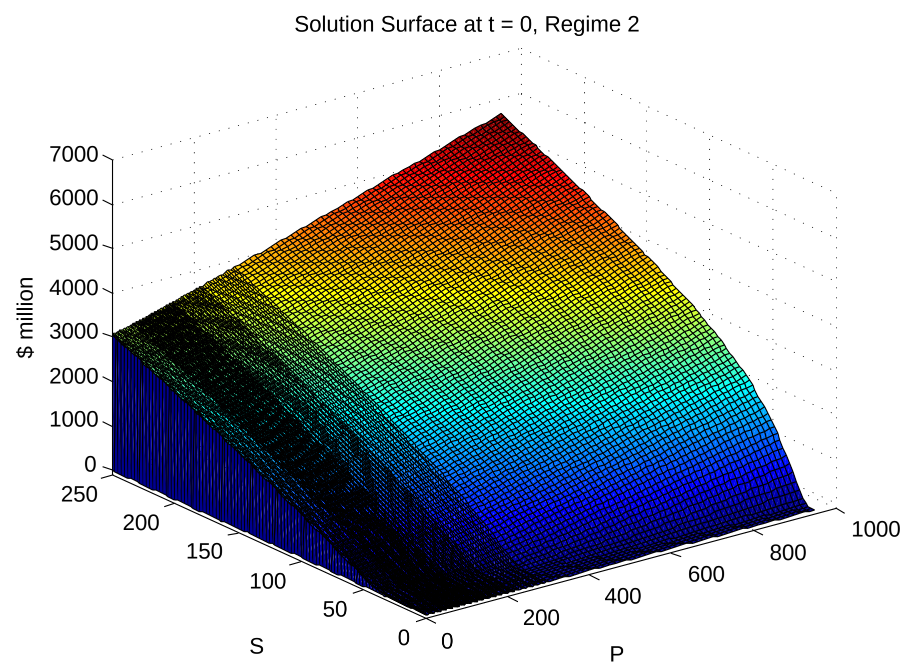
*CERI (2008, 2009, 2012) & Plourde (2009, Energy Journal)

- Royalty rates are based on pre-payout rate.
- Adds considerable complexity to calculate post-payout royalties, as it depends on price, which is stochastic.
- Assume bitumen price is 65% of the price of WTI.

Case 1: Project value pre-construction versus price and reserves

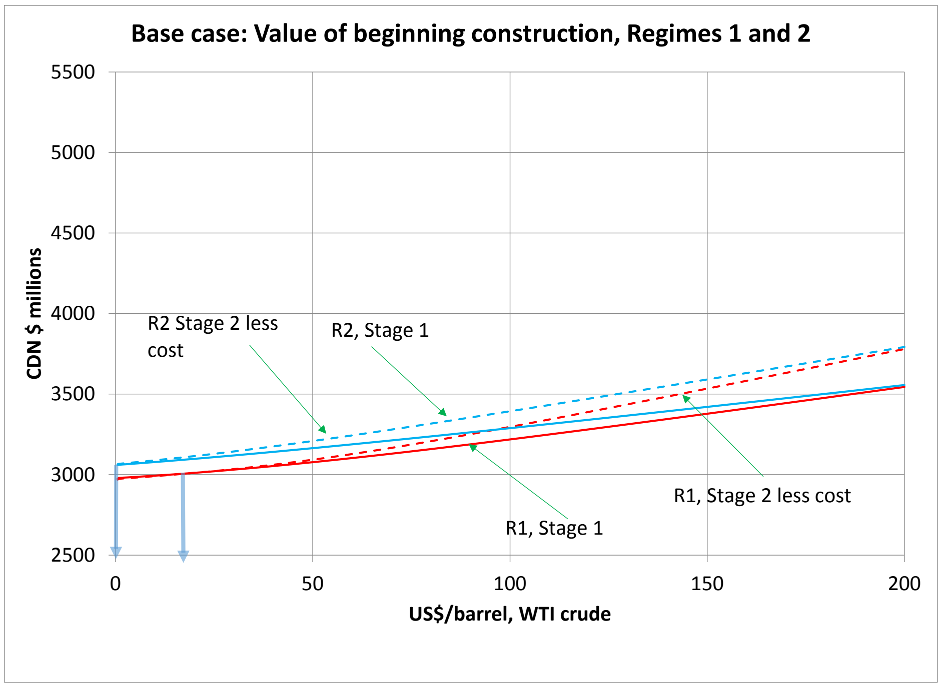


(a) Regime 1

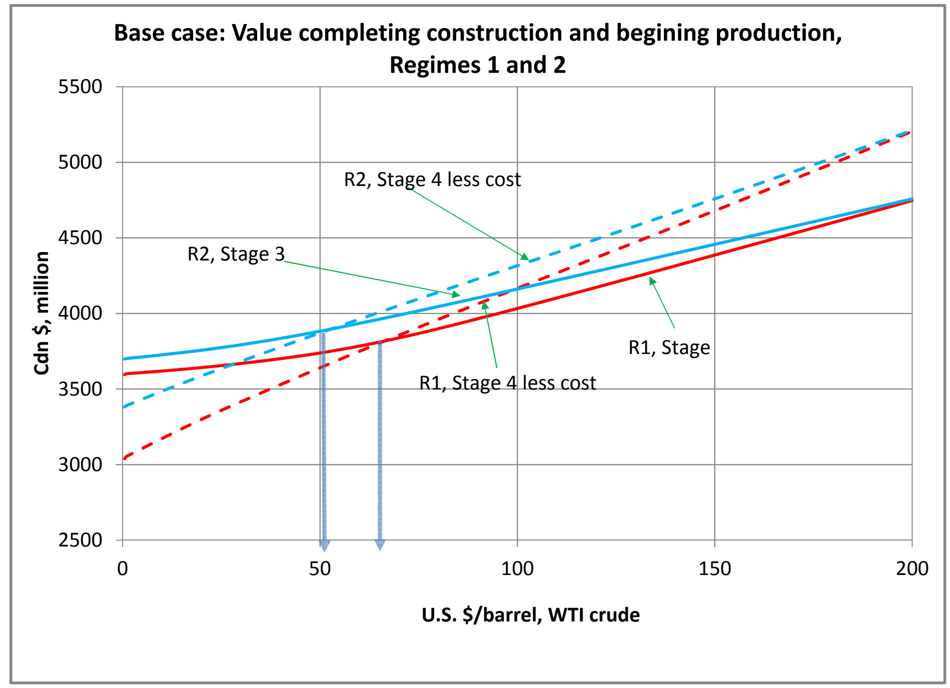


(b) Regime 2

Value of beginning construction (left) and finishing construction (right)



(c) Stage I - II



(d) Stage III - IV

R1: $\eta = 0.29,\ \bar{P} = 50,\ \lambda^{12} = .45$; R2: $\eta = 0.49,\ \bar{P} = 98,\ \lambda^{12} = .47$

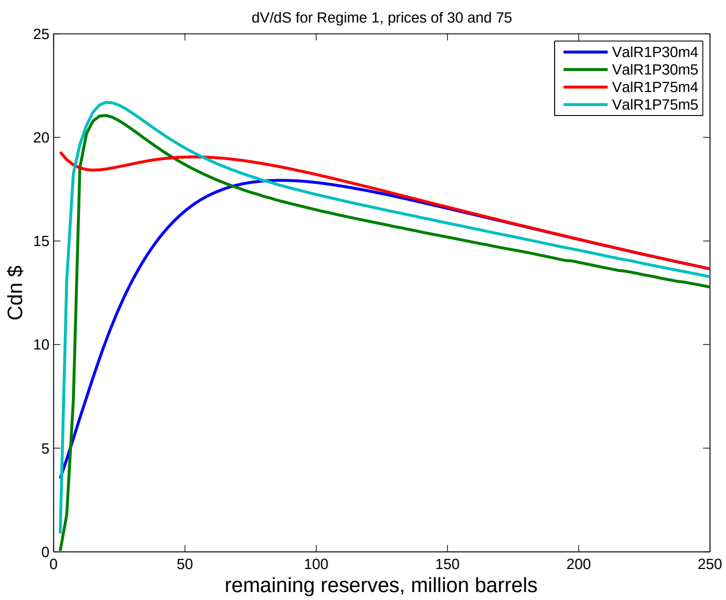
	$S_0 = 250$		$S_0 = 125$	
Critical Prices for Transition from:	R1	R2	R1	R2
Stage I to Stage II: Begin construction	20	0	62	32.5
Stage II to Stage III: Continue	40	15	68	45
Stage III to Stage IV: Finish, Begin production	66	52	88	74
Stage IV to Stage V: Mothball	52	37.5	69	55
Stage V to Stage IV: Reactivate	54	40	71	57
Stage IV or V to Stage VI: Abandon	NA	NA	NA	NA

- Critical prices are lower in regime 2 - higher long run price and more rapid speed of MR.
- Critical prices to reopen are higher than critical prices for mothballing - hysteresis.

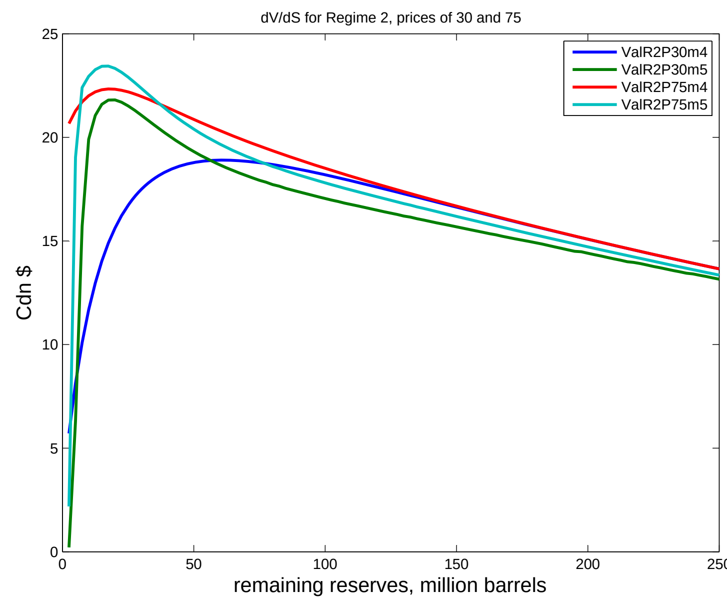
- At these levels of reserves there is no price at which the resource would be abandoned. (To be further discussed later.)
- Critical prices are higher when stock is lower
- Critical prices rise as construction proceeds.

Why do critical prices rise as reserves fall?

These figures show $\frac{\partial V}{\partial S}$ versus remaining reserves for two prices levels.



(e) Regime 1, Vertical axis: Million dollars, Horizontal: millions of barrels



(f) Regime 2, Vertical axis: Million dollars, Horizontal: millions of barrels

Why do critical prices rise as construction proceeds?

- Compare benefits versus costs of delaying the next stage of capital investment
- Benefits of delay
  - Delay in construction spending
- Costs of delay
  - Delay in receiving revenue from production
  - Maintenance costs while construction is mothballed

Why do critical prices rise as construction proceeds?

- Construction is begun at a critical price lower than that at which it would be optimal to begin production.
- Getting construction underway is like exercising an option which moves the firm one step closer to production.
- Costs of delay are higher at an earlier stage of construction since the firm is unable to quickly finish the project and get production underway in the event of a sudden surge in oil prices.

Why do critical prices rise as construction proceeds?

- This pattern of critical prices is not a general result - depends on the nature of price process involved.
- Cost of delaying construction depends on the stochastic price process.
- This pattern is typical for prices following a mean reverting process - want to be able to respond quickly to temporary upswings.
- For GBM process, critical prices start high and then fall as construction proceeds.

Importance of regime switching

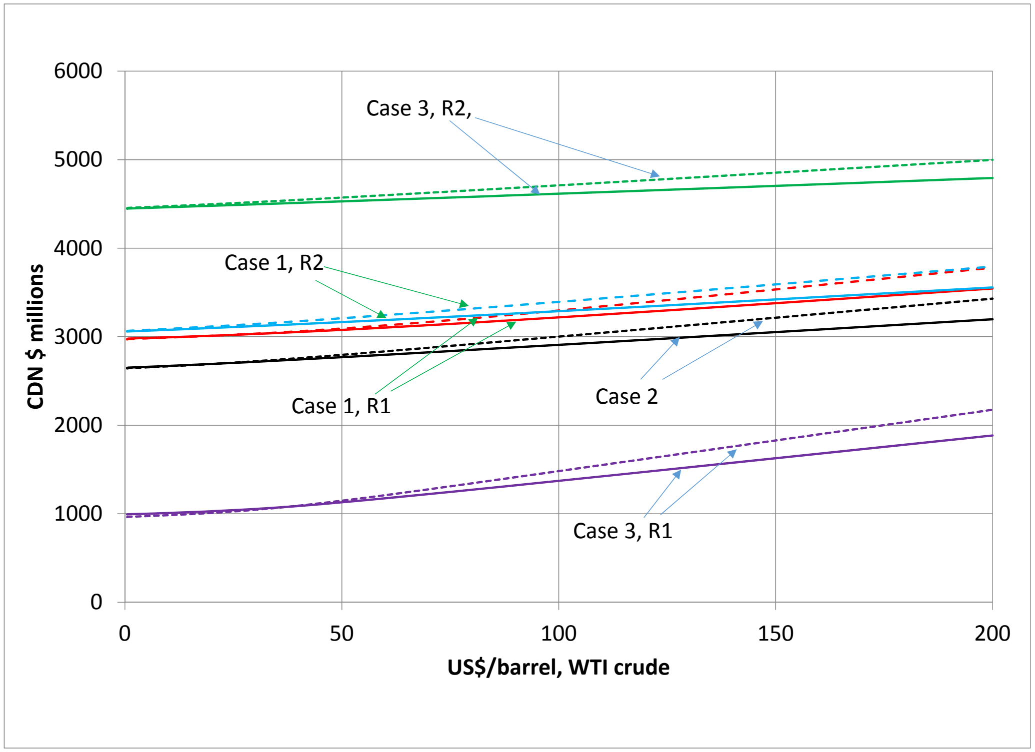
Weighted Average Price (Case 2) and
Zero Probability of Switching Regimes (Case 3)

	Case 1	Case 1	Case 2	Case 3	Case 3
	Regime 1	Regime 2	Weighted Average	Regime 1	Regime 2
η	0.29	0.49	0.39	.29	.49
$\bar{P}$	50	98	73	50	98
λ^{jl}	.45	0.47	NA	0	0
σ	0.28	0.34	0.31	0.29	0.34

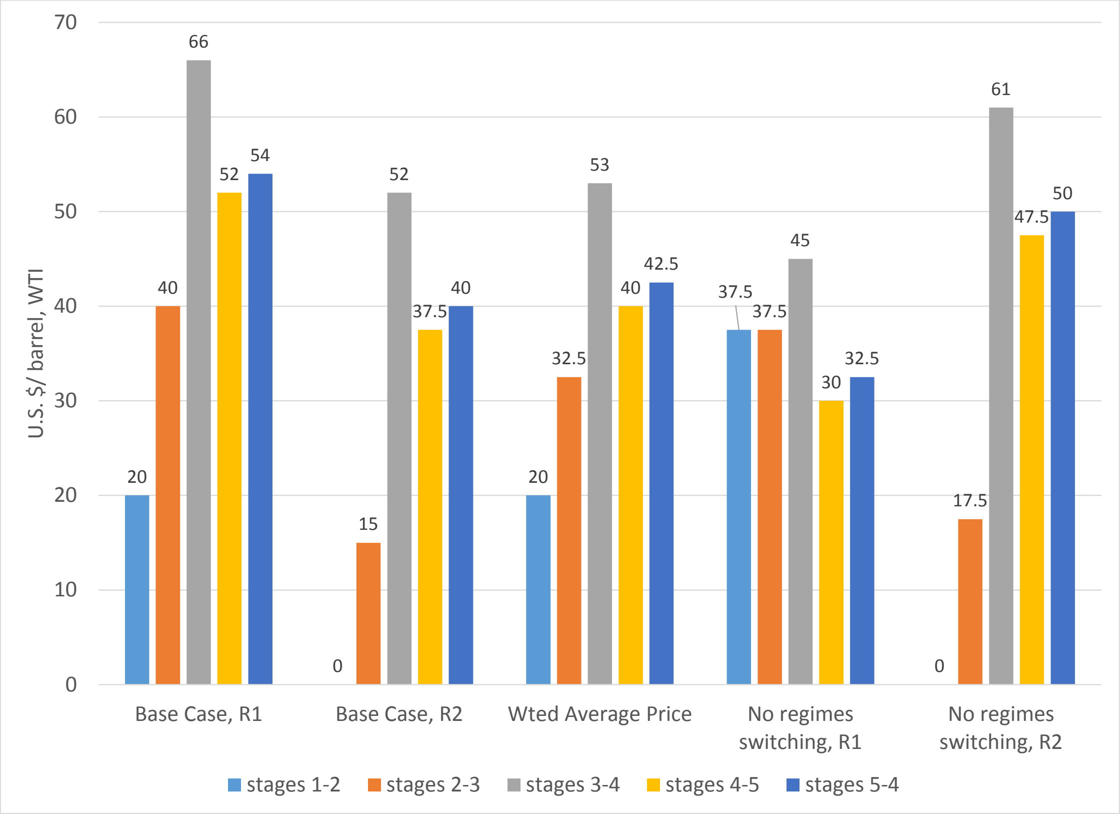
Cases 1, 2, and 3 parameter values. $dP = \eta^j(\bar{P}^j - P)dt + \sigma^j P dz, j = 1, 2.$

Importance of regime switching

Weighted Average Price (Case 2) and
Zero Probability of Switching Regimes (Case 3)



Comparing critical prices, Cases 1, 2 and 3



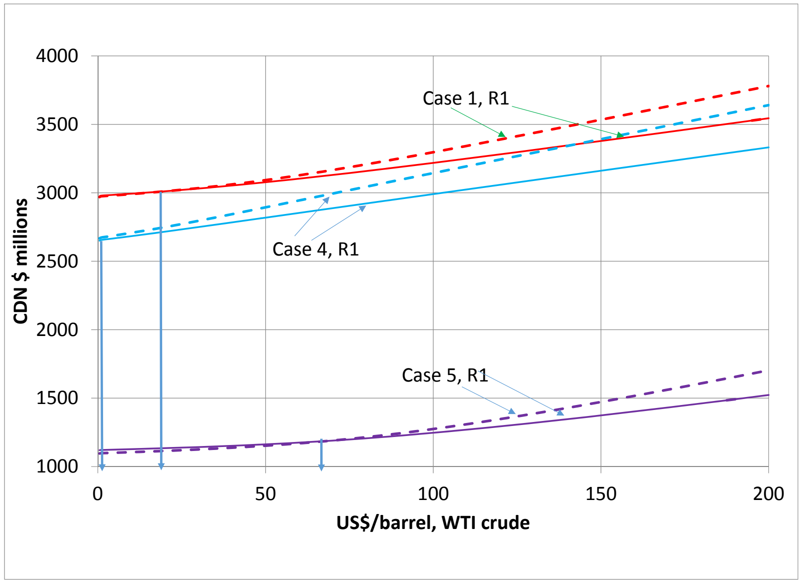
Comparing critical prices, Cases 1, 2 and 3

- Project values are lower in Case 2 (weighted average) compared to the base case.
- Critical prices differ across the three cases - ignoring price regimes would result in non-optimal decisions.

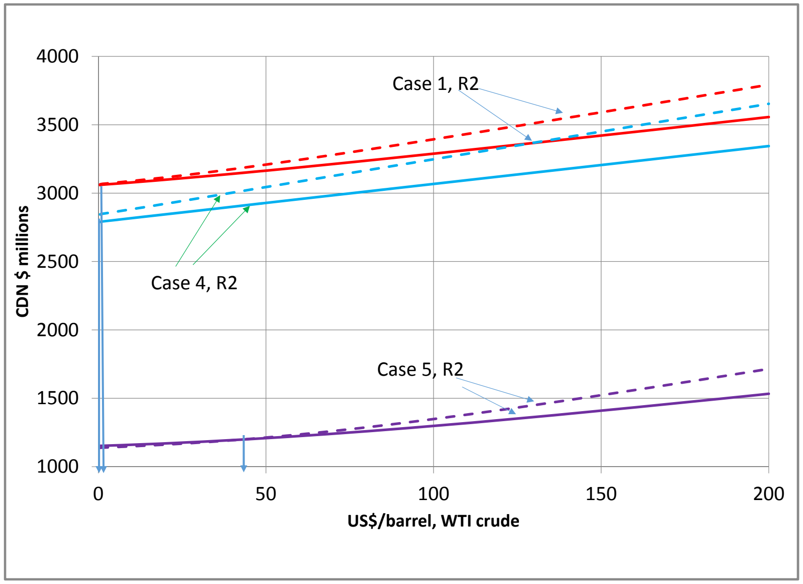
Impact of a carbon tax

- IPCC has suggested a global carbon price that increases to around $200 per tonne of CO_2 is needed by the middle of this century.
- Consider two additional cases:
  - Case 4: Tax increasing gradually from $40 to $200 per tonne over 15 years
  - Case 5: Tax increasing immediately to $200 per tonne

Impact of a carbon tax: Project value

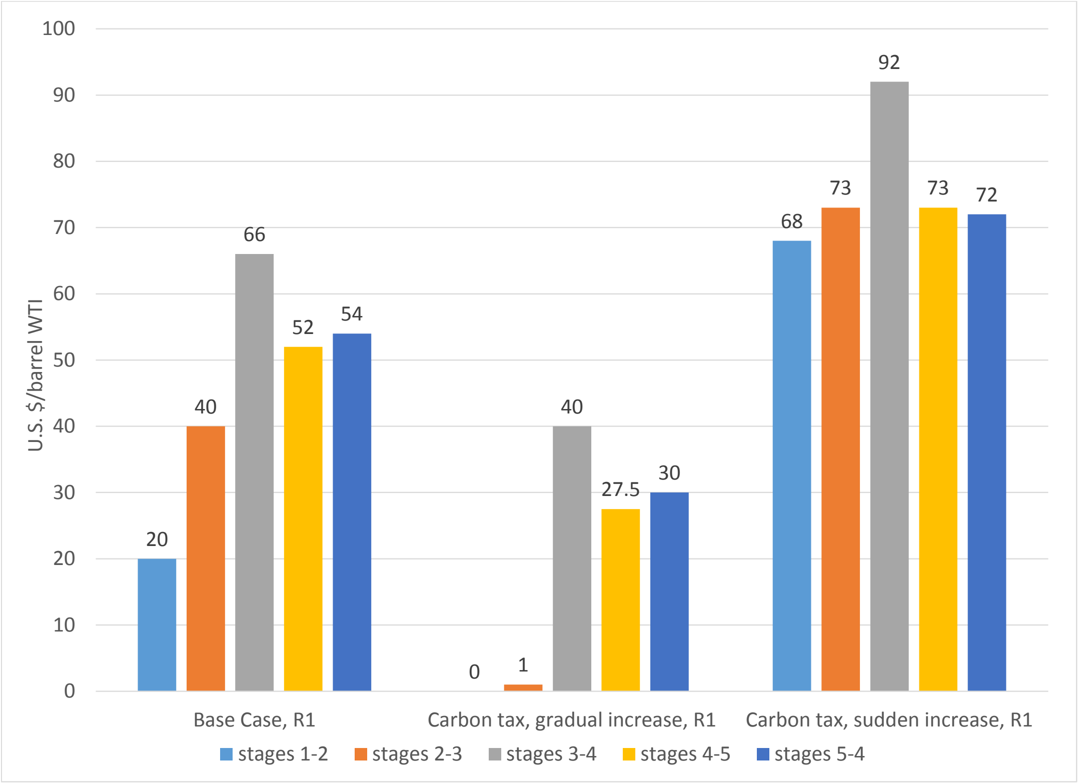


(g) Regime 1

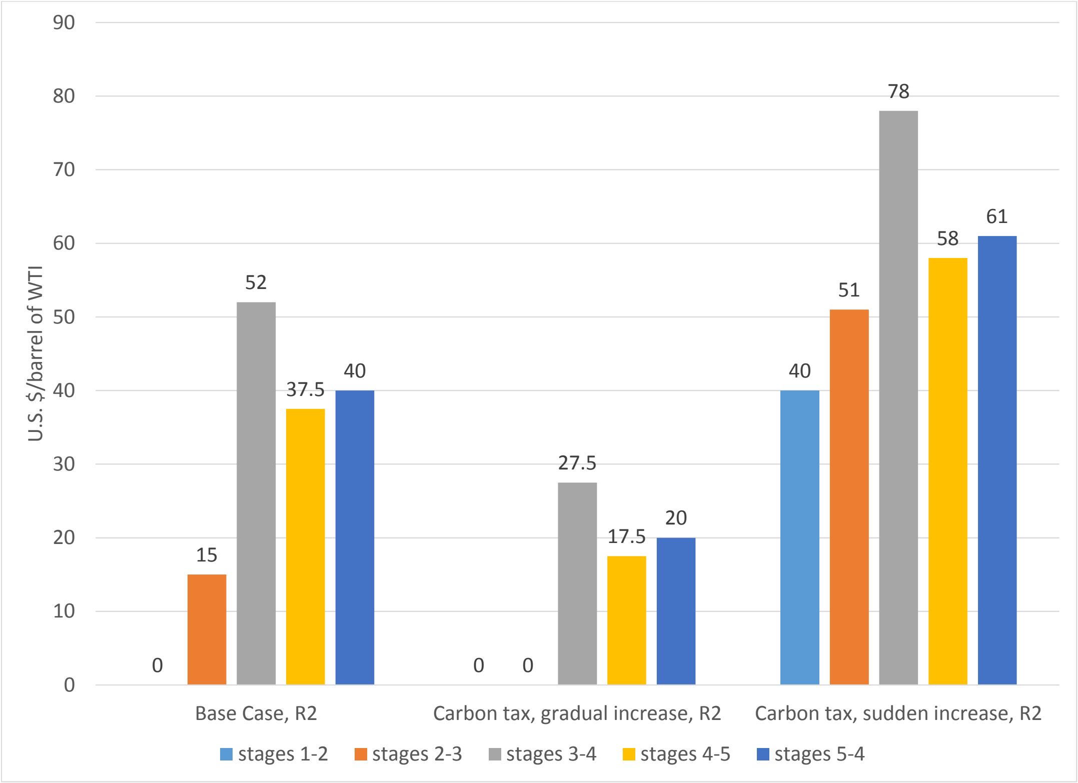


(h) Regime 2

Impact of a carbon tax: Critical prices, R1



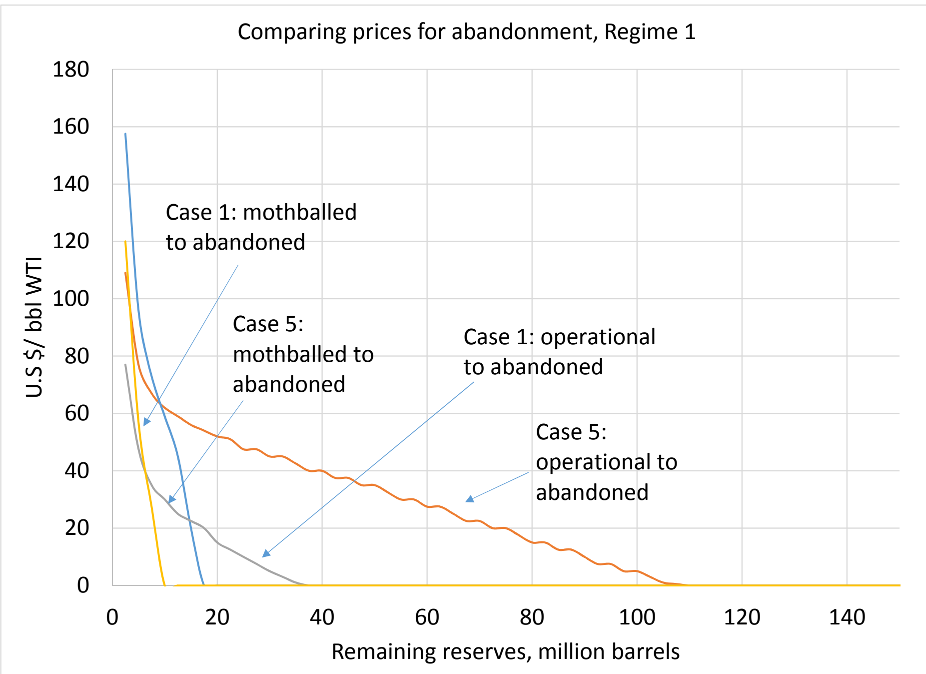
Impact of a carbon tax: Critical prices, R2



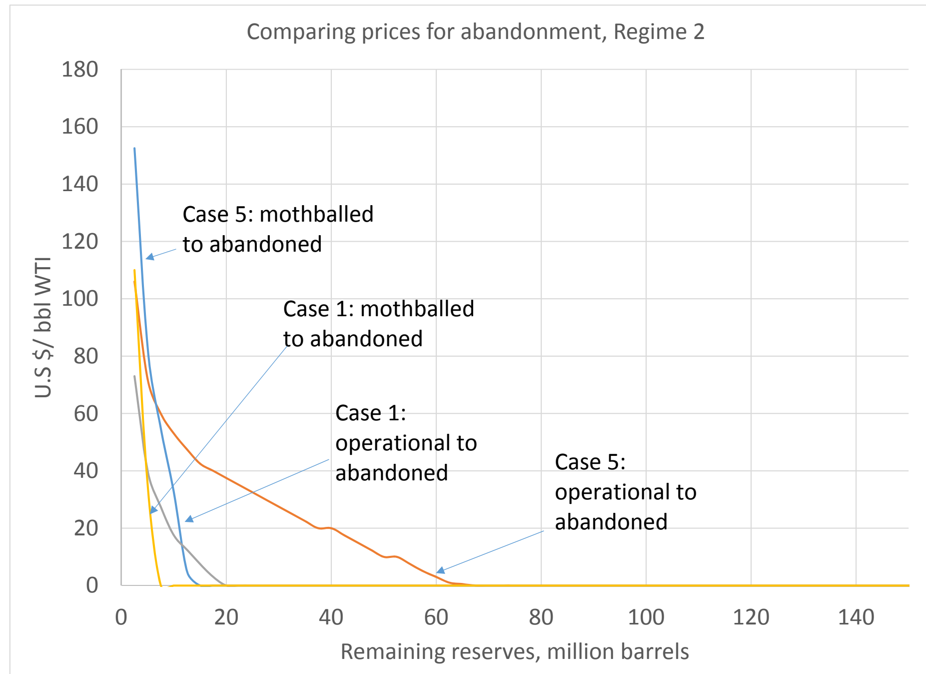
Carbon tax

- With a gradually increasing tax, critical prices are markedly lower. Construction and production will be speeded up.
- With a sudden tax increase, critical prices increase at all stages. Construction and production are delayed.
- As in the base case, there are no prices for abandonment at full reserves. This changes for lower reserve levels.

Critical prices for abandonment versus reserves



(i) Regime 1



(j) Regime 2

Critical prices for abandonment

- Critical prices for abandonment rise as reserve level falls.
- Critical prices for abandonment under a carbon tax of $200 are higher than under a carbon tax of $40.
- The higher carbon tax may cause some reserves to be left in the ground.

Sensitivity on volatility

Base case: $\sigma_1 = 0.28$, $\sigma_2 = 0.34$.
Case 7 (high volatility): $\sigma_1 = 0.84$, $\sigma_2 = 1.02$

	Case 1: Base case		Case 6: High volatility	
Transition from :	R1	R2	R1	R2
Stages 1 to 2: Begin construction	20	0	15	0
Stages 2 to 3: Continue	40	15	35	15
Stages 3 to 4: Finish, Begin production	66	52	121	110
Stages 4 to 5: Mothball	52	37.5	85	69
Stages 5 to 4: Reactivate	54	40	87	71
Stages 4 or 5 to 6: Abandon	na	na	na	na

Sensitivity on mean reversion speed

Base case: $\sigma_1 = 0.28$, $\sigma_2 = 0.34$.
Case 8 (low mean reversion speed): $\eta_1 = 0.02$, $\eta_2 = 0.02$.

	Case 1: Base case		Case 7: Low speed of mean reversion	
Transition from :	R1	R2	R1	R2
Stages 1 to 2: Begin construction	20	0	83	83
Stages 2 to 3: Continue	40	15	79	78
Stages 3 to 4: Finish, Begin production	66	52	83	86
Stages 4 to 5: Mothball	52	37.5	58	59
Stages 5 to 4: Reactivate	54	40	59	61
Stages 4 or 5 to 6: Abandon	na	na	na	na

Conclusions

- Modelling resource prices as regime switching stochastic processes can give insight into optimal investment decisions in natural resource industries.
- A myopic investor ignoring possibility of regime change can make suboptimal decisions.
- Uncertainty affects the pace of development. This has implications if environmental costs are unevenly distributed over the lifetime of the project.
- The timing of an environmental tax has a significant effect on the pace of development and how much of the total resource is extracted.