Static Single Assignment

Idea: Make each variable have only one definition.

\[
\begin{align*}
a &= 2; \\
b &= a + 1; \\
a &= 3; \\
b &= a + 1;
\end{align*}
\]

\[
\begin{align*}
a_1 &= 2; \\
b_1 &= a_1 + 1; \\
a_2 &= 3; \\
b_2 &= a_2 + 1;
\end{align*}
\]
Idea: Make each variable have only one definition.

```plaintext
a = 2;
b = a + 1;
a = 3;
b = a + 1;

if() {
    a = 2;
} else {
    a = 3;
}
b = a + 1;

if() {
    a_1 = 2;
b_1 = a_1 + 1;
    a_2 = 3;
b_2 = a_2 + 1;
} else {
    a_3 = \phi(a_1, a_2);
b_1 = a_3 + 1;
```
What does $\phi$ really mean?

- $\phi$ with $n$ arguments only makes sense at the beginning of a basic block with $n$ incoming edges. Incoming control-flow edges are implicit parameters of $\phi$ “function”.
- All $\phi$’s in a BB are “executed” simultaneously, NOT sequentially.

Example

$\begin{align*}
B1 & \quad B2 \\
x &= \phi(x, y); \\
y &= \phi(y, x);
\end{align*}$

- If coming from B1, keep $x$ and $y$ unchanged.
- If coming from B2, swap $x$ and $y$. 
SSA Algorithm 1 ("maximal", "really crude")

1. At every basic block with multiple predecessors, insert $\phi$ node for every variable.

2. Do reaching definitions analysis. Every use will have one reaching def. (Why?)

3. Number each def, and assign the same number to the uses it reaches.
Do SSA Algorithm 1.

Remove $\phi$ functions of the following forms:

- $x_i = \phi(x_i, x_i, \ldots, x_i)$
- $x_i = \phi(x_K, x_K, \ldots, x_K)$, where each $K$ is either $i$ or $j$;
  replace all uses of $x_i$ with $x_j$.

until there are no such $\phi$ functions left.
SSA Algorithm 3 (Dominance Frontier)

Dominance frontier criterion
Whenever basic block \( x \) defines variable \( a \), every node \( z \) in the dominance frontier of \( x \) needs a \( \phi \) function for \( a \).

Algorithm \( \text{SSA}3() \):

1. \textbf{while} \( \exists x, z, a \) satisfying the criterion and \( z \) has no \( \phi \) for \( a \) \textbf{do}
2. \hspace{1em} add \( \phi \) for \( a \) to \( z \)

Note: adding \( \phi \) adds a definition of \( a \)
Dominance frontier criterion with liveness

Whenever basic block $x$ defines variable $a$, every node $z$ in the dominance frontier of $x$ such that $a$ is live before $z$ needs a $\phi$ function for $a$.

**Algorithm SSA4()**:  
1. perform live variables analysis  
2. while $\exists x, z, a$ satisfying the criterion and $z$ has no $\phi$ for $a$ do  
   3. add $\phi$ for $a$ to $z$
DJ graph

Dominator tree (D edges) augmented with J edges – CFG edges that are not already D edges.

Useful Properties of SSA Form

Property

Every variable has exactly one definition.
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Corollaries:

- If $v$ is live at statement $s$, then def of $v$ dominates $s$.  

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Corollaries:
- If \( \nu \) is live at statement \( s \), then def of \( \nu \) dominates \( s \).
- If \( \nu, w \) are live at the same statement, then def of \( \nu \) dominates def of \( w \) or vice versa.
Applications of SSA

Dead code elimination

\[ a = b + c \] is dead code iff there are no uses of variable \( a \).
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Constant propagation

Whenever the single assignment to \( x \) is a constant \( c \), all uses of \( x \) can be replaced with \( c \).
Applications of SSA

Dead code elimination
a = b + c is dead code iff there are no uses of variable a.

Constant propagation
Whenever the single assignment to x is a constant c, all uses of x can be replaced with c.

Copy propagation
x = y can be deleted and uses of x replaced with y.
Basic idea: insert assignments at end of predecessors.

Difficulties:

- $\phi$ nodes must be “executed” simultaneously.
- Predecessor may have multiple successors.
  - insert nodes on edge from predecessor ("edge splitting")