**Definition**

In a CFG, node $a$ **dominates** $b$ if every path from the start node to $b$ passes through $a$. Node $a$ is a **dominator** of $b$.

**Property**

The dominance relation is a partial order.

**Definition**

Node $a$ **strictly dominates** $b$ if $a \neq b$ and $a$ dominates $b$. 


Theorem

IF $a$ and $b$ both dominate $c$, THEN either $a$ dominates $b$ or $b$ dominates $a$. 
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Corollary
Every node $n$ has at most one immediate dominator $\text{idom}(n)$ such that
- $\text{idom}(n) \neq n$
- $\text{idom}(n)$ dominates $n$, and
- $\text{idom}(n)$ does not dominate any other dominator of $n$. 
Dominator Example
As a dataflow analysis

1. Forwards
2. Lattice is \( (\mathcal{P}(Stmts), \supseteq) \)
3. \( \cap \)
4. \( \text{out}_\ell = \text{in}_\ell \cup \{\ell\} \)
5. Start node value is \( \{\} \)
6. \( \bot = \{\text{all statements}\} \)
Computing Dominators

As a dataflow analysis

- Forwards
- Lattice is \((\mathcal{P}(Stmts), \supseteq)\)
- \(\cap\)
- \(\text{out}_\ell = \text{in}_\ell \cup \{\ell\}\)
- Start node value is \(\{\}\)
- \(\bot = \{\text{all statements}\}\)

More efficient approaches

- Lengauer-Tarjan: see Appel book section 19.2
A node $w$ is in the dominance frontier of $x$ if:
- $x$ does not strictly dominate $w$, and
- $x$ dominates a predecessor of $w$. 
\( DF_{\text{local}}(x) \): the successors of \( x \) not strictly dominated by \( x \).
\( DF_{\text{up}}(y) \): nodes in \( DF(y) \) not strictly dominated by \( \text{idom}(y) \).
\[ DF(x) = DF_{\text{local}}(x) \cup \bigcup \{ y \mid \text{idom}(y) = x \} \ DF_{\text{up}}(y). \]
Algorithm \( \text{DF}(x) \):

1. \( S = \{\} \)
2. for all nodes \( w \in \text{succ}(x) \) do
3. \quad if \( \text{idom}(w) \neq x \) then
4. \quad \quad \( S \cup = \{w\} \)
5. \quad /* \( S \) is now \( \text{DF}_{\text{local}}(x) \) */
6. for all nodes \( y \) for which \( \text{idom}(y) = x \) do
7. \quad /* below we compute \( \text{DF}_{\text{up}}(y) \) */
8. \quad for all nodes \( w \in \text{DF}(y) \) do
9. \quad \quad if \( x \) does not dominate \( w \) or \( x = w \) then
10. \quad \quad \quad \( S \cup = \{w\} \)
11. return \( S \)
Restatement of definition of DF

\[ w \in DF(x) \] for every \( x \) that dominates a predecessor of \( w \), but does not strictly dominate \( w \).

**Algorithm** \texttt{COMPUTE DFS}():

1. \texttt{for all} nodes \( w \) \texttt{do}
2. \texttt{for all} \( p \in \text{preds}(w) \) \texttt{do}
3. \hspace{1em} \( x = p \)
4. \hspace{1em} \texttt{while} \( x \neq \text{idom}(w) \) \texttt{do}
5. \hspace{2em} \( DF(x) \cup = \{ w \} \)
6. \hspace{1em} \( x = \text{idom}(x) \)