Register Allocation

- IR: arbitrary number of variables
- machine: limited number of registers

- Ideal Goal: allocate every IR variable to a register
- Secondary Goal: allocate many IR variables to registers; spill the rest, minimizing spill costs

When can two variables share a register?

- Value of a variable only matters while it is live.
- Two variables can share a register if they are never live at the same time.

Definition

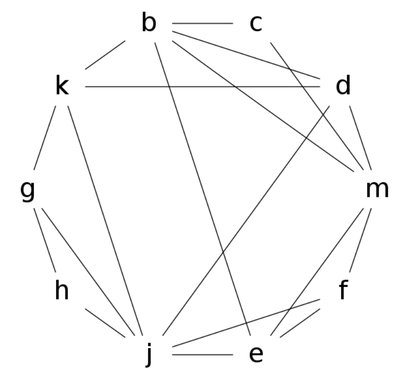
A pair of variables interfere if there is a program point at which both are live.

Interference Graph

- One vertex for each variable.
- Edge connects two variables if they interfere.

Example [Appel]

```
live: k, j
g = *(j+12)
h = k - 1
f = g * h
e = *(j+8)
m = *(j+16)
b = *(f)
c = e + 8
d = c
k = m + 4
j = b
live: d, k, j
```



Register Allocation by Graph Colouring

Goal

Assign a colour (register) to each vertex (variable) so that:

- no two interfering vertices have the same colour, and
- no more than k colours used (k = number of registers)

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NP-complete for $k > 2$.

Heuristic: Simplification

Definition

The degree of a vertex v is the number of edges incident to v.

Theorem

Let v be a vertex of degree $< k$ in graph G. Let G' be the graph obtained by removing v and all its incident edges from G. If G' is k-colourable, then so is G.

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Algorithm

Algorithm $\textsc{Colour}(G)$:

1: find vertex v of degree $< k$

2: $\textsc{Colour}(G \setminus v)$

3: assign v a colour distinct from all its neighbours

What if Simplification fails?

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When there is no vertex of degree $< k$, choose a vertex to spill, remove it from the graph, and continue.

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Coalescing

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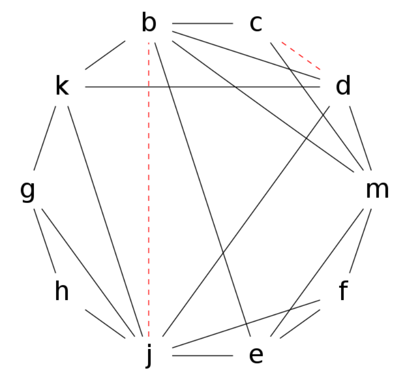

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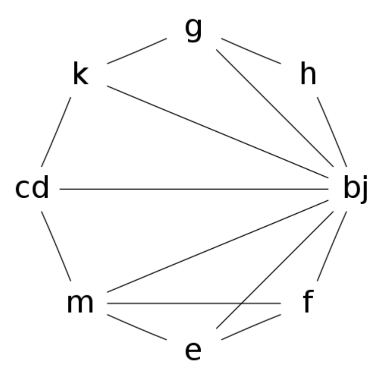

“Safe” 1 Coalescing Rules

Safe: Coalescing will not change semantics.

It is safe to coalesce *a* and *b* if:

- *a* and *b* do not interfere, OR
- *a* and *b* are never written after the copy

"Safe" 2 Coalescing Rules

Safe: Coalescing will not cause additional spills.

Option 1 [Briggs]

Coalesce if the coalesced node would have $< k$ neighbours of degree $\geq k$.

Option 2 [George]

Coalesce a and b if every node c of degree $\geq k$ interfering with a also interferes with b.