The Penn Treebank and Statistical Parsing

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Overview

1. Background
2. Penn Treebank
3. Probabilistic Context-Free Grammars
4. Maxiumum-Entropy Models
5. Semantic Parsing
6. Modern Statistical Parsing

Overview

Part of Speech Tags

- POS tagging is a process of marking each word in a corpus with a POS tag based on meaning, context, etc
- Initially, tagging was done by hand or by simple rules
- Over the last 20 years, more automated ways have been discovered, usually in the form of supervised learning
- Nine "categories" of tags (noun, verb...), corpora contained anywhere from 50-200 tags

Parse Tree

```
(S
  (NP
    (NNP  John)
  )
  (VP
    (VBZ  loves)
    (NP
      (NNP  Mary)
    )
  )
  (.  .)
)
```

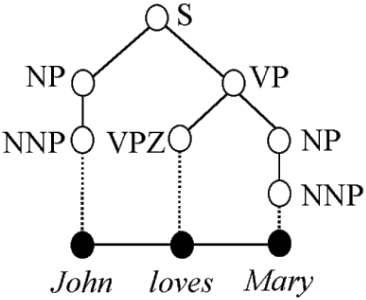

Figure 1: Example structure for *John loves Mary*.

Why Parse?

- Machine translation
- Information retrieval
  - Question-answering
  - Search
- Information extraction
  - Sentiment analysis
  - Text classification
  - Summarization

Brown Corpus

- Brown corpus was the first major POS tagged corpus available, developed by Kucera and Francis at Brown in the 60s
- Roughly 500 English works, 1 million words
- Used 87 different tags and allows compound tags
  - Such as *I'm* is PPSS+BEM for non-third person nominative pronoun and *am*
- Brown corpus is now dwarfed in size compared to modern corpora, which usually contain millions and millions of words

Context-Free Grammars

Context-Free Grammar

A context-free grammar (CFG) is a 4-tuple $G = (N, \Sigma, R, S)$ where:

N is a finite set of non-terminal symbols

Σ is the set of terminal symbols

R is the set of rules of the form $n \rightarrow \sigma$ where $n \in N$ and $\sigma \subset \Sigma \cup N$

$S \in N$ is the start symbol

Context-Free Grammars

CFG Example

$$
\begin{array}{rcl}
\text{S} & \rightarrow & \text{NP VP} \\
\text{NP} & \rightarrow & \text{NNP} \\
\text{NNP} & \rightarrow & \textit{John} \\
\vdots & &
\end{array}
$$

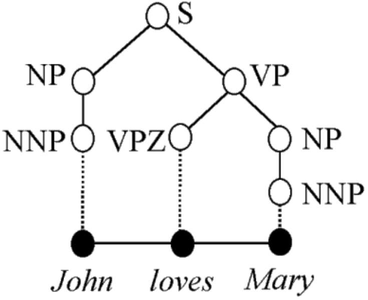

Figure 2: Example structure for *John loves Mary*.

Shift-Reduce Parsing

- Uses the "current" parse tree, the words of the sentence in a queue and partial parse trees in a stack
- Applies different state transitions until the queue is empty and the stack only contains the completed parse tree
- Possible transitions:
  - Shift: move a word from queue to the stack
  - Unary reduce: label on the top of the stack changes
  - Binary reduce: top 2 nodes on the stack are combined with a new label

Shift-Reduce Parsing

- Stanford parser uses a multiclass perceptron to determine the next transition
- POS tags are not assigned, but rather used as features
- Trained by iterating over the parse trees until "converged"
  - Start from base state and apply states until the actual tree can no longer be rebuilt
  - Once wrong, each transition's weights can be adjusted

CYK Algorithm

- Cocke–Younger–Kasami (CYK) algorithm is a parser for CFGs
- CYK is a dynamic programming algorithm with run time $\Theta(n^3|G|)$, where n is the sentence length
- Considers every possible consecutive subsequence of words $(i, \ldots, j)$ if the sequence can be generated by a rule r
- Does so for subsequences of length 1, 2, ...
- For length 2 and greater, also consider all possible partitions into two parts and check for rules that can lead to such a production

Classical Parsing

- Context-Free Grammars were used as symbolic parsing tools
- Did not scale well, many possible parses due to ambiguities
- Issues with this strategy:
  - Constrained grammars limit weird parses, but lead to many sentences having no parse
  - Less constrained grammars have a broad search space, simple sentences will end up with many possible parses
- Solution: need mechanism that finds the most likely parse out of all the possible parses, statistical parsing!

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- Solution: need mechanism that finds the most likely parse out of all the possible parses, statistical parsing!
- But datas!

Overview

The Penn Treebank

- Several projects have extended the Brown corpus tagset
- These other projects include anywhere from 100 to 200 tags, the rationale being that more tags would lead to better classifications of words
- The Penn treebank consists of over 4.5 million words, but only 48 tags
- Their goal was to reduce redundancies by considering lexical and syntactic information
- Created by Marcus, Marcinkiewicz at U. Pennsylvania and Santorini at Northwestern
- Most recent release is 20 years old now and still requires a licensing fee and a cd-drive

Recoverability

- Brown corpus distinguishes five forms of verbs, VB (singular, past tense etc)
- The same paradigm is followed for *have* – although *have* is assigned it's own base tag, HV
- The Brown corpus also distinguishes 3 forms of *do* and 8 forms of *be*
- However, all these distinctions are lexically recoverable, so they are not included in the Penn treebank tagset

Consistency

- The Penn treebank also removed some inconsistent tags
- For example, *there* and *now* are always tagged as adverb, but *here* and *then* are tagged as adverb or as nominal adverb

Syntactic Function

- The Penn treebank encodes a word's syntactic function in the tag when possible
- For example, *one* is always labeled as a number by the Brown corpus, but it is labeled as a noun when appropriate in the Penn treebank
- Another example, the word *both* receives different tags depending on context, such as *the boys both* (postnominal), *both the boys* (prenominal) or *both of the boys* (noun phrase head)

Indeterminacy

- In some cases, annotators may simply not know how to tag a word
- To account for this, the treebank allows for disjunctions of tags
- Any combination of tags is allowed, however the vast majority of cases are restricted to a small set of two-tag options

POS Tagging

- The corpus is POS-tagged by an automated stage and a manual correction stage
- Initially used a stochastic algorithm called PARTS (Church, 1988), but now uses a “cascade of stochastic and rule-driven taggers”
- Manual correction stage uses output of automated stage
- Four annotators with graduate training in linguistics
- Experiment showed that accuracy and inter-annotator agreement rates were higher when correcting the automated output versus tagging from scratch

Bracketing

- Bracketing is a “skeletal syntactic structure” using an “impoverished flat context-free notation”

```
( (S
    (NP Battle-tested industrial managers here)
     always
    (VP buck
    up
    (NP nervous newcomers)
    ...))
```

- Basically just a relaxed parse tree for a CFG

Bracketing

- *Fidditch* is a deterministic parser developed at the University of Pennsylvania in 1983
- The parser produces a bunch of chunks that must be "glued" together manually
- Manual correction done on over half the corpus
- Special *null elements* included because they can be used to infer additional information like *predicate-argument structure*

Annotations

- Penn Treebank introduced the idea of an "annotated parse tree" to circumvent problems with ambiguity and/or consistency
- The X label is used whenever unsure of syntactic category
- Global ambiguity in determining the correct attachment point handled by "pseudo-attachment" notation

a boatload of warriors $\underbrace{\text{blown ashore 375 years ago}}_{\text{pseudo-attached}}$

Future of Penn Treebank

- In 1994, released paper on making the predicate-argument structure more explicit
- No other literature could be found
- Used as a training corpus for POS taggers and parsers
- Gold standard for evaluating new parsers
- Used especially within University of Pennsylvania
- Don't hear about it as much anymore, perhaps due to restricted availability, and many free alternatives [1]

[1] `http://en.wikipedia.org/wiki/Treebank#Syntactic_treebanks`

Overview

Motivating Example

Example from Ratnaparkhi (1999)

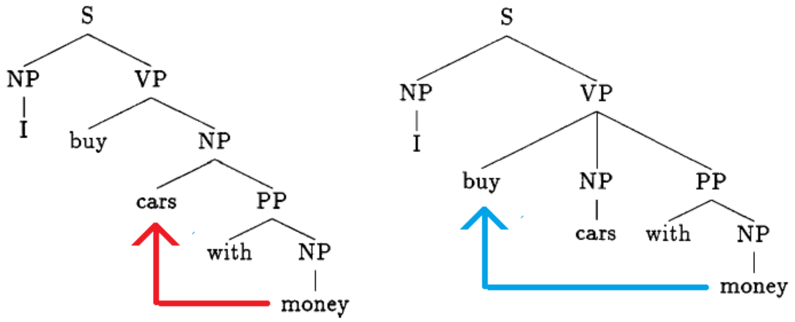

Figure 3: Unlikely parse (left); Likely parse (right)

- *Money* refers to *buy* (verb-phrase), not *cars* (noun-phrase)
- Both parses are legal, but we want the one that is more likely

Statistical Parsers

- Distinguishing the more likely parse requires semantic knowledge
- Or, for instance, the likelihood of someone buying a car *that contains* money versus the likelihood of someone buying a car *using* money
- Superior performance achieved using statistical parsers to learn a probabilistic context-free grammar from a treebank

Probabilistic Context-Free Grammars

- This section was adapted from course notes by Collins (2011)
- Key idea in probabilistic context-free grammars (PCFG) is to extend the definition of CFGs by assigning a probability $p(t)$ to each possible parse tree t that the grammar produces

$$p(t) \geq 0, \quad \sum_t p(t) = 1$$

- This seems an impossible task at a glance, the set of possible parse trees is large if not infinite
- Turns out there is a nice trick!

PCFG

- A PCFG consists of:
  - A CFG $G = (N, \Sigma, R, S)$
  - A parameter $q(\alpha \to \beta)$ for each rule $\alpha \to \beta \in R$
- $q(\alpha \to \beta)$ is the conditional probability of producing through the rule $\alpha \to \beta$ given the non-terminal being expanded is α
- Then, we can define $p(t)$ as

$$p(t) = \prod_{i=1}^{n} q(\alpha_i \to \beta_i)$$

PCFG Training

- CFG parameters are taken from all the trees in the corpus
  - eg Σ is the set of all terminals found
- The maximum likelihood estimate of q is

$$q(\alpha \rightarrow \beta) = \frac{\text{Count}(\alpha \rightarrow \beta)}{\text{Count}(\alpha)}$$

where Count($\alpha \rightarrow \beta$) is the number of times the rule $\alpha \rightarrow \beta$ is seen and Count(α) is the number of times the non-terminal α is seen in the corpus

Parsing with PCFGs

- Given all possible parse trees T of a sentence, we can use the PCFG to find the most likely parse with

$$\arg\max_{t \in T} p(t)$$

- CYK algorithm can be extended for use with PCFGs
- Most likely parse is sometimes called the "Viterbi parse"
  - Viterbi algorithm finds most likely sequence of states in a hidden Markov model
  - The term became popular, and started being used simply to mean the "most likely parse"

Problems with PCFGs

- PCFGs exhibit two mean weaknesses:
  1. Lack of sensitivity to lexical information
  2. Lack of sensitivity to structure preferences

Lack of Sensitivity to Lexical Information

$p(t) =$
$q(S \rightarrow NP\ VP) \cdot q(NP \rightarrow NNP) \cdot$
$q(NPP \rightarrow John) \cdot q(VP \rightarrow VPZ\ NP) \cdot$
$q(VPZ \rightarrow loves) \cdot q(NP \rightarrow NNP) \cdot$
$q(NNP \rightarrow Mary)$

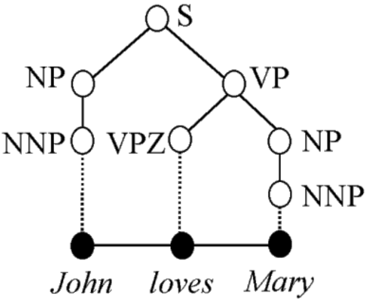

Figure 4: Example structure for *John loves Mary*.

Lack of Sensitivity to Lexical Information

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$q(NPP \rightarrow John) \cdot q(VP \rightarrow VPZ\ NP) \cdot$
$q(VPZ \rightarrow loves) \cdot q(NP \rightarrow NNP) \cdot$
$q(NNP \rightarrow Mary)$

- PCFG makes a strong independence assumption

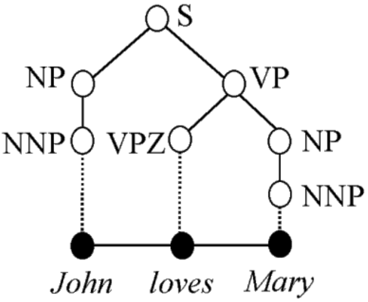

Figure 4: Example structure for *John loves Mary*.

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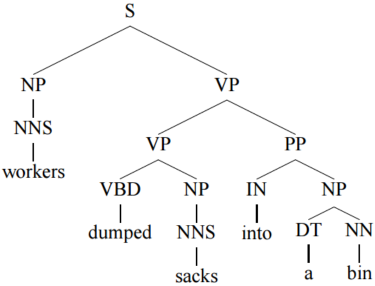
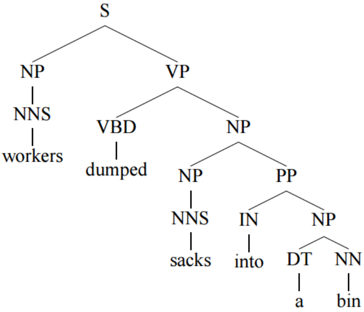


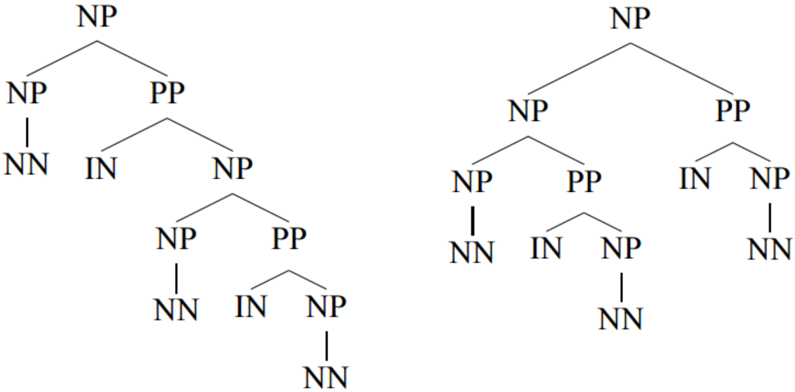


Lack of Sensitivity to Lexical Information

S	$\rightarrow$	NP VP		S	$\rightarrow$	NP VP
NP	$\rightarrow$	NNP		NP	$\rightarrow$	NNP
VP	$\rightarrow$	VBD NP		VP	$\rightarrow$	VBD NP
NP	$\rightarrow$	NNS		NP	$\rightarrow$	NNS
PP	$\rightarrow$	IN NP		PP	$\rightarrow$	IN NP
NP	$\rightarrow$	DT NN		NP	$\rightarrow$	DT NN
VP	$\rightarrow$	**VP PP**		**NP**	$\rightarrow$	**NP PP**

- Parse tree picked depends only on $q(\text{VP} \rightarrow \text{VP PP})$ and $q(\text{NP} \rightarrow \text{NP PP})$

Lack of Sensitivity to Structural Preferences



Lack of Sensitivity to Structural Preferences

- Searching the corpus, we find that the first structure is roughly twice as common as the second structure
- However, the PCFG assigns an identical probability to both trees since they have the same rules

Lexicalized PCFG

- Lexicalized PCFGs address the first of the two weaknesses
- Requires lexicalization of the corpus
- LPCFGs are essentially PCFGs were each non-terminal in each rule includes additional lexical information

$$\text{S} \rightarrow \text{NP VP}$$

becomes

$$\text{S(questioned)} \rightarrow \text{NP(lawyer) VP(questioned)}$$

- Otherwise, LPCFGs are the same as PCFGs
    - Larger set of non-terminals
    - Larger set of rules

Lexicalization of the Corpus

- For each context-free rule in the corpus, identify the “head” of of the rule
- Then, lexical information can be propagated bottom-up through the parse tree from the head to the parent

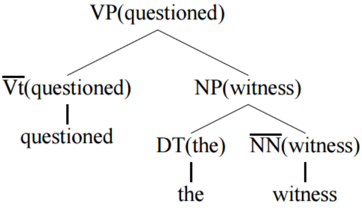


Figure 5: Lexicalized parse tree for *question the witness*.

Formal Definition

Lexicalized Probabilistic Context-Free Grammar

A lexicalized probabilistic context-free grammar (LPCFG) is a 6-tuple $G = (N, \Sigma, R, S, q, \gamma)$ where:

N is a finite set of non-terminal symbols

Σ is the set of terminal lexical items

R is the set of rules in one of the following forms:

$$X(h) \rightarrow_1 Y_1(h) Y_2(m), \quad X(h) \rightarrow_2 Y_1(m) Y_2(h), \quad X(h) \rightarrow h$$

where $X, Y_1, Y_2 \in N$ and $h, m \in \Sigma$

$S \in N$ is the start symbol

Formal Definition

Lexicalized Probabilistic Context-Free Grammar

For each $r \in R$, there is a $q(r) \geq 0$, and for any $X \in N, h \in \Sigma$

$$\sum_{LHS(r)=X(h)} q(r) = 1$$

For each $X \in N, h \in \Sigma$, there is a $\gamma(X, h) \geq 0$ and

$$\sum_{X,h} \gamma(X, h) = 1$$

Finally, the probability of a derivation of $r_1, \ldots, r_n$, $r_i \in R$, is

$$\gamma(LHS(r_1)) \times \prod_i q(r_i)$$

LPCFG Training

- The number of rules, and therefore parameters, is huge
- Make use of smoothing techniques to estimate each $q(r)$

$$q(\text{S(questioned)} \rightarrow_2 \text{NP(lawyer) VP(questioned)})$$
$$= P(R = \text{S} \rightarrow_2 \text{NP VP}, m = \text{lawyer} | X = S, h = \text{questioned})$$

- Use chain rule to decompose into two terms

$$= P(R = \text{S} \rightarrow_2 \text{NP VP} | X = S, h = \text{questioned})$$
$$\times P(m = \text{lawyer} | R = \text{S} \rightarrow_2 \text{NP VP}, X = S, h = \text{questioned})$$

LPCFG Training

- Find smoothed estimates for both terms
- Use MLE for $q(\mathrm{S} \rightarrow_2 \mathrm{NP\ VP} | S, \mathrm{questioned})$ and $q(\mathrm{S} \rightarrow_2 \mathrm{NP\ VP} | S)$ as derived from the corpus
- Then an estimate of the first term can be

$$a_1 \cdot q(\mathrm{S} \rightarrow_2 \mathrm{NP\ VP} | S, \mathrm{questioned}) + b_1 \cdot q(\mathrm{S} \rightarrow_2 \mathrm{NP\ VP} | S)$$

- A similar method can be used to estimate the second term
- Putting both together yields

$$(a_1 \cdot q(\mathrm{S} \rightarrow_2 \mathrm{NP\ VP} | S, \mathrm{questioned}) + b_1 \cdot q(\mathrm{S} \rightarrow_2 \mathrm{NP\ VP} | S)) \times$$
$$(a_2 \cdot q(\mathrm{lawyer} | \mathrm{S} \rightarrow_2 \mathrm{NP\ VP}, \mathrm{questioned}) + b_2 \cdot q(\mathrm{lawyer} | \mathrm{S} \rightarrow_2 \mathrm{NP\ VP}))$$

Parsing with LPCFGs

- Parsing strategy remains the same as for with PCFGs, an extended version of the CYK algorithm can be used to find the most likely parse tree
- Slower than PCFGs since grammar is much larger

Performance

- Collins (1996) compared lexicalized PCFGs to non-lexical PCFGs and showed that lexicalization in this fashion achieved significantly better parsing results; 85% compared to 75%
- Magerman (*Statistical Decision-Tree Models for Parsing*, 1995) showed similar results with a decision-tree learned parser
- Later, Collins (1997) improved his results with more sophisticated models to get 88% correct rates

Overview

History

- Ratnaparkhi (1999) provides the core model used by Charniak (2000)
- Interesting facts: Ratnaparkhi is also from the University of Pennsylvania, but Charniak is from Brown University

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"Charniak (1997) and Collins (1997) do not use general machine learning algorithms, but instead develop specialized statistical estimation techniques for their respective parsing tasks." Ratnaparkhi (1999)

- Idea of applying maximum entropy to NLP first proposed by Berger (1996) for machine translation
- Perhaps the first example of a general machine learning technique applied to parsing

Model Intuition

- Idea: *score* a parse by examining its derivation
- Derivations are built from the bottom up by adding *actions* to a list (similar to *shift-reduce* parsing)
- Every derivation corresponds to a unique parse tree (a complete parse T has exactly one derivation $d = \{a_1, \ldots, a_n\}$)
- Each action is scored using the maximum entropy approach
- Parsing is again just a standard search problem for the highest scoring parse

Maximum Entropy Framework

- Also referred to as a *log-linear* approach (a kind of *discriminative* model)
- The probability of taking an action a is conditioned on H, which includes some chosen *history* and *surrounding context* from the partial derivation

$$P(a|H) = \frac{1}{Z(H)} e^{\lambda_1 f_1(a,H) + \ldots + \lambda_m f_m(a,H)}$$

- $f_1, \ldots, f_m$ provide the scalar features of a, given H (fixed)
- λ_i are weights in $(-\infty, \infty)$ indicating the relative importance of feature i, depending on a and H (estimated)
- $Z(H)$ simply normalizes so that $\sum_a p(a|H) = 1$, for each H

Maximum Entropy Framework

- Advantages:
  - Model is easily changeable
  - Can incorporate arbitrarily diverse information
  - No independence assumption required for features
  - Smoothing comes for free
- Disadvantages:
  - Can be difficult to incorporate prior linguistic knowledge in a predictable manner
  - Feature selection is an "art"; evidence should have little noise (otherwise need smoothing)
- In other words, to design a parser, one only needs intuition about what evidence is "useful" to distinguish likely actions from unlikely actions in a derivation.

Alternative Representations

- Again, the model is $P(a|H) = \frac{1}{Z(H)} e^{\lambda_1 f_1(a,H) + \ldots + \lambda_m f_m(a,H)}$
- Taking the log of both sides,

$$\ln P(a|H) = \sum_{i=1}^{m} \lambda_i f_i(a, H) - \ln(Z(H))$$

gives a more familiar form (logistic regression).

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  gives a more familiar form (logistic regression).
- Let $g_0(a, H) = 1/Z(H)$, $g_i(a, H) = e^{\lambda_i f_i(a,H)}$, then

$$p(a|H) = g_0(a, H) \cdots g_m(a, H)$$

  is the form used by Charniak (2000) to connect each feature's contribution back to conditional probabilities.

Parameter Estimation

- *Generalized Iterative Scaling* algorithm (Darroch & Ratcliff, 1972)
- Let $\alpha_i = e^{\lambda_i}$, then

$$P(a|H) = \frac{1}{Z(H)} \prod_{i=1}^{m} \alpha_i^{f_i(a,H)}$$

- Introduce a "correction" feature $f_{m+1} = C - \sum_{i=1}^{m} f_i(a, H)$
- Iteratively update all α_i by computing the expectation of each f_i,

$$\sum_a P(a|H) f_i(a, H)$$

- Guaranteed to converge monotonically to the optimal solution

Maximum Likelihood Estimation

- Berger (1996) states that this algorithm gives the maximum likelihood estimate for the set of models under the given form

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"The duality is appealing since as a maximum entropy model, it will not assume anything beyond the evidence, and as a maximum likelihood model, it will have a close fit to the observed data." Ratnaparkhi (1999)

Training and Searching

- Trained using the Penn Treebank
- Beam-search
- Achieves state of the art performance

Charniak's Model

- Uses a top-down generative parser to narrow down the search space to a set of candidate parses
- Then uses the methods from Ratnaparkhi (1999)
- However, instead of feature selecting only the common features, includes all features and smooths with deleted interpolation
- Also uses a less flexible formulation with a specific decomposition of probabilities, equivalent (?) to using 6 particular features and $Z(H) = 1$ (see eq. 7 from Charniak, 2000)

Overview

Semantic Parsing

- Often a traditional parse tree is not enough for an application
- Parse trees only reflect a language's grammar, which gives few clues as to the meaning of a sentence
- For instance, in question-answering tasks, it is much easier to work with *logical forms*
- The task of parsing to a logical form is called "semantic parsing"

Logical Forms

- A *logical form* can be thought of as a parse tree for a special grammar where the rules in the derivation encode logical relationships[2]
- Most often this is written using lambda calculus,

Example from Kwiatkowski et. al (2010)

Sentence: Which states border Texas?
Meaning: $\lambda\ x.state(x) \wedge next\_to(x, Texas)$

- In 2007, Collins went on to coauthor a paper titled *Online Learning of Relaxed CCG Grammars for Parsing to Logical Form.*

[2] Formalized as something called a *combinatory categorial grammar* (CCG)

Dependency-Based Compositional Semantics

- A hot topic in the field of semantic parsing is *Dependency-based Compositional Semantics* (DCS), first introduced by Liang et al (2011)
- Fun fact: Michael Jordan was a coauthor!
- Inspired by *Discourse Representation Theory* (DRT)
- They define a new semantic representation called DCS, which is a tree corresponding to a *latent* logical form, that is much less stringent than lambda calculus

Dependency-Based Compositional Semantics

- Interestingly, they use a discriminative log-linear feature model and beam-search, much like the Ratnaparkhi (1999) model discussed earlier
- Model trained using an EM-like algorithm and evaluated on a question-answer corpus
- This approach outperforms all existing semantic parsers, without even learning from annotated logical forms
- Success owed to using a flexible representation in a latent factor space

Overview

Charniak Extension

- Charniak and Johnson (*Coarse-to-Fine n-Best Parsing and MaxEnt Discriminative Reranking*, 2005) build a discriminative ranker of parses produced by the maximum entropy parser
- Get top 50 parses, exploiting a "course-to-fine" heuristic they defined
- Improvements over the vanilla maximum entropy model, achieving about 91% on sentences up to 80 words

Stanford's Factored Parser

- *Fast Exact Inference with a Factored Model for Natural Language Parsing* (Klein and Manning, 2003)
- Version that is combined with a PCFG parser is available online for play: `http://nlp.stanford.edu:8080/parser/`
- Source code also available for download, including some more recent models

Neural Networks

- Compositional Vector Grammars (CVG)
  - *Parsing With Compositional Vector Grammars* (Socher et al., 2013)
  - Andrew Ng and Stanford
  - Combines PCFG with a recurrent neural network to learn a "syntactico-semantic" representation
  - Faster and more accurate than Stanford's factored parser
- Dependency Parser
  - *A Fast and Accurate Dependency Parser using Neural Networks* (Chen and Manning, 2014)
  - Dependency parsing establishes relationships between head words and the words that modify those heads
  - Their parser is transition based, where decisions are made by a neural network

Conclusions

- CFGs served as a decent foundation for parsing, but were fundamentally flawed
- Development of PCFGs "was one of the biggest breakthroughs in natural language processing in the 1990s"
- Modern parsers improved by use of discriminative techniques (maximum entropy)
- Stanford's newest neural network models are state of the art with 92% accuracy, fast, and freely available

Bioinformatics

- PCFGs are used to predict RNA structure
- Trained using a database of RNA structures in a similar fashion to using a treebank
- A maximum probability parse corresponds to a maximum probability RNA structure
- Can model long range interactions, pairwise structure and nested structures
- Pseudoknots (essentially loops in the structure) cannot be modeled however

Bioinformatics

- Generally a grammar is chosen by modeling RNA sequence structure[3]

$$\begin{aligned} S &\rightarrow LS \mid L \\ L &\rightarrow s \mid dFd \\ F &\rightarrow dFd \mid LS \end{aligned}$$

- Transition probabilities learned from training data
- Used to rank most likely structures
- *Biological Sequence Analysis: Probabilistic Models of Proteins and Nucleic Acids* (Durbin, 1998)

[3] `http://en.wikipedia.org/wiki/Stochastic_context-free_grammar#Building_a_PCFG_model`