CS341 ASSIGNMENT 3 NOTES

Question 1

Part A

Many people did not get this proof right. Since you have a counter-example in part b, you should make sure that your proof fails on this counter-example or something is wrong.

A very common mistake was to say that let $(g_1, g_2, g_3 \ldots, g_{n+1})$ be the greedy solution and let $(o_1, o_2, \ldots, o_n)$ be an optimal solution. Let i be the first position at which g_i and o_i differ. Since the greedy algorithm takes as much as it can at each step, we know that $g_i > o_i$. So far so good. Thus the total value for the optimal solution is currently less than the greedy solution and so the optimal solution will have to make up this difference with smaller valued coins. But since the coins are smaller valued, it will take more of them and thus the optimal solution will take at least as many coins as the greedy. This is incorrect! Consider the coin system $(4, 3, 1)$ and you wish to make change for 6. The greedy solution is $(1, 0, 2)$ while the optimal is $(0, 2, 0)$. The first position where they differ is on the 4 coin (greedy takes it, optimal doesn't). So the optimal solution has to make up the difference of 4 in smaller coins. But since it uses 3-valued coins whereas the greedy solution uses 1-valued, it can make up this difference without using more coins. In other words, it is erroneous to assume that when transforming the greedy solution into an optimal solution, we can only replace one coin at a time (eg $(1, 0, 2) \rightarrow (0, 0, 6)$).

Another common error was to say that greedy works because all coins are integer multiples of the next smaller coin (not true for dimes-quarters) or because all coins are linear combinations of smaller coins (true in $(4, 3, 1)$ system).

Part B

Was very well done.

Part C

Similar to part A

Question 2

This question was quite well done. The only common error I encountered was that quite a few students didn't understand the criteria of part c. The pair M_i, M_{i+1} of maximum k_{i+1} means the pair that share the largest dimension (ie the number of columns of the first matrix or, equivalently, the number of rows of the second matrix).

Question 3

Though many students knew that induction was the way to go, very few were able to complete the induction correctly. The most common line of reasoning was to remove a leaf from the graph. Now by the induction hypothesis we know that the greedy algorithm on the remaining graph ($n - 1$ verticies) gives a maximal matching. Now add the leaf back in. Two cases:

- case 1, we can add the leaf edge to the matching, so we do so. In this case there are two things to prove, 1) that what you have just done is the same as what the greedy algorithm outputs. 2) that this is optimal. Many students just stated these without justification.
- case 2, we add the leaf edge to the matching and then run the algorithm on the graph to "fix" the rest so that it is a legal matching. But if we do this, we've basically thrown out the induction hypothesis since we no longer have the output of the greedy algorithm on the subgraph. Many students claim that this is now a maximal matching, but did not justify why 1) this is the same as the greedy algorithm's output or 2) this is maximal.

Many students also tried to argue some kind of proof using the theory of matchings in bipartite graphs from C&O. Only a few were able to do so successfully. A correct proof along these lines is: suppose the greedy solution is not maximal. Then there exists an augmenting path from v_1 to v_2 where v_1 and v_2 are unmatched vertices. Call this path: $v_1 u_1 u_2 u_3 u_4 \ldots u_{2k-1} u_{2k} v_2$. For $i = 1 \ldots k$, the edge (u_{2i-1}, u_i) is in the matching and the other edges in the path are not. Since (u_1, u_2) is in the matching, at some point during the algorithm u_1 or u_2 must have been a leaf. But v_1 is unmatched so it was never deleted, so u_1 cannot have ever been a leaf. But in order for u_2 to be a leaf, u_4 must have been a leaf first. Continuing like this we see that u_{2k} must have been a leaf. But v_2 is unmatched so this is impossible. Contradiction.

Questions 4 and 5

Both of these questions were very well done.