Assignment 4. CS341, Winter 2011

Distributed Tuesday, March 1, due March 15, 1pm, 2011. Hand in to the assignment boxes on the 3rd floor of MC, before 1pm.

1. (10 marks) In the deterministic linear time selection algorithm, we partitioned the elements into groups of 5. What if

   (a) we partition the elements into groups of 3?

   (b) we partition the elements into groups of 7?

   For each of above situations, can you repeat the analysis similar to that of 5? In each case, will the time complexity remain linear? (If the time complexity is linear, prove it. If it is not, you only need to give the recurrence relationship.)

2. (5 marks) Give a simple adversary argument to show that its takes n comparisons, in the worst case, to check if an array of length n contains a 1.

   When $n = 3$, this is a street game called "Three card Monte" played by street dealers to steal tourists' money. The dealer has three cards, say the Queen of Hearts and two clubs. The dealer shuffles the cards face down on a table (usually slowly enough that you can follow the Queen), and then asks the tourist to bet on which card is the Queen. In principle, the tourist's odds of winning are at least 1/3. In practice however, the tourist never wins, because the dealer cheats. There are actually 4 cards; before he even starts shuffling the cards, the dealer palms the queen or sticks it up his sleeve. No matter what card the tourist bets on, the dealer turns over a black card. If the tourist gives up, the dealer slides the queen under one of the cards and turns it over, showing the tourist 'where the queen was all along'. If the dealer is really good, the tourist won't see the dealer changing the cards and will think maybe the queen was there all along, and he just wasn't smart enough to figure that out. As long as the dealer doesn't reveal all the black cards at once, the tourist has no way to prove that the dealer cheated! The dealer is playing the adversary here.

3. (10 marks) A *scorpion* graph is an undirected graph G of the following form: there are three special vertices called the *sting*, the *tail*, and the *body*, of degree 1, 2, and $n - 2$, respectively. The sting is connected only to the tail; the tail is connected only to the sting and the body; and the body is connected to all vertices except the sting. The other vertices of G (the feet, $n - 3$ of them) may be connected to each other arbitrarily.

   Give an algorithm that determies whether G is a scorpion graph, using only $O(n)$ probes to the adjacency matrix of G.

   Hint: Start with an arbitrary vertex v. If degree of v is $n - 2$, then it is body; if degree of v is 1 or 2, then either v is sting (degree 1) or tail (degree 2) or one of its neighbors is body. If degree of v is between 3 to $n - 3$, then this is one of the feet. Then let N be the set of neighbors of v, and S be the set of vertices not connected to v. Then the body is in N, and the sting and tail are in S. Take $x \in N$ and $y \in S$. If they are connected, y cannot be the sting. If they are not, and this happens to x twice (for two y's), then x cannot be the body.

4. (10 marks) Prove that if A is unsolvable, then $\bar{A} = \Sigma^* - A$ is also unsolvable.