

## Lec 2: Underlying Concepts

2-1

4] Pseudocode: \* Careful some lines may not be  $O(1)$   
eg  $A[n] = 0$  // initialize all  $n$  to 0  
 $\in \Theta(n)$

5] A number  $a$  (base 10)  $\log a + 1$   
• requires  $\sim \log a$  bits to represent it in binary

$a * b$  takes  $O((\log a)(\log b))$   $\sim$  school method  
 $O((\log a)(\log b)^{0.59})$   $\sim$  efficient mult  
• see later

Often set a word size (real world: 64 bits)  
• elementary word size ops are then  $O(1)$  <sup>constant</sup>  
• add/sub, compare, read/write a word,

Numbers may still get big:

Product( $A, n$ )

$S \leftarrow 1$

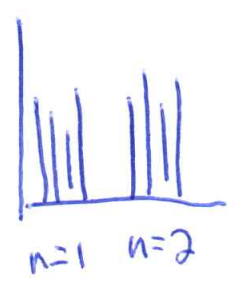
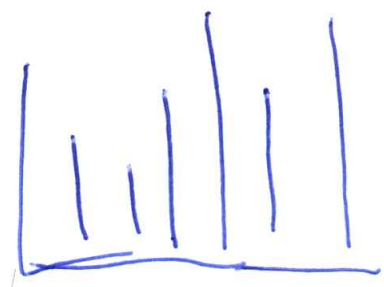
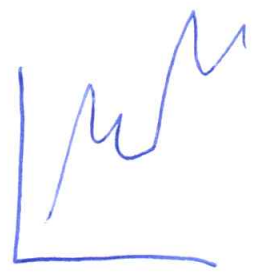
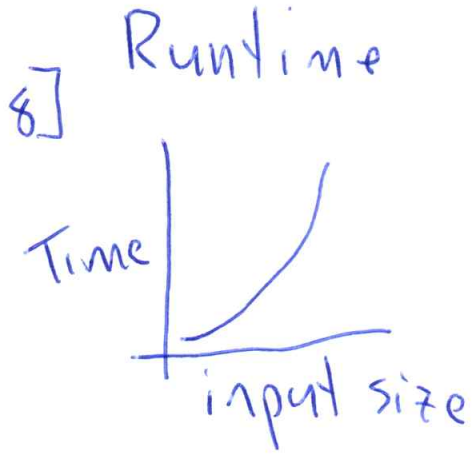
for  $i = 1$  to  $n$

$S = S * A[i]$

what if  $A$  stores  
 $INT\_MAX$ ,

what does C/C++  
• overflow  $O(n)$  do?

what if we allow  
to have as much space  
as needed?  $O(n^2)$



a] Random Algs => expected runtimes

$$f(n) = 7n^2 + 13n + 27 \in O(n^2)$$

$$10^{100} n \in O(n)$$

$$2^{n+1} \in O(2^n)$$

$$(n+1)! \in O(n!) ? \quad \times \quad \in O(n+1)!$$

$$\log(n!) \in \Theta(n \log n)$$

$$(n-1)! \in O(n!) ? \quad \text{yes but also } \overset{\text{little}}{o}(n!) \text{ its } O((n-1)!)$$

Challenge: If you have a RAM model and allow unlimited # bits for each memory location and charge 1 for arithmetic and shifts, how can you "cheat" and sort in  $O(n)$  steps?

Hint: Do all  $n^2$  pairwise comparisons in 1 subtraction

14) Suppose Alg A is  $O(n^2)$   
Alg B is  $O(n \log n)$

Which is faster?

• This is like  $x \leq 5, y \leq 10$   
which is smaller?

=> To compare Algs, need tight bounds  
 $\Theta(n^2)$  vs  $\Theta(n \log n)$  *better*

2-Sum Alg 3

(sorted) Target: 23

A: 2 3 5 11 12 20 22 24

*i* ↑     ↗     ↑

sum 22 too small *i++*     sum 24     j

sum 26 2+24 too big

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3-Sum

Faster algs?

For many years NO!  
but now 2014, 2017.