Kleene's Theorem

A language L is regular $\iff L = L(M)$ for some DFA M.

Recall: the notation $L(M)$ denotes the language of automata M.

A DFA $M = (\Sigma, Q, q_0, A, \delta)$.

We will show part of this proof over the next few lectures - CS 360 provides a complete proof.

Simulating a DFA

```
int state = q0
char ch
while NOT EOF do
  read ch
  case state of
    q0:
      case ch of
        a:  state = ...
        b:  state = ...
        ...
    q1:
      case ch of
      ...
    qn:
      case ch of
        a:  state = ...
        b:  state = ...
        ...
  end case
end while
return state memberof A
```

DFAs with Actions

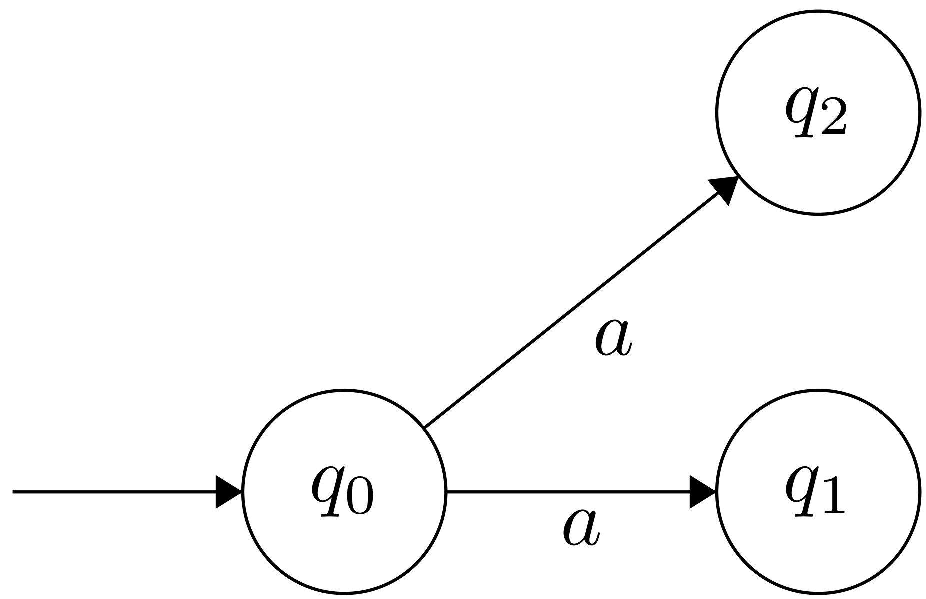
Typically, when working with finite automata in a theory class, we are mainly concerned with determining if a given input string is accepted or rejected.

In CS 241, we are using the finite automata to recognize the patterns for valid tokens (a Scanner); i.e. we want to recognize if the input would be accepted or ERROR and also have actions on the transitions.

- Compute the value of a number as we read it in digit by digit.
- Build a token character by characer, emit the token.

Nondeterministic Finite Automata (NFA)

An NFA allows more than 1 transition on a given alphabet symbol from a state.



More complex? More powerful? Which path to take?

Formal Defintion - NFA

Formally an NFA M is a 5-tupple $M = (\Sigma, Q, q_0, A, \delta)$

- Σ - non-empty, finite alphabet
- Q - non-empty, finite set of states
- q_0 - start state
- $A \subseteq Q$ - set of accepting states
- $\delta : (Q \times \Sigma) \rightarrow$ **Subset of** Q, $2^{|Q|}$ possible subsets.

Accept if there exists at least one path through M that leads to an accepting state; reject if no such path exists.

Note: this doesn't help us choose the correct path.

Acceptance in an NFA

Since the NFA transition function is different than a DFA, we must fix the extended transitions function and the definition of acceptance.

δ^* for NFAs: (Subset of states) $\times \Sigma^* \rightarrow$ (Subset of states)

Base Case: $\delta^*(q_{subset}, \epsilon) = q_{subset}$

Recursive Case:

$$\delta^*(q_{subset}, cw) = \delta^*(\left(\bigcup_{q \in q_{subset}} \delta(q, c), w\right)$$

Acceptance: NFA M accepts w if $\delta^*(\{q_0\}, w) \cap A \neq \emptyset$

Simulating an NFA

```
states = {q0}
while NOT EOF do
  read ch
  states = Union(delta(q, ch) for each q in states)
end while
if states INTERSECT A != NULL
  Accept
else
  Reject
```

Conversion: NFA to a DFA

We will use the **Subset Contruction Method**.

Idea: having read in part of the input string, we trace all paths that could be followed in the NFA; i.e. we have a set of states of the NFA that we could be in.

This “set of states" will be a single state in the DFA.

Label each DFA state as a “set of states" from the NFA.

Note: In CS 241, it is okay to skip drawing the state for $\{\emptyset\}$ and the transitions that would lead there.

- The start state of the DFA is $\{q_0\}$.
- For a state q_{subset} of the DFA and a given symbol c in the alphabet, $c \in \Sigma$, determine the “set of states" in the NFA that can be reached from each $q \in q_{subset}$ on c.

  Create a new state in the DFA if this new “set of states" does not currently exist.
- Repeat the previous step until all DFA states have a transition on each alphabet symbol.
- Mark all states in the DFA as accepting if the state includes an accepting state from the NFA.