

# CS 484/W13: Assignment 1, Image Formation and Lighting

Due: 13:00, Tues. Jan 29th, 2013.

## 1. Perspective projection [4 marks]

### (a) Trucco and Verri, Question 2.3. [2 marks]

Show that, in the pinhole camera model, three collinear points in 3-D space are imaged onto three collinear points on the image plane.

Recall the equations of perspective projection:

$$x = \frac{f}{Z}X; \quad y = \frac{f}{Z}Y$$

where  $\mathbf{P} = (X, Y, Z)^T$  is the 3-D point and  $\mathbf{p} = (x, y)^T$  is the 2-D point, and  $f$  is the focal length.

### (b) Horn, Exercise 2-12. [2 marks]

Straight lines in the three-dimensional world are projected as straight lines into the two-dimensional image. The projections of parallel lines intersect at a *vanishing point* in the image plane. When does the vanishing point of a family of parallel lines lie at infinity?

In the case of a rectangular object, a great deal of information can be recovered from lines in the images and their intersections. The edges of a rectangular solid fall into three sets of parallel lines, and so give rise to three vanishing points. In technical drawing one speaks of one-point, two-point, and three-point perspective. These terms apply to the cases in which two, one, or none of the three vanishing points lie at infinity. What alignment between the edges of the object and the image plane applies in each case? (Draw a diagram and/or describe in words.)

## 2. Combining images of different exposure [8 marks]

**Finding the camera response function.** Suppose we have  $P$  images taken at different exposure times  $\Delta t_j, j = 1 \dots P$ . Let  $I_{ji}$  be the brightness of pixel  $i$  in image  $j$ . In each image

$i = 1 \dots N$  pixels are measured. The brightness of each pixel is

$$I_{ji} = f(E_i \Delta t_j)$$

where  $I$  is the image brightness,  $E$  is the (unknown) scene brightness, and  $f()$  is the (unknown) camera response function that maps the image brightness onto a discrete set of pixel values. Note that  $f$  is a function of the *product* of the scene brightness and the exposure time. That is, if the exposure time is doubled, the input to  $f$  doubles. The image brightness, however, is generally a nonlinear function of scene brightness.

Given a series of measurements, we solve for  $f$  as follows:

$$\begin{aligned} f^{-1}(I_{ji}) &= E_i \Delta t_j \\ \log f^{-1}(I_{ji}) &= \log E_i + \log \Delta t_j \end{aligned}$$

Letting  $g = \log f^{-1}$ , gives

$$g(I_{ji}) = \log E_i + \log \Delta t_j$$

where  $i = 1 \dots N$  (pixels) and  $j = 1 \dots P$  (images).  $g$  maps image pixels  $I$  onto log brightness.

To solve for  $g$  we select some subset of pixels (eg., 200 points) in all frames and minimize the following quadratic function.

$$\varepsilon^2 = \sum_{i=1 \dots N} \sum_{j=1 \dots P} w(I_{ji}) (g(I_{ji}) - \log E_i - \log \Delta t_j)^2 + \lambda \sum_{z=2}^{n-1} w(z) g''(z)^2$$

where  $g''(z) = g(z-1) - 2g(z) + g(z+1)$  and  $n$  is the number of grayscale levels in the image. The first term fits the camera response function, while the second term acts as a smoothing term that prefers functions with low curvature.  $\lambda > 0$  trades off the effects of these two terms. A value of  $\lambda = 100$  was used in our experiments.  $w$  is a weighting function that emphasizes the fit when pixels are near the midpoint of their range.

$$w(z) = \begin{cases} z & \text{if } 0 \leq z \leq n/2 \\ n - z & \text{if } n/2 \leq z \leq n \end{cases}$$

Finally, we add a constraint  $g(n/2) = 0$ . This constrains the function  $g$  to be zero at the midpoint of its domain. This is necessary since the above equations only determine  $g$  up to a constant factor.

The above function is minimized using the singular value decomposition. The result is a camera response function  $g(z)$ ,  $z = 1 \dots n$  and a set of brightness values  $\log E_i$ ,  $i = 1 \dots N$ .

### (a) Equation counting [2 marks]

Write the number of *equations* and *unknowns* for the above equations in terms of  $N$ ,  $P$ , and  $n$ . Compute these numbers when  $N = 200$ ,  $P = 3$ , and  $n = 256$ , the case in our experiments below.

**Combining images of different exposure.** Since we can only determine the image response function  $g$  up to a scale factor, we let  $\Delta t_j = 1$  for a reference image.

For each image pixel  $i$  of image  $j$  we can compute the estimated scene brightness:

$$\log E_{ji} = g(I_{ji}) - \log \Delta t_j$$

Using all image measurements, we can perform an average for the log brightness  $E$  at pixel  $i$ :

$$\log E_i = \frac{\sum_{j=1\dots P} w(I_{ji}) \log E_{ji}}{\sum_{j=1\dots P} w(I_{ji})}$$

To display the composite image, we apply a “tone map”, a mapping from (log) scene brightness  $\log E$  to pixel values  $I$ . We could map the overall brightness  $\log E$  onto (any of the) input image(s) using the computed response function  $g$ . This would provide better (lower noise) images, but they would still saturate at extreme values. Instead, we use the *logistic* function:

$$s(\log E) = \frac{1}{1 + \exp(-\alpha \log E)}$$

$s$  maps the (infinite) domain from zero to one, has a mid-point value of  $s(0) = 0.5$ , and has a slope of  $\alpha/4$  at the midpoint. This function is linear in the middle, with compression at the extreme values. As long as the compression is mild, the over and underexposed details will still be visible in the composite image. Note that the composite image will not be directly comparable to any of the input images.

## (b) Experiments [6 marks]

In the program, `wyckoff.m`, I use three images, taken with *exposure values* (EV) as follows:  $(2, 1, \frac{1}{2\sqrt{2}})$ . A unit change in exposure value corresponds to a doubling or halving of exposure times. It may be achieved by either changing the exposure time, the camera aperture, or both. Please do the following experiments:

1. Run the program, print out, and comment on the composite image. In particular, what details does it capture that are not visible in the input images?
2. Modify the program to display  $s(\log E_j)$ ,  $i = 1 \dots 3$ , the estimated radiance from each input image. To compare the individual images to the composite image  $s(\log E)$  above. To what parts of the composite does each image contribute? Hint: you can also look at the “weighting” images,  $w(I_j)$ .

3. What is the role of the smoothing term  $\lambda$ ? To demonstrate its effect, try fitting the response curve with  $\lambda = 10$  (low value) and  $\lambda = 1000$  (high value). Print out the plots of the response functions and comment on your findings.

### 3. Lightness and shading [12 marks]

#### (a) Lambertian blocks [4 marks]

Suppose we observe a trihedral junction (ie., a vertex where three faces meet) of a rectangular block, with known orientation wrt the camera. If the block has a Lambertian surface of constant albedo, we can determine the direction of a single (distant) light source from the relative brightness of the three faces of the block.

If the orientation of each face in the camera's reference frame is given by unit normal vectors  $\hat{\mathbf{n}}_1$ ,  $\hat{\mathbf{n}}_2$ , and  $\hat{\mathbf{n}}_3$ , respectively, show how to obtain the lighting direction and albedo of the surface.

Note: You can ignore the  $\cos^4 \theta$  term in the imaging equation and that we are using orthographic projection.

#### (b) Derivation of photometric stereo [3 marks]

(Adapted from Horn, Exercise 10-16) Suppose we have three images of a Lambertian surface under three different light source directions.

The three images  $E_1$ ,  $E_2$ , and  $E_3$  are given by the following equations,

$$E_1 = \rho(\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{n}}), \quad E_2 = \rho(\hat{\mathbf{s}}_2 \cdot \hat{\mathbf{n}}), \quad E_3 = \rho(\hat{\mathbf{s}}_3 \cdot \hat{\mathbf{n}})$$

where  $\rho$  is the surface albedo (may vary with position  $(x, y)$ ),  $\hat{\mathbf{n}}$  is the surface normal (unit vector), and

$$\hat{\mathbf{s}}_i = \frac{(-p_i, -q_i, 1)^T}{\sqrt{1 + p_i^2 + q_i^2}}$$

for  $i = 1, 2, 3$  give the directions of the light source.

By subtracting pairs of equations, show that:

$$\rho \hat{\mathbf{n}} \cdot (E_1 \hat{\mathbf{s}}_2 - E_2 \hat{\mathbf{s}}_1) = 0 \quad \text{and} \quad \rho \hat{\mathbf{n}} \cdot (E_2 \hat{\mathbf{s}}_3 - E_3 \hat{\mathbf{s}}_2) = 0.$$

The above equation shows that  $\hat{\mathbf{n}}$  is perpendicular to both  $(E_1\hat{\mathbf{s}}_2 - E_2\hat{\mathbf{s}}_1)$  and  $(E_2\hat{\mathbf{s}}_3 - E_3\hat{\mathbf{s}}_2)$ . Show that  $\hat{\mathbf{n}}$  must be parallel to

$$(E_1\hat{\mathbf{s}}_2 - E_2\hat{\mathbf{s}}_1) \times (E_2\hat{\mathbf{s}}_3 - E_3\hat{\mathbf{s}}_2) = E_2(E_1(\hat{\mathbf{s}}_2 \times \hat{\mathbf{s}}_3) + E_2(\hat{\mathbf{s}}_3 \times \hat{\mathbf{s}}_1) + E_3(\hat{\mathbf{s}}_1 \times \hat{\mathbf{s}}_2))$$

Conclude that

$$\rho\hat{\mathbf{n}} = k(E_1(\hat{\mathbf{s}}_2 \times \hat{\mathbf{s}}_3) + E_2(\hat{\mathbf{s}}_3 \times \hat{\mathbf{s}}_1) + E_3(\hat{\mathbf{s}}_1 \times \hat{\mathbf{s}}_2))$$

for some constant  $k$ .

By taking the dot product of this relationship with  $\hat{\mathbf{s}}_1$ , and remembering that  $E_1 = \rho(\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{n}})$ , show that

$$\rho(\hat{\mathbf{n}} \cdot \hat{\mathbf{s}}_1) = kE_1 [\hat{\mathbf{s}}_1\hat{\mathbf{s}}_2\hat{\mathbf{s}}_3]$$

where  $[\hat{\mathbf{s}}_1\hat{\mathbf{s}}_2\hat{\mathbf{s}}_3]$  is the triple product  $\hat{\mathbf{s}}_1 \cdot (\hat{\mathbf{s}}_2 \times \hat{\mathbf{s}}_3)$ .

Finally, show that  $k = \frac{1}{[\hat{\mathbf{s}}_1\hat{\mathbf{s}}_2\hat{\mathbf{s}}_3]}$ , so that

$$\rho\hat{\mathbf{n}} = \frac{(E_1(\hat{\mathbf{s}}_2 \times \hat{\mathbf{s}}_3) + E_2(\hat{\mathbf{s}}_3 \times \hat{\mathbf{s}}_1) + E_3(\hat{\mathbf{s}}_1 \times \hat{\mathbf{s}}_2))}{[\hat{\mathbf{s}}_1\hat{\mathbf{s}}_2\hat{\mathbf{s}}_3]}$$

### (c) Photometric stereo experiment [5 marks]

I have given you three images of an unknown surface illuminated from three lightsource directions:  $\mathbf{s}_1 = [0, 0, 1]^T$ ,  $\mathbf{s}_2 = [-1, 1, 2]^T$ ,  $\mathbf{s}_3 = [1, 0, 3]^T$ . Assuming the light source has the same radiance at each direction (ie.,  $E_1 = E_2 = E_3$ ), use the expression in part (b) to compute the surface normal and albedo at each point. To accomplish this, complete the program `photometric.m`, and show plots of surface normals and albedo  $\rho$ . Note: Since we don't know  $E$ , we can only determine  $\rho$  only up to a constant factor. *Remember to normalize the light source directions  $\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3$  to unit vectors in your code.*