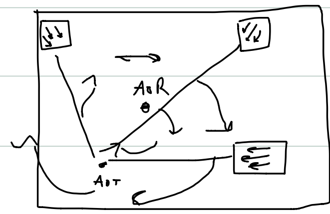
ASST #4 Questions

heading from motion parallax (snow)

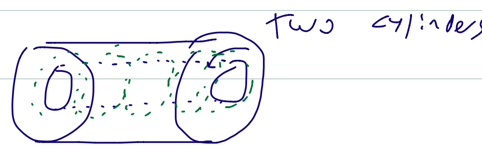
Discrete SFM



SFM Discrete case

2 Frame Motion (8 pt. algorithm stereo)

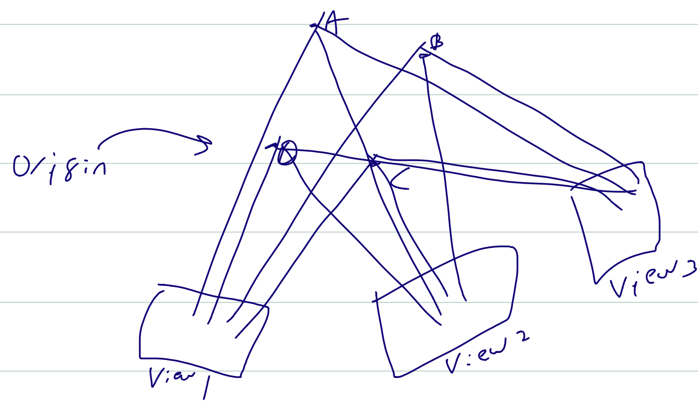
Multiframe motion : "Kinetic depth effect"
(Ullman 1979)



SFM Theorem (Ullman 1979, Marr's book pp 205-212)

Given 3 distinct (orthographic) views of
4 (non-coplanar) points in rigid configuration
the motion is uniquely* determined

* upto depth reversal *upto single depth uncertainty



Counting argument

		frame 1
	O	2 dof depth unknown
dof for rigid	A,B,C wrt O	9 dof 3 x 3 dof
motion of 4 pts.	frame 2 and 3	6 dof rotation wrt frame 1
	posn in 2D	4 dof translation wrt frame 1
		$\boxed{21 \text{ dof}}$

Measurement	A, B, C, O every frame gives	8 dof x 3 frames
		$\boxed{24 \text{ dof}}$

$$|\text{Measurement}| > |\text{dof}|$$

- Random sets of (4 image pts. in 3 views) highly unlikely to generate images of rigid motion.
- Images of rigid pts occupy a measure zero set of random sets of 2d points

- If I see collections of image pts consistent with rigid motion, we should infer they are truly rigid motion.

David Lowe, UBC

Jepson and Richards "key features"

Rigidity Assumption (Ullman 1979)

any set of pts undergoing 2d motion that is consistent with 3d rigid motion should be interpreted as such.

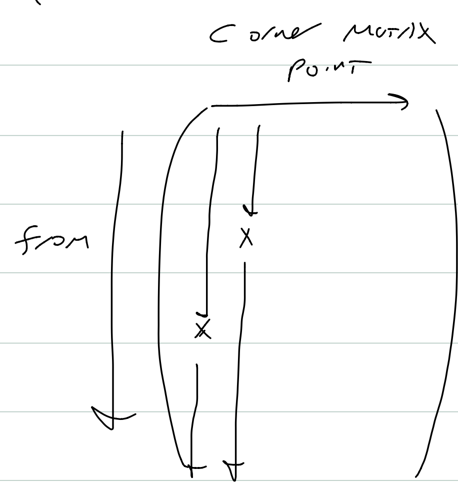
Ullman did "RANSAC"; choose set of 4 pts

keep adding points as long as they are consistent.

Modern Tracking: KLT tracker

corners

(Kanade-Lucas-Tomasi)

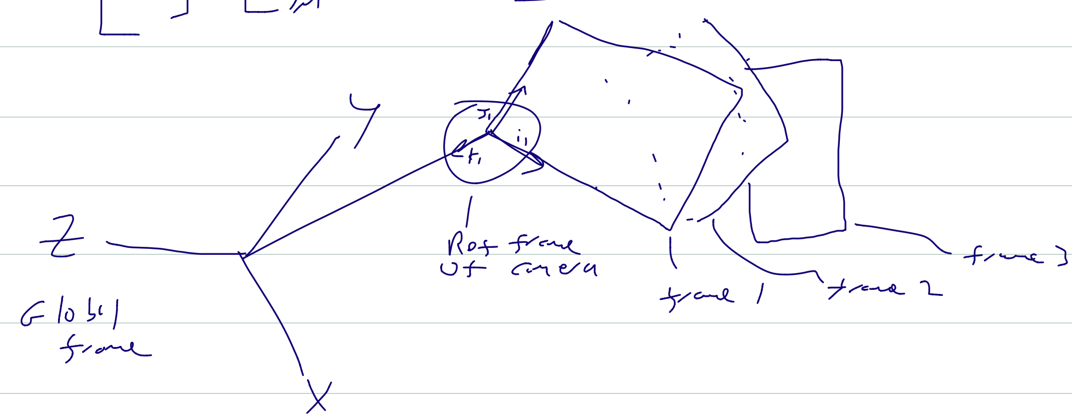


Factorization Method (Tomasi and Kanade 1992, T and V §8.5)

$n \geq 4$ points (non coplanar)

$N \geq 3$ views

$$W = \begin{bmatrix} X \\ \hline Y \end{bmatrix} = \begin{array}{c} \xleftarrow{\quad n \text{ points} \quad} \\ \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & & & \\ \vdots & & & \\ x_{N1} & & & \\ \hline y_{11} & y_{12} & \cdots & y_{1n} \\ \vdots & & & \\ y_{N1} & & & y_{Nn} \end{bmatrix} \end{array} \; 2N \quad N \text{ views}$$



WLOG set $(i_1, j_1, k_1) = (X, Y, Z)$

Now first frame is taken to be global reference frame.

Let $\begin{cases} \tilde{x}_{ij} = x_{ij} - \bar{x}_i \\ \tilde{y}_{ij} = y_{ij} - \bar{y}_i \end{cases}$ $(\bar{x}_i, \bar{y}_i)$ mean pixle value in frame i

$\tilde{W} = \begin{bmatrix} \tilde{X} \\ \tilde{Y} \end{bmatrix}$ Same as W, just subtract centroid of each frame "measurement matrix"

Claim

$$\tilde{W} = \underbrace{R}_{\text{rotation}}\ \underbrace{S}_{\text{shape}}$$

$$
\begin{array}{c}
\\ 2N
\end{array}
\overbrace{\begin{bmatrix} \tilde{X} \\ \hline \tilde{Y} \end{bmatrix}}^{n}
=
\begin{bmatrix} i_1^T \rightarrow \\ i_2^T \rightarrow \\ \vdots \\ \hline j_1^T \rightarrow \\ j_2^T \rightarrow \\ \vdots \\ j_N^T \rightarrow \end{bmatrix}
\underbrace{\begin{bmatrix} p_1 & p_2 & \cdots & p_n \\ \downarrow & \downarrow & & \downarrow \end{bmatrix}}_{n}
\begin{array}{c} 3 \end{array}
$$

rigid pts 3d

Rank 3 matrix!!!

Problem: Given measurement matrix $2N \times n$, find R, S

Algorithm

Measure W

subtract mean from each frame $\rightarrow \tilde{W}$

factor

$$\tilde{W} = U D V^T \qquad (SVD)$$

$$\text{Let } D' = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \quad \text{top 3 singular values.}$$

Let $\hat{R} = U' D'^{1/2}$

$\hat{S} = D'^{1/2} V'^T$

$$U' = \overbrace{\begin{bmatrix} u_1 & u_2 & u_3 \\ \downarrow & \downarrow & \downarrow \end{bmatrix}}^{3} \; 2N \qquad V'^T = 3 \underbrace{\begin{bmatrix} v_1^T \rightarrow \\ v_2^T \rightarrow \\ v_3^T \rightarrow \end{bmatrix}}_{n}$$

Solve for

$\underline{R} = \hat{R} Q$, $S = Q^{-1} \hat{S}$ problem Q can be any (pos def) matrix

Choose Q s.t.

$$\boxed{\begin{aligned} i_i^\dagger Q Q^T i_i &= 1 \\ J_i^\dagger Q Q^T J_i &= 1 \\ i_i^\dagger Q Q^T J_i &= 0 \end{aligned}}$$

I now choose Q s.t. basis vectors in R are orthonormal

tricks needed in alg.