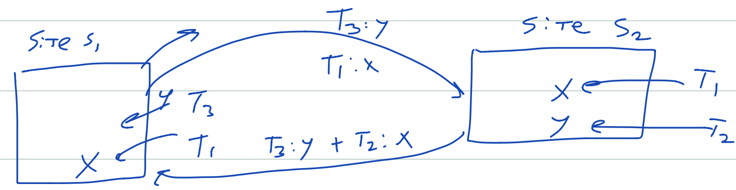
Asst #4 hints

Tutorial Wed 5-6 pm

Epipdun Finish

Today: Structure from motion (SFM)

epidemic / gossip protocol



Vector version:

	S_1	S_2	
X	T_1	T_2	
Y		V	

	S_1	S_2	
X	T_1	T_2	
Y			

Essential Matrix

$$p_r^T E p_l = 0$$

points in right image

points in left image

$$p_l = (p_{lx} \;\; p_{ly} \;\; 1)$$

$$p_r = (p_{rx} \;\; p_{ry} \;\; 1)$$

$$E = RS$$

Rotation, Translation

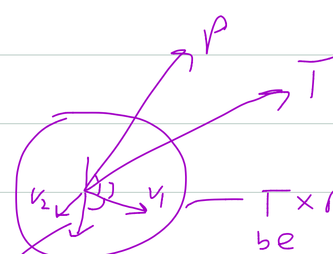
(Recall $SP = T \times P$)

$$\sum_K \left(p_r^{K\,T} E \, p_r^K\right)^2$$

Understand: $EP = R(SP)$

Claim E has rank 2

$$SP = T \times P$$

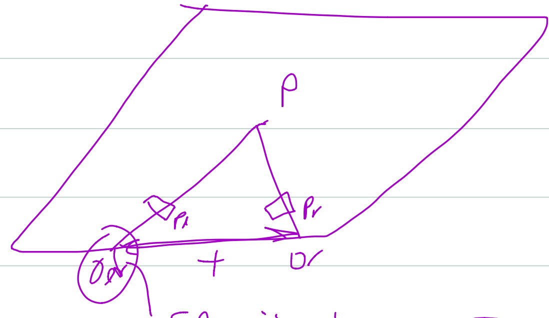


$T \times P$ must be $\perp$ to T

SP is $\perp$ T

SP is $\perp$ P

v_1 and v_2 define space of SP



Sp is ⊥ to T
Sp is ⊥ to P

Fundamental Matrix
Uncalibrated Cameras

$$x = \frac{1}{f}(x_{im} - O_x)\, s_x$$
$$y = \frac{1}{f}(y_{im} - O_y)\, s_y$$

(x_{im}, y_{im}) pixel in camera
(x, y) pt in space
(O_x, O_y) center camera
(s_x, s_y) scale

T and V ch 2

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -f/s_x & 0 & O_y \\ 0 & -f/s_y & O_x \\ 0 & 0 & 1 \end{bmatrix}}_{M} \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix}$$

Camera coords

$$P_l = M_l^{-1} p_l$$
$$P_r = M_r^{-1} p_r$$

True coords   camera coords

$$p = \begin{pmatrix} x_{im} \\ y_{im} \end{pmatrix} = \begin{bmatrix} x_1/x_3 \\ x_2/x_3 \end{bmatrix}$$

true coords

$$p_r^T E p_l = 0$$

$$(M_r^{-1} p_r)^T E (M_l^{-1} p_l) = 0$$

$$p_r^T \boxed{(M_r^{-1})^T E M_l^{-1}} p_l$$

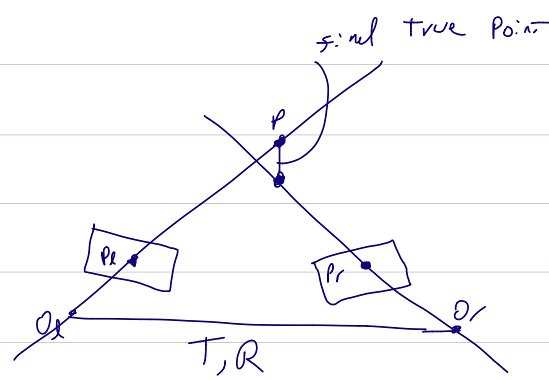
$$\| $$

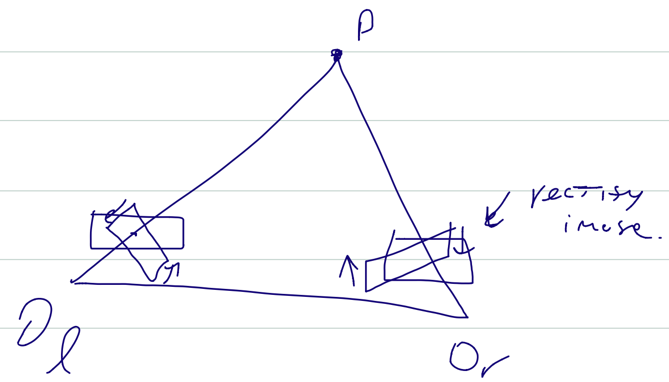
$$p_r^T \boxed{F} p_l$$ F: fundamental matrix

Summary

- Get 8 or more corresponding pts (e.g. SIFT)
- Compute E (or F)
- (opt: bundle adjustment)
- rectify images
- baseline stereo

* triangulate to get depth.





Structor from Motion

Continuous approach

- Given (image) pts + velocities (eg. optical flow)

find heading T, ω — translational velocity, angular velocity

depth.

Longuet — Higgins (1981)

Now different fields in CV!

Discrete approach

- Given (corresponding) pts. in multiple frames.

- find T, R (rigid transformation) (depth)

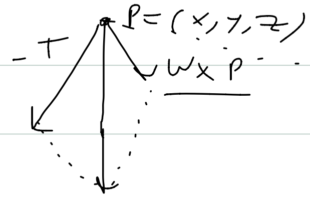
Note: Stereo is "2 frame" motion

Longuet-Higgins 198x

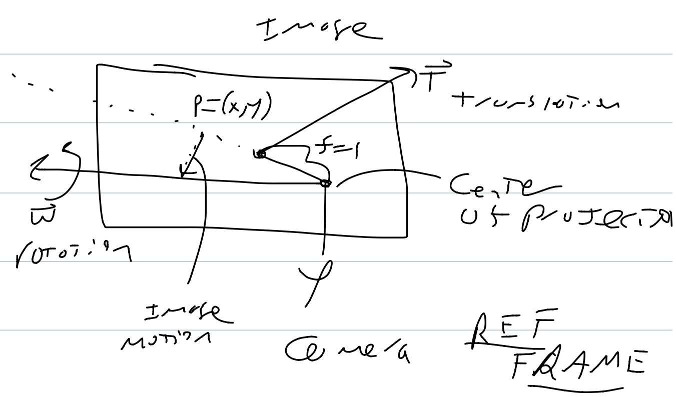
Ego motion (self motion)

Camera moves with translation velocity $\vec{T} = (T_x, T_y, T_z)$

" " " rotation " $\vec{\omega} = (\omega_x, \omega_y, \omega_x)$



$$\boxed{\frac{d}{dt} P = -T - \omega \times P}$$



Sorry for the math

$$\frac{dP}{dT} \triangleq \underbrace{-T}_{(T_x, T_y, T_z)} - \underbrace{\omega}_{(\omega_x, \omega_y, \omega_z)} \times \underbrace{P}_{P=(x,y,z)}$$

$$\omega \times P = \begin{vmatrix} i & j & k \\ \omega_x & \omega_y & \omega_z \\ X & Y & Z \end{vmatrix}$$

$$V_x = -T_x - \omega_y Z + \omega_z Y$$

$$V_y = -T_y + \omega_z X + \omega_x Z$$

$$V_z = -T_z - \omega_x Y + \omega_y X$$

Image motion due to T, ω

$p = \frac{f}{Z} P$ perspective projection

$\vec{v} = \frac{dp}{dt} = Pf \frac{d}{dt}\left(\frac{1}{Z}\right) + \frac{f}{Z}\frac{d}{dt}P$ product rule

image pt

$= Pf\left(\frac{-1}{Z^2}\right)\frac{dZ}{dt} + \frac{f}{Z} V$ — 3d velocity from before

$\vec{v} = \frac{fZ\vec{V} - V_z\vec{P}}{Z^2}$ $V_z \triangleq \frac{dZ}{dt}$ depth change

$V = \frac{dP}{dt}$ point velocity.

busy work

After some steps

image point velocity

$$v_x = \frac{T_z x - T_x f}{Z} \quad - \omega_y f + \omega_z y + \omega_x \frac{xy}{f} + \frac{\omega_y x^2}{f}$$

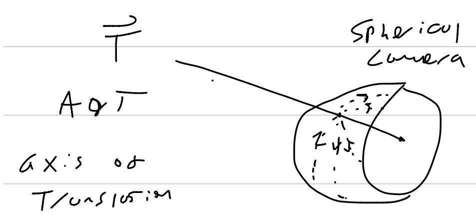
$$v_y = \frac{T_z y - T_y f}{Z} \quad + \omega_x f - \omega_z x - \frac{\omega_y xy}{f} + \frac{\omega_x y^2}{f}$$

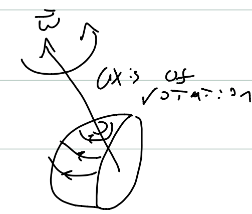
transl. comp. rotational comp.

$$\boxed{\vec{v} = \vec{u}^T + \vec{u}^\omega}$$

depends on T and $\frac{1}{Z}$ depends on ω but not Z

Intuition (LHP 1980, TV)





$\vec{v}^T$ velocity vector

length inversly

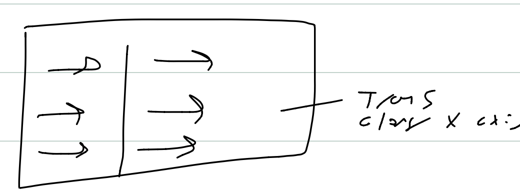
prop to depth.

$\vec{v}^\omega$ axial motion

speed map of depth

If $\vec{T} = 0$ easy to solve for w | LSQ

If $\vec{\omega} = 0$ " " " " $\vec{T}$ |



camera rotation about y axis