Office hours: Friday 3-5 PM.

Tutorial: Monday, Next Week

Assn #3 (Out Friday): LSQ, Robust Mixture

Today: Optical Flow

Vectors converge, but speeds depend on \( \frac{1}{z} \) in depth.

Applications

- "Egomotion" get camera motion from flow field in a rigid environment.
- "Scene Reconstruction" depth map
  - object tracking - image registration - segmentation

Original notes from Gibson 1966 "Ecological Optics"
Optical Flow (TV Ch 8, Ch 12)

Fleet Unit

Measurement of Image Velocity

Fourier Methods

\[ \mathbf{i} = (x, y, z) \]

Assumptions

1. Uniqueness - every image point corresponds to a unique scene point. Violated at occlusion boundaries, or transparency.

2. Observable Image Motion

3. Correspondence between image features and scene markings

\[ \text{moving light source} \]

\[ \text{induces shadow change not due to motion.} \]
1. **Matching Techniques** (e.g., track corners, edges)

2. **Differential Techniques**

3. **Fourier methods**

**Differential Techniques**

**Intuition**

\[ I(x) \]

\[ V = \frac{dx}{dt} = -\frac{\partial I}{\partial x} \]

**Total derivative**

\[ 0 = \frac{\partial I}{\partial t} = \frac{\partial I}{\partial x} \frac{dx}{dt} + \frac{\partial I}{\partial y} \frac{dy}{dt} + \frac{\partial I}{\partial y} \quad \text{(Chain rule)} \]

Optical *5.10* (2d *5.810*1)

**I(x,y,t)**

\[ 0 = \frac{\partial I(x,y,t)}{\partial t} = \frac{\partial I}{\partial x} \frac{dx}{dt} + \frac{\partial I}{\partial y} \frac{dy}{dt} + \frac{\partial I}{\partial y} \quad \text{(Chain rule)} \]

\[ \nabla I = \left[ \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right]^T \quad \mathbf{V} = \left( \frac{dx}{dt}, \frac{dy}{dt} \right) \]
\[ 0 = D \nabla \cdot \mathbf{v} + \frac{\partial \mathbf{v}}{\partial t} \]  

**Brightness Constancy Condition (BCC)**

Problem: 1 equation, but 2 unknowns

\[ \frac{\partial x}{\partial t}, \frac{\partial y}{\partial t} \]

**Comments**

1. \[ \frac{\partial \mathbf{v}}{\partial t} = 0 \]  
   - More transients only
   - Does not change brightness
   - Does not desaturate
   - BCC
   - More final constraint equation

2. \[ \mathbf{v} = \left( \frac{\partial x}{\partial t}, \frac{\partial y}{\partial t} \right) \] is under constrained

\[ f(x, y, t) \quad f(x, y, t + \Delta t) \]

3. Brightness often sensitive to lighting, shadows, etc.  Often use filtered outputs (e.g., phase of a color filter)
Aistence problem

\( \mathbf{F}(x,y,t) \) \hspace{1cm} \( \mathbf{F}(x,y, t+\Delta t) \)

Local region

where BC is applied

Possible neighbors

Normal component of motion

Can only observe normal motion

Constraint Lines

\[ \nabla \mathbf{F} = \left( \frac{\partial F}{\partial x}, \frac{\partial F}{\partial y} \right) \]

Line of possible motion

\[ \nabla \mathbf{F} \cdot \mathbf{V} = -F_x \]

Constant
Some use regions of images (with similar velocities)
but more than one image derivative direction

Intersection constraints by LSA

Intersection constraints

Region $\rightarrow$ not enough orientations
Too small $\rightarrow$ noisy

Region $\rightarrow$ multiple orientations
Too big $\rightarrow$ reflections
Transparency
Periallax

Method #1 Horn & Schunk (1980)

Regularization of the flow field

\[
\mathcal{E}_C = \iint (E_x(u) + E_y(v) + \mathbf{f}_0)^2 \, dx \, dy
\]

BCC constraint \( u = \frac{dx}{dt}, \) \( v = \frac{dy}{dt} \)

\[
\mathcal{E}_S = \iint \left[ (u_x^2 + u_y^2) + (v_x^2 + v_y^2) \right] \, dx \, dy
\]

Smoothness regularizes down value of velocity
\[ \varepsilon = \sum c + \lambda \varepsilon_5 \]

(smoothness term "regularization"

**Method #2** (Lucas & Kanade)

\[
\begin{bmatrix}
I_x(p_i) & I_y(p_i) \\
I_x(p_i) & I_y(p_i) \\
\vdots & \vdots \\
I_x(p_n) & I_y(p_n)
\end{bmatrix}
\begin{bmatrix}
U \\
V
\end{bmatrix} =
\begin{bmatrix}
-I_x(p_i) \\
-I_y(p_i) \\
\vdots \\
-I_x(p_n)
\end{bmatrix}
\]

\[
\begin{align*}
A & = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} \\
\varepsilon & = \frac{1}{n} \sum_{i=1}^{n} \|AV - b\|_2^2
\end{align*}
\]

\[
\begin{align*}
\varepsilon & = \frac{C}{\sum_{i=1}^{n} \|AV - b\|_2^2} \\
C & = \sum_{i=1}^{n} \begin{bmatrix}
I_x^2 & I_x I_y \\
I_x I_y & I_y^2
\end{bmatrix}
\end{align*}
\]

If \( \varepsilon = 0 \) then no deviations

1. Normal flow
2. 2D flow

"Burton, Bedewany, Fleet, Jepson" (performance of optical flow techniques)
\[ \mathbb{Z}^{n} = \mathbb{Z} \times \mathbb{Z} \times \cdots \times \mathbb{Z} \]

\[ \mathbb{C} = \mathbb{R} + i\mathbb{R} \]

\[ (\nu_0, \nu_{\gamma}) = c \]