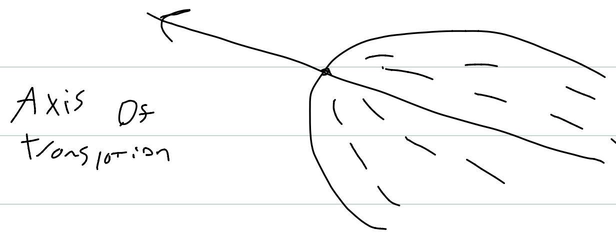
Office hours: Friday 3-5 pm.

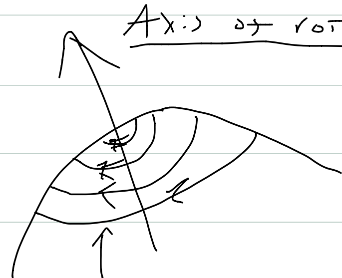
Tutorial: Monday next week

Asst #3 (out Friday): LSQ, Robust, Mixture

Today: Optical flow



Vectors converge, but speeds depend on $\frac{1}{Z}$ in depth



"Egomotion"

Originates from Gibson 1966 "Ecological optics"

applications

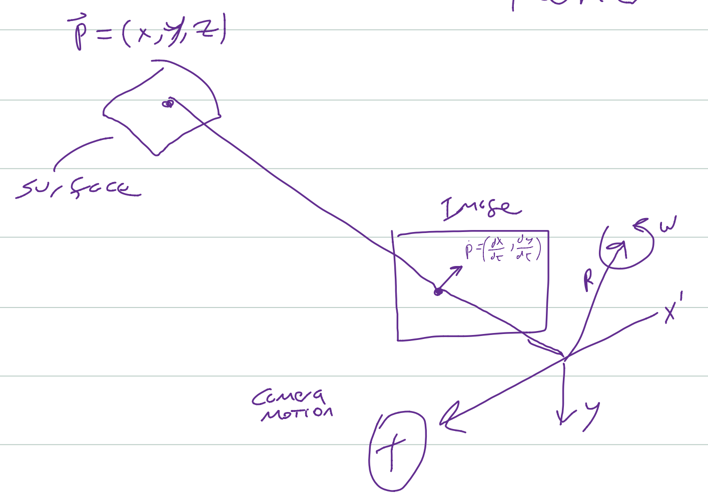
- "Egomotion" get camera motion from flow field in a rigid environment.
- "Scene reconstruction" depth map
  - object tracking - Image registration - Segmentation

Optical flow (T&V ch8, Ch 12)

(Fleet UofT

"Measurement of image velocity"

Fourier methods)



5 dof : 3R

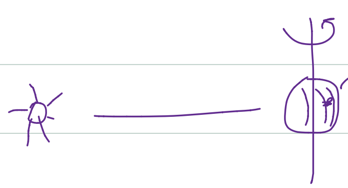
: 2 T

upto scale factor.

assumptions

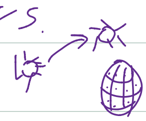
1. Uniqueness every image pt corresponds to a unique scene pt. Violated at occlusion boundaries, or transparency

2. Observable image motion



uniform sphere with no markings

3. Correspondence between image features and scene markings.



Moving light source induces shadow change not due to motion.

Approaches

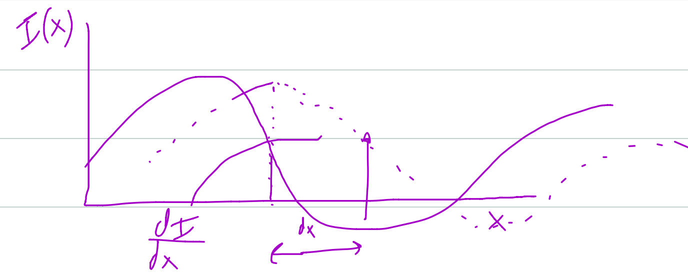
1. Matching Techniques (eg. Track corners, edges, ect.)

$\boxed{\text{2. Differential Techniques}}$

3. Fourier Methods

Differential Techniques

Intution 1D Signal



Total derivative

$$0 = \frac{dI}{dt} = \frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial t}$$

$$V = \frac{dx}{dt} = \frac{-\partial I/\partial t}{\frac{\partial I}{\partial x}}.$$

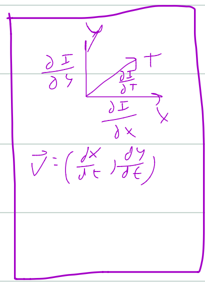
Optical flow (2d signal)

$I(x,y,t)$

$$0 = \frac{d}{dt} I(x,y,t) = \frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t}$$ (Chain rule)

Let $\nabla I = \left[\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right]^T$ $\vec{V} = \left(\frac{dx}{dt}, \frac{dy}{dt}\right)$

$$\boxed{0 = \nabla I \cdot \vec{v} + \frac{\partial I}{\partial t}}$$ Brightness Constancy Condition (BCC)



Problem: 1 equation, but 2 unknowns $\left(\frac{dx}{dt}, \frac{dy}{dt}\right)$

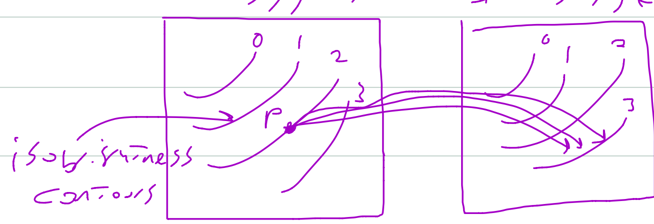
Comments

(1) $\frac{dI}{dt} = 0$ image translates only
- does not change brightness
- does not deform.
- BCC

Image flow constraint Equation

(2) $\vec{V} = \left(\frac{dx}{dt}, \frac{dy}{dt}\right)$ is underconstrained

$I(x,y,t)$ $I(x,y,t+\Delta t)$

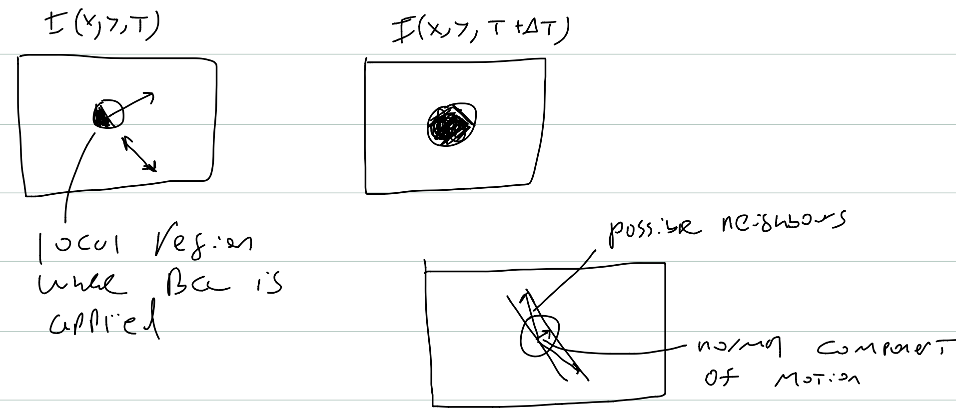


3. Brightness often sensitive to lighting shadows, ect. Often use filtered outputs (eg. Phase of a Gabor filter)

1

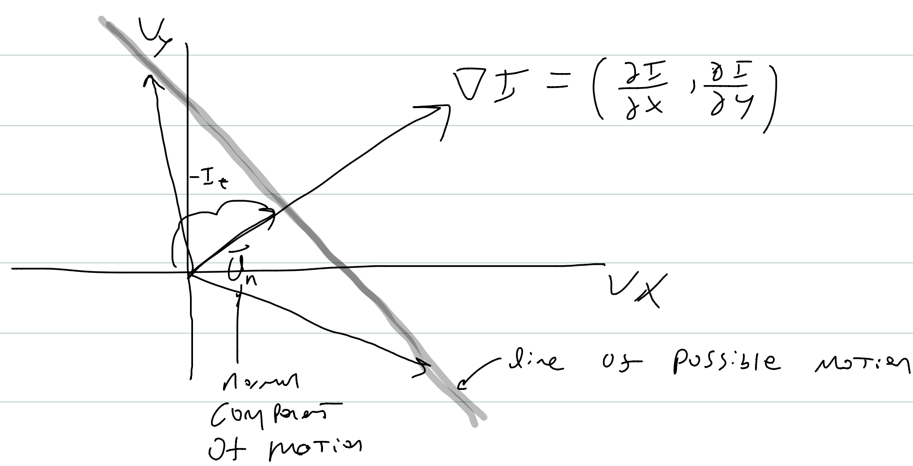
Fleet's book

Aperture problem



Can only observe normal motion

Constraint Lines



$$\boxed{\nabla I \cdot \vec{V} = -I_t}$$ Constant

Soln use regions of images (with similar velocities) but more than one image derivative direction

Intersect constraint lines by LSQ

Intersecting constraints

Region too small $\rightarrow$ - not enough orientations
- noisy

Region too big $\rightarrow$ - multiple motions
- reflections
- transparency
- parallax

Method # 1 Horn & Schunk (1980)

Regularization of the flow field

measurements I_x, I_y, I_t
unknowns $u(x,y), v(x,y)$

$$E_c = \iint (I_x u + I_y v + I_t)^2 \, dx\, dy$$

BCC constraint $u = \frac{dx}{dt}, \; v = \frac{dy}{dt}$

$$E_s = \iint \left[(u_x^2 + u_y^2) + (v_x^2 + v_y^2) \right] dx\, dy$$

Smoothness penalizes derivative of velocity

$E = E_c + \lambda E_s$

(smoothness term "Regularization"

Method #2 (Lucas & Kanade)

$$\begin{bmatrix} I_x(P_1) & I_y(P_1) \\ I_x(P_2) & I_y(P_2) \\ \vdots & \\ I_x(P_n) & I_y(P_n) \end{bmatrix} \begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} -I_t(P_1) \\ -I_t(P_2) \\ \vdots \\ -I_t(P_n) \end{bmatrix}$$

$$\underline{\underline{A}} \qquad \underline{v} = \underline{b}$$

$$\hat{\underline{v}} = \arg\min_{\underline{v}} \|\underline{\underline{A}}\,\underline{v} - \underline{b}\|^2 = \sum_{i=1}^{N} (\nabla I(P_i) \cdot \underline{v} + I_T)$$

(removed to fast)
(missing notes)

$$C = \underline{\underline{A}}^T \underline{\underline{A}} = \begin{bmatrix} \Sigma I_x^2 & \Sigma I_x I_y \\ \Sigma I_x I_y & \Sigma I_y^2 \end{bmatrix}$$

If rank $(C) == 0$ then no derivatives

1 normal flow

2 2d flow

Barron, Beauchemin, Fleet, Jepson"
"Performance of optical flow techniques"

$\vec{C}^n = (C^n_x, C^n_y, C^n_T)$ set of constraint lines

$\vec{C} \cdot (v_x, v_y, 1) = 0$