Least Squares (Con't)

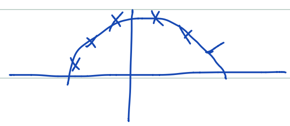
Review:

Geometric Constraint

Standard

$$\arg\min_{\underline{a}} \sum_k (\underline{a}^T \underline{x}_k - d_k)^2$$

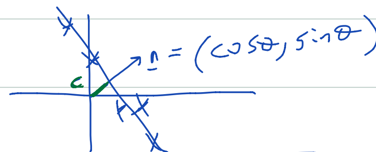
model, data



TLS

$$\min_{\underline{a}} \sum_k (\underline{n}^T \underline{x}_k - c)^2$$

s.t. $\|\underline{n}\| = 1$



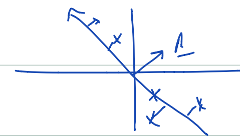
$$\boxed{\tilde{\underline{x}}_k \leftarrow \underline{x}_k - \underline{\mu}}$$

Homogenious LSQ

$$\min \sum_k (\underline{n}^T \underline{x}_k)^2$$

ST $\|\underline{n}\| = 1$

$\underline{n} \leftarrow$ smallest eigenvector of $\underline{\underline{X}} = \sum (\underline{x}_k \underline{x}_k^T)$



Ellipse fitting (Trocco & Veri)

$$\varepsilon(a, \underline{x}) = \underline{a}^T \underline{x}$$

$$= [a_1 x^2 + a_2 xy + a_3 y^2 + a_4 x + a_5 y + a_6] = 0$$

Let $\underline{x}_k = [x_k^2, x_k y_k, y_k^2, x_k, y_k, 1]$

$\underline{a} = [a_1, a_2, a_3, a_4, a_5, a_6]$

such that $b^2 - 4ac > 0$

$$a_2^2 - 4 a_1 a_3 > 0$$

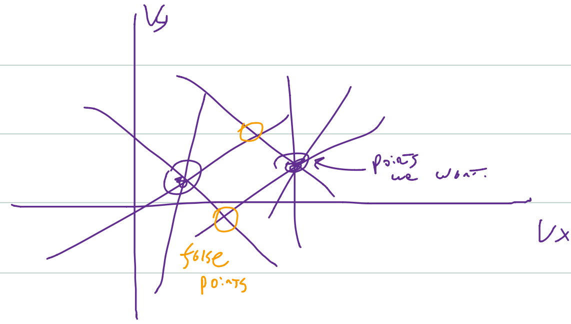
$$\boxed{\min_{\underline{a}} \sum_k (\underline{a}^T \underline{x}_k)^2}$$

Algebraic Constraint

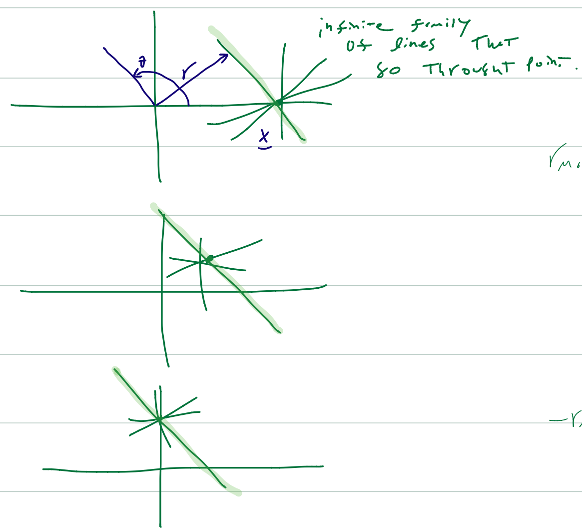
Subject to scaling problems.

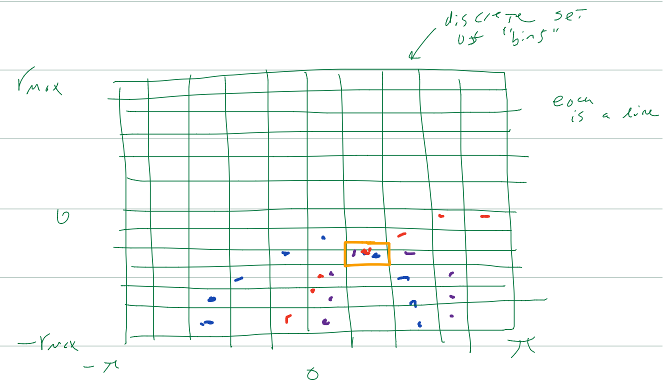
Note this doesn't do the right thing. This is linear, and should be non-linear.

asst #3 Preview



Hough Szeliski (p221)





$$r_i(\theta) = (x_i, y_i) \cdot \hat{n} = x_i \cos\theta + y_i \sin\theta$$

RANSAC "Random Sample Consensus"

- Choose random subset of features that over constrain model
  e.g. 3 points for one line
  4 points in 3D for plane.
- fit model, keep if residual is low.
- add "nearby" points to improve fit.
- optionally, remove points.

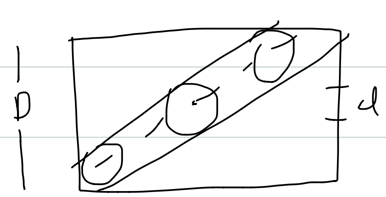
Let m be # of features needed to fit model
(e.g. $m=2$ for line)
k be # of samples taken.

probability of "accidental" colinear triplet decreases as $O(\varepsilon^{\varepsilon-m})$ where $\varepsilon = \frac{d}{D}$

measurement range

measurement accuracy.

$k > m$ $\quad k - m > 1$ will result in lower probabilitys of finding model.



Measurement resolution

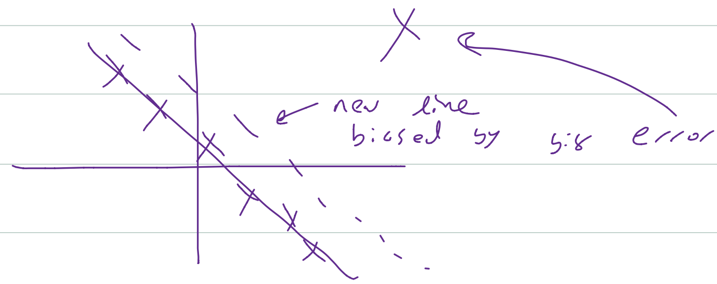
Robust Fitting — Several Methods (Meer et al, ICCV 1991)

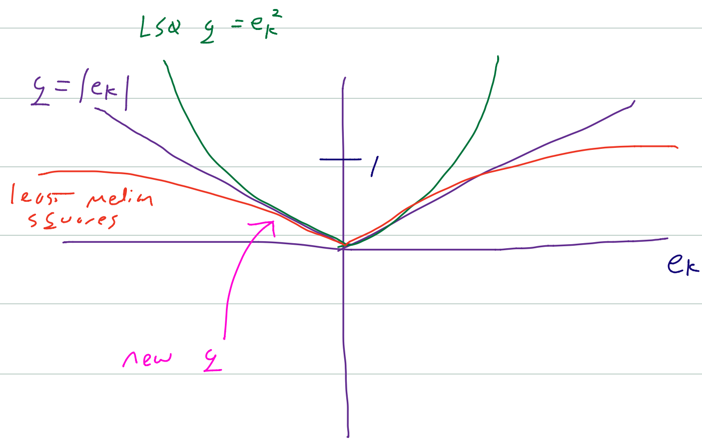
M-estimation

e_k error in k^{th} point

$$\min_{\underline{a}} \sum_{k=1}^{K} \rho\left(\underbrace{\underline{a}^T \underline{x}_k - d_k}_{\text{regular LSQ}}\right)$$

RHO error measurement





Convenient form:

$$\rho(e) = \frac{e^2}{\sigma^2 + e^2}$$

if $e \ll \sigma$

$$\rho(e) \approx \frac{e^2}{\sigma^2}$$

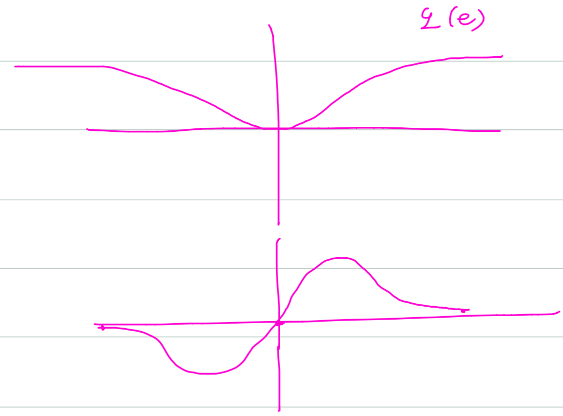
if $e \gg \sigma$

$$\rho(e) \approx 1$$

Let $\psi(e) = \frac{d}{de}\rho(e)$

$$= \frac{d}{de}\left(e^2(\sigma^2 + e^2)^{-1}\right)$$

$$\Rightarrow \psi(e) = \frac{2e\sigma^2}{(\sigma^2 + e^2)^2}$$ "influence function"



Robust

$$\mathcal{E}(\underline{a}) = \sum_k \rho\,(\underbrace{\underline{a}^T \underline{x}_k - d_k}_{e_k})$$

$$\frac{\partial \mathcal{E}(\underline{a})}{\partial \underline{a}} = \sum_k \dot{\rho}(e_k)\,\frac{\partial e_k}{\partial \underline{a}} = 0$$

$$\boxed{0 = \sum_k e_k \left[\frac{2\sigma^2}{(\sigma^2+e_k^2)^2}\right] x_k}$$

weighting function
$w(e_k)$

Robust

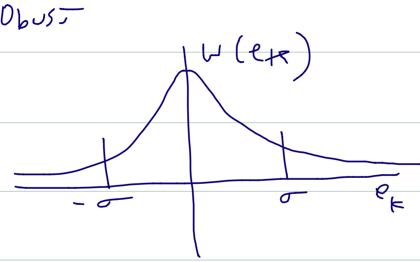
LSQ

$$\mathcal{E}(\underline{a}) = \sum_k (\underline{a}^T \underline{x}_k - d_k)^2$$

$$\frac{\partial \mathcal{E}(\underline{a})}{\partial \underline{a}} = \sum_k \frac{1}{2}(e_k \cdot x_k)$$

$$\boxed{0 = \sum 2 e_k x_k}$$

LSQ



$$w(e) = \frac{2\sigma^2}{(\sigma^2+e_k^2)^2} \qquad w(\sigma) = \frac{2\sigma^2}{(\sigma^2+\sigma^2)^2} = \frac{1}{2\sigma^2}$$

$$w(0) = \frac{2\sigma^2}{\sigma^4} = \frac{2}{\sigma^2}$$

$$0 = \frac{\partial}{\partial \underline{a}}\mathcal{E}(\underline{a}) = \sum_k w(e_k)\,(\underline{a}^T \underline{x}_k - d_k)\, x_k$$

$$0 = \underbrace{\left[\sum_k w(e_k)\, x_k x_k^T\right]}_{\underline{\underline{A}}} \underline{a} - \underbrace{\left[\sum_k w(e_k)\, d_k\, x_k\right]}_{\underline{b}}$$

$$\underline{\underline{A}}\,\underline{a} = \underline{b}$$

$$\boxed{\hat{\underline{a}} = \underline{\underline{A}}^{-1}\underline{b} \quad \text{LSQ sol'n}}$$

But wait, w(e_k) need a model

<u>Iterative alg</u>

Choose initial $\underline{a}_0$

for $n = 1:N$ steps

compute $A(\underline{a}_n)$, $b(\underline{a}_n)$

solve for $\underline{a}_{n+1} \leftarrow \underline{\underline{A}}_n^{-1} b_n$

each step in (weighted) LSQ