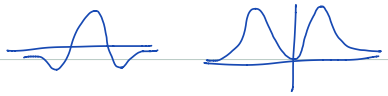
Fourier, part II

- ASST #2, TUES, NEXT week
- PROJECT COMMENTS, Thursday, next week
- Books on reserve, (see website)

Szeliski (p 137) FT pairs



Convolution Theorem (Last day)

$$\mathcal{F}\{\underbrace{f(x,y)}_{\text{filter}} \otimes \underbrace{g(x,y)}_{\text{image}}\} = \mathcal{F}\{f(x,y)\}\,\mathcal{F}\{g(x,y)\}$$
$$= F(u,v) .* G(u,v)$$

$$f(x,y) \otimes g(x,y) \leftarrow \mathcal{F}^{-1}\{\mathcal{F}\{f(x,y)\} .* \mathcal{F}\{g(x,y)\}\}$$

$M \times M$ $\quad n \times n$

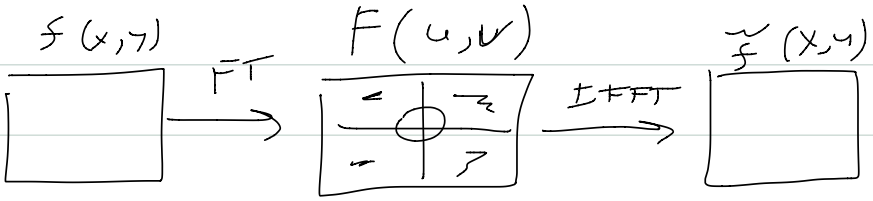
$O(M^2 n^2)$

$O(N^4)$

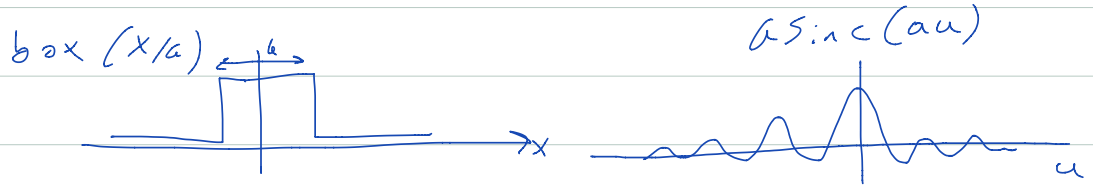
Let $N = M \Rightarrow$ $\underbrace{N^2 \log(N)}_{\text{FFT}} + \underbrace{N^2 \log(N)}_{\text{FFT}} + N^2 + \underbrace{N^2 \log(N)}_{\text{I FFT}}$

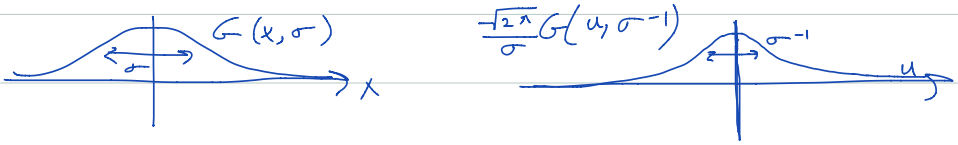
$O(N^2 \log(N))$

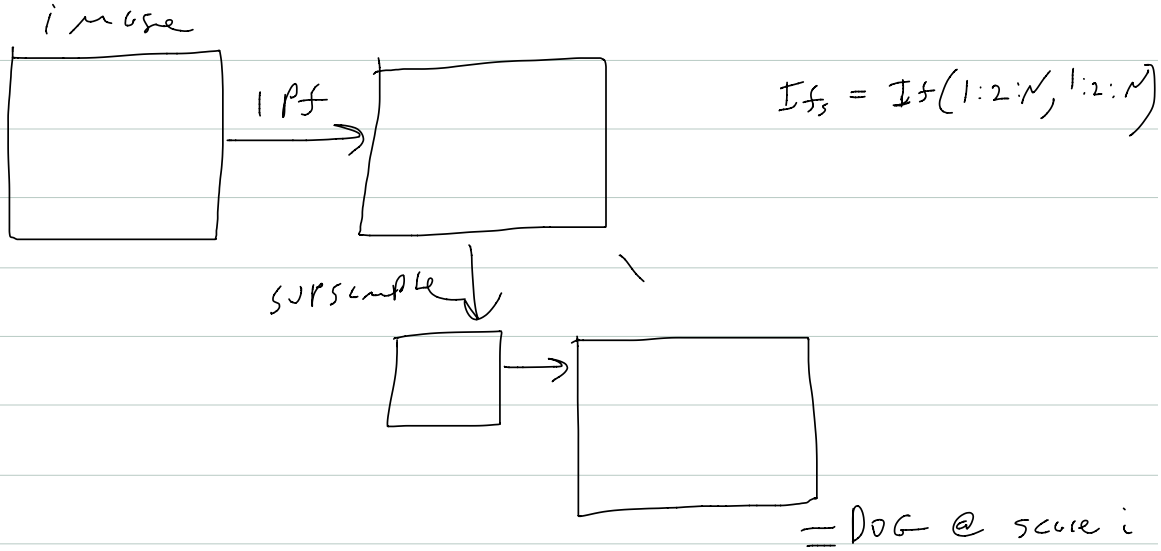
"Ideal" low-pass filter Idea



→ produces ripples.







K. Continued

Digital Image Processing

Similarity Transforms

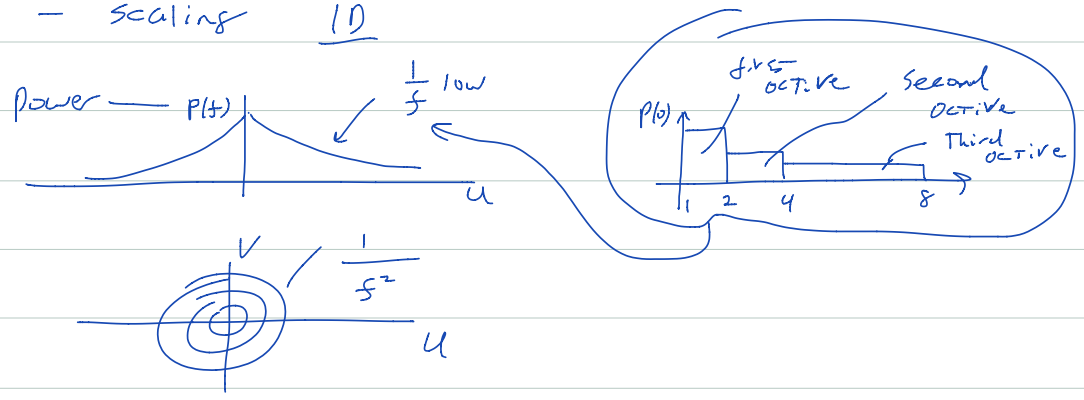
$f(ax, by) \longleftrightarrow \frac{1}{|ab|} F\left(\frac{u}{a}, \frac{v}{b}\right)$ "Scaling"

$f(x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta) \longleftrightarrow F(u\cos\theta + v\sin\theta, -v\sin\theta + v\cos\theta)$ "Similarity"

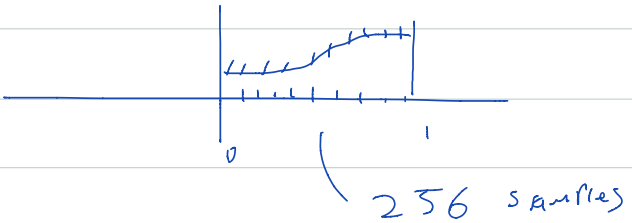
$\iint |f(x,y)|^2 \, dx\,dy \longleftrightarrow \iint |F(u,v)|^2 \, du\,dv$ "Rayleigh's Theorem"

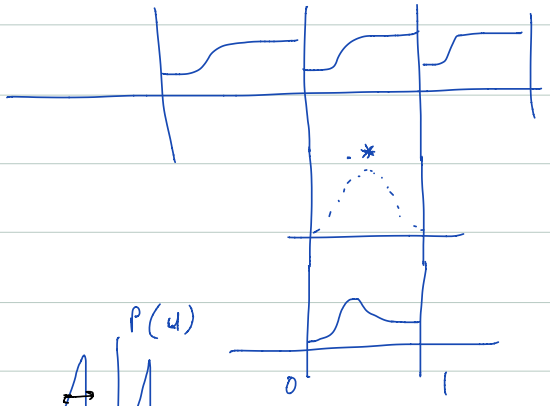
Practical Issues

1. Power distn in Fourier Domain
   - large DC component
   - scaling 1D

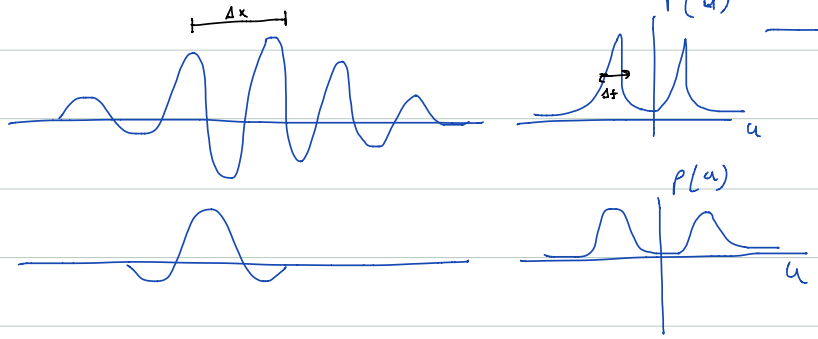


2. Windowing





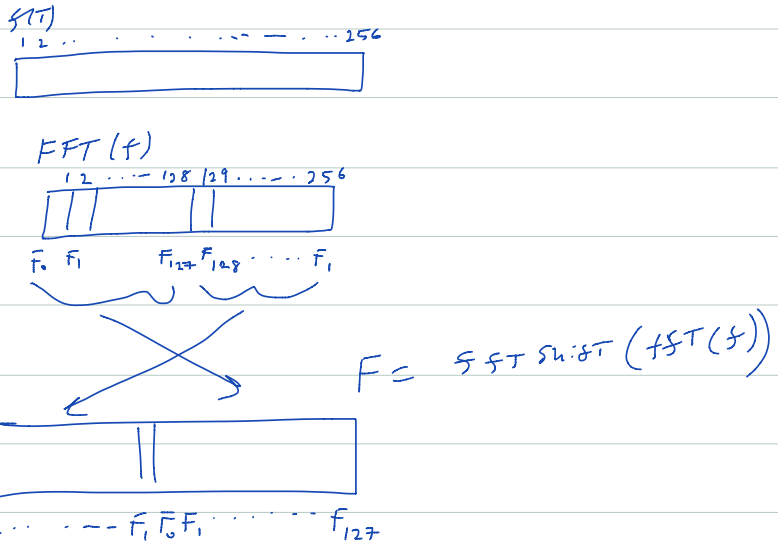
3. Uncertainty



$\Delta x \, \Delta f = \text{constant}$

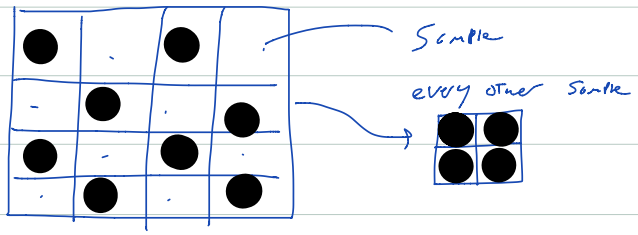
Image width — Bandwidth of filter

3. MATLAB



F = fftshift(fft(f))

4. Aliasing (Forsyth and Porce)



Checker board

or anything with

sharp edges.