Fourier Analysis

- Library books (Nalwa, Truco, Verri, Horn)

- Project proposal:

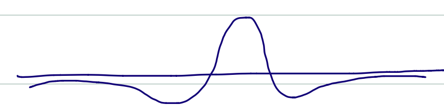
  UW id, hard copy, propose algorithm

  - Eigen faces
  - Optical flow

$\frac{1}{16}[1, 4, 6, 4, 1]$

Pyramid Szeliski

$$\nabla^2 G = \left(\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} \right)$$

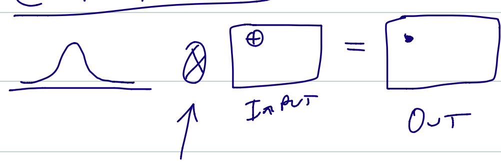


Mexican hat fn

Linear Shift Invariant? $\longleftrightarrow$ Fourier filter

Convolution

element wise multiply

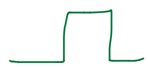
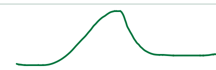


$(Im - (Im \otimes G))$ high pass

Fourier Analysis

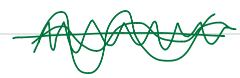
- Explicitly represent frequency of signals
- Characterize filters

e.g. box filters — Gaussian filter

- Use fourier features for recognition and analysis

(- Computational tool)



2D Signals

amplitude — phase shift

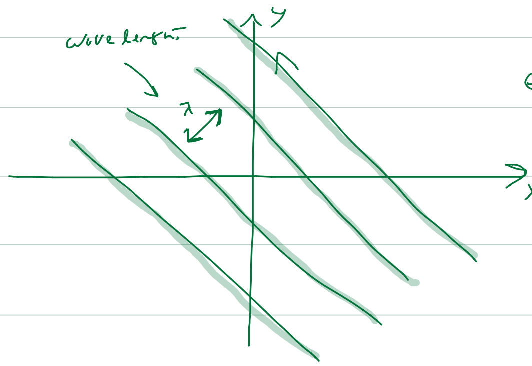
$$f(x,y) = A\cos(2\pi(ux+vy)+\phi)$$

(u, v) → spatial frequency $= \sqrt{u^2+v^2}$

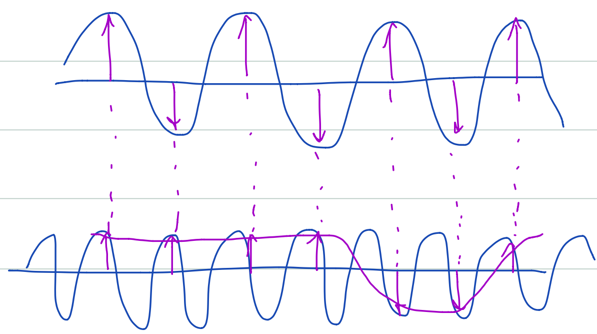
hor. vert.

Orientation $= \theta = \mathrm{atan2}(u,v)$

(x,y) image pos'n



$$e^{i2\pi(ux+vy)} = \cos(2\pi(ux+vy)) + i\sin(2\pi(ux+vy))$$



aliasing

Def'n (2D) Fourier Transform

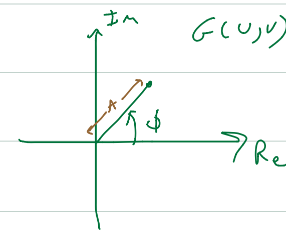
$$G(u,v) = \mathcal{F}\{g(x,y)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underbrace{g(x,y)}_{\text{image}}\ \underbrace{e^{-i2\pi(ux+vy)}}_{\text{basis}}\,dx\,dy$$

Complex # for each spatial frequency (u,v)

$|G(u,v)|$ magnitude

$\angle G(u,v)$ phase shift

$|G(u,v)|^2$ power/energy of u,v



Properties of F.T.

linearity $\mathcal{F}\{f(x,y)+g(x,y)\} = \mathcal{F}\{f(x,y)\} + \mathcal{F}\{g(x,y)\}$

inversion $g(x,y) = \mathcal{F}^{-1}\{G(u,v)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \underbrace{G(u,v)}\ \underbrace{e^{i2\pi(ux+vy)}}_{\text{sinusoidal "grating"}}\,du\,dv$

$$g(x,y) = \mathcal{F}^{-1}\{\mathcal{F}\{g(x,y)\}\}$$

Properties cont

$G(u,v)$ is complex fn

$G(u,v)$ is defined for both positive and negative freq.

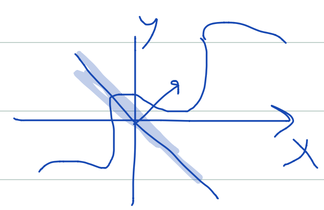
If $g(x,y)$ is even $\rightarrow G(u,v)$ is real & even

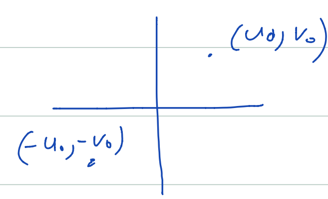
If $g(x,y)$ is odd $\rightarrow G(u,v)$ is imaginary & odd

Example: Single component (u_0, v_0).

if $g(x,y) = \cos(2\pi(u_0 x + v_0 y))$

$$\rightarrow G(u,v) = \frac{1}{2}\delta(u-u_0)\,\delta(v-v_0) + \frac{1}{2}\delta(u+u_0)\,\delta(v+v_0)$$



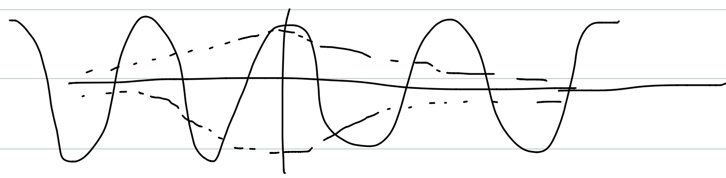


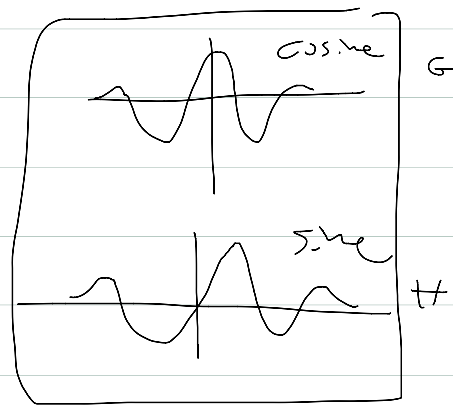
Example

got removed too fast...

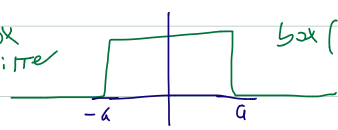
Gabor filter

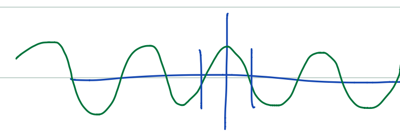
feature detection for sinusoids





Spatial domain	frequency domain
$f(x,y)$	$G(u,v)$
$\delta(0,0)$ impulse at origin	1 all frequencies
box filter, $\text{box}\left(\frac{x}{a}\right)$	$a \sin(au)$
$f(x-a, y-b)$ Shift	$e^{-2\pi(au+bv)} F(u,v)$ shifting phase same power spectrum.
$e^{-\pi(x^2+y^2)\frac{1}{a}}$ Gaussian	$e^{-\pi(u^2+v^2)a}$





Convolution Theorem

$$\mathcal{F}\{f(x,y) \otimes g(x,y)\} = \mathcal{F}\{f(x,y)\}\,\mathcal{F}\{g(x,y)\}$$

$$\boxed{f(x,y) \otimes g(x,y) = \mathcal{F}^{-1}\{F(u,v) \;.*\; G(i,v)\}}$$

Image

Gaussian filter.

Matlab notation