Asst # 1, due Tuesday (in class)

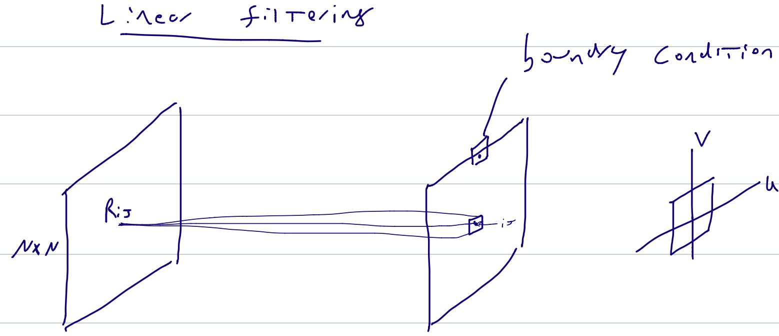
Project (1 page) due Thurs (in class)

Tutorial Mon 5–6pm DC 2306C

(Linear) Filtering

- image processing (eg. noise removal)
- image compression
- features (points + lines)
  - → SIFT: Scale invariant feature T...

Linear filtering



Output image R

Image F

$$R_{ij} = \sum_{u,v \in M} H_{u,v} \cdot F_{i+u,\, j+v}$$

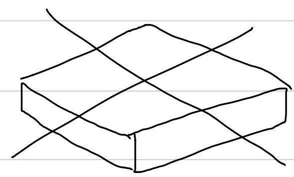
in MATLAB: $R_{ij} = H \,.\!*\, F\left(i-\frac{M}{2}-1 : i+\frac{M}{2},\; j-\frac{M}{2}-1 : j+\frac{M}{2}\right)$

Notes

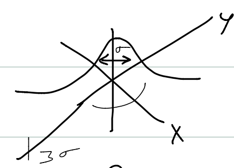
1. Finite (usually small) filter.

eg Gaussian $G_\sigma(x,y) = \left(\frac{1}{2\pi\sigma^2}\right)\exp\left(\frac{1}{2\sigma^2}(x^2+y^2)\right)$

eg $\sigma = 1$, filter goes $-3\sigma \le x, y \le 3\sigma$



"Box" filter

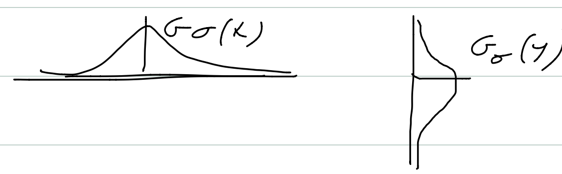


2. Finite images → Boundary conditions

"help conv2" in Matlab

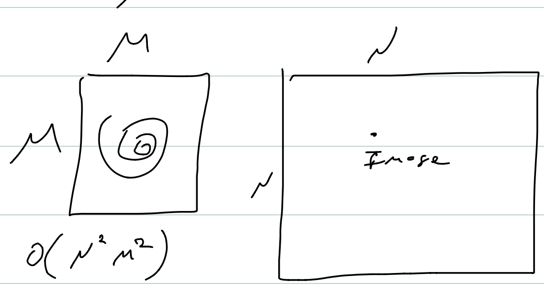
3. Seperable filters

$$G_\sigma(x,y) = \frac{1}{\sqrt{2\pi}\,\sigma}\exp\left(-\frac{1}{2\sigma^2}x^2\right)\frac{1}{\sqrt{2\pi}\,\sigma}\exp\left(-\frac{1}{2\sigma^2}y^2\right)$$



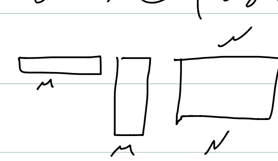
convolution

$G_\sigma(x,y) \otimes I$



$O(N^2 M^2)$

$G_\sigma(x) \otimes (G_\sigma(y) \otimes I)$



$2 \times M \times N^2$

Linear systems Theory

Convolution (or correlation)
represents a class of operators
called "Linear Systems"

Linearity

f, g images
R operator
(eg. blurring)

$$R(f+g) = R(f) + R(g)$$

eg $\quad blur(Im1 + Im2) = blur(Im1) + blur(Im2)$

Scaling $\quad R(kf) = kR(f)$

Linearity: $\quad R(\alpha f + \beta g) = \alpha R(f) + \beta R(g)$

Correlation

$$R_{ij} = \sum_{u,v \in M} H_{uv} \cdot F_{i+u, j+v}$$

Convolution

flipped order

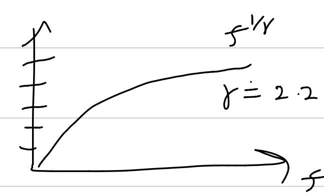
$$= \sum H_{uv} \cdot F_{i-u, j-v}$$

Note $\quad$ Non-linear operators

→ eg median filter

→ eg gamma collection

$$R(f) = \beta f^{\gamma}$$

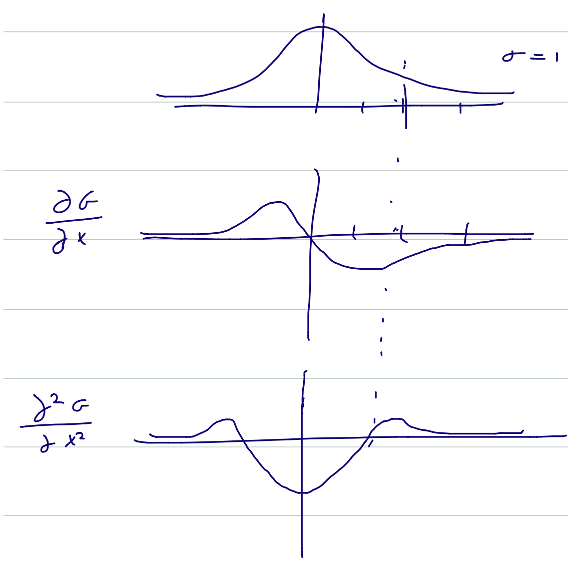


Shift Invariance
filter coefficients don't depend on position $\Rightarrow$ order of operations does not matter

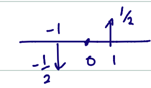
"Linear Shift invariant System"

$$D \otimes (G \otimes I) = (D \otimes G) \otimes I$$

$G_x \otimes I \quad G_x \otimes I$



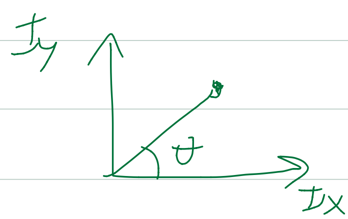
$D \triangleq$



$$G \propto \exp\left(\frac{1}{2\sigma^2} x^2\right)$$

$$\frac{\partial G}{\partial x} = \exp\left(\frac{-1}{2\sigma^2} x^2\right) \frac{2x}{2\sigma^2}$$

$$= \frac{x}{\sigma^2} G(x)$$



$$\boxed{\begin{array}{l} \|\nabla I\| \text{ edge magnitude} \\ \theta = \operatorname{atan2}(I_y, I_x) \text{ angle} \end{array}}$$

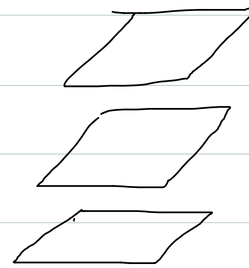
Example #2 Blurring

$$\text{blur}(x, 2\sigma) = \text{blur}(\text{blur}(x,\sigma),\sigma)$$ exercise

"Scale space"

$$g(x,y,\sigma) = \frac{1}{2\pi\sigma} e^{-(x^2+y^2)/2\sigma^2}$$

$$L(\cdot,\cdot,\sigma) = \underbrace{g(\cdot,\cdot,\sigma)}_{\text{Gaussian}} \otimes f(\cdot,\cdot)$$

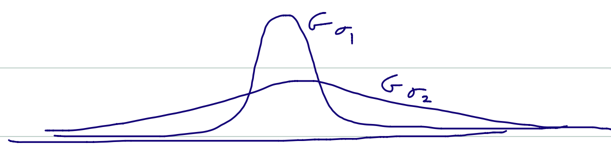


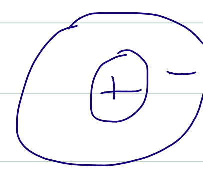
$\sigma = 0.5$

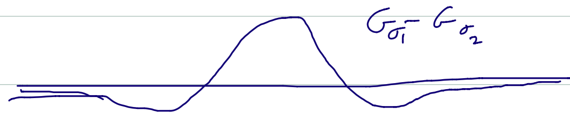
$\sigma = 2$

$\sigma = 8$

Example #3 Center Surround







DoG: difference of Gaussians

- very close to $\nabla^2 G = \frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y}$
- also similar to windowed cosine

"Gabor filter"