CS 489 / 698: Software and Systems Security

Module: Bug Finding Tools and Practices
Lecture: symbolic execution

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Outline

Motivation

Q: Why research on symbolic execution when we have unit testing or even fuzzing?

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A: A more complete exploration of program states.

Illustration

```
fn foo(x: u64): u64 {
    if (x * 3 == 42) {
        some_hidden_bug();
    }
    if (x * 5 == 42) {
        some_hidden_bug();
    }
    return 2 * x;
}
```

Illustration

Unit Test

```
foo(0);
foo(1);
```

```
fn foo(x: u64): u64 {
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Illustration

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fn foo(x: u64): u64 {
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}
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Unit Test

```
foo(0);
foo(1);
```

Fuzzing

```
foo(0);
foo(1);
foo(12);
foo(78);
......
foo(9,223,372,036,854,775,808);
```

Illustration

```
fn foo(x: u64): u64 {
    if (x * 3 == 42) {
        some_hidden_bug();
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```

Unit Test

foo(0);

foo(1);

Fuzzing

foo(0);

foo(1);

foo(12);

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......

foo(9,223,372,036,854,775,808);

Symbolic execution

foo(x)

aborts when $x = 14$

returns $2x$ otherwise

Satisfiability Modulo Theories (SMT)

Definition: A procedure that decides whether a mathematical formula is satisfiable.

Example:

- $3x = 42$
- $2x \geq 2^{64}$
- $5x = 42$

Satisfiability Modulo Theories (SMT)

Definition: A procedure that decides whether a mathematical formula is satisfiable.

Example:

- $3x = 42 \longrightarrow$ satisfiable with $x = 14$
- $2x \geq 2^{64} \longrightarrow$ satisfiable with $x \geq 2^{63}$
- $5x = 42 \longrightarrow$ unsatisfiable, cannot find an x

Ask two question whenever you see a symbolic execution work:

- How does it convert code into mathematical formula?
- What does it try to solve for?

Program Modeling Desiderata

- Control-flow graph exploration
- Loop handling
- Memory modeling
- Concurrency

Outline

An example of a pure function

```
fn foo(
    c1: bool, c2: bool,
    x: u64
) -> u64 {
    let r = if (c1) {
        x + 3
    } else {
        x + 4
    };

    let r = if (c2) {
        r - 1
    } else {
        r - 2
    };

    r
}
spec foo {
    ensures r > x;
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An example of a pure function

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fn foo(
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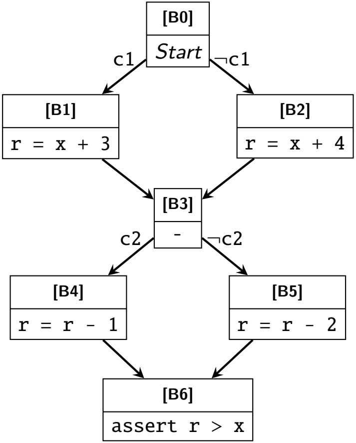

The example in SSA form

```
fn foo(
    c1: bool, c2: bool,
    x: u64
) -> u64 {
    let r = if (c1) {
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    } else {
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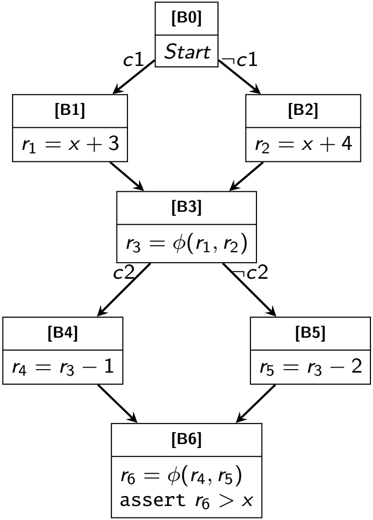
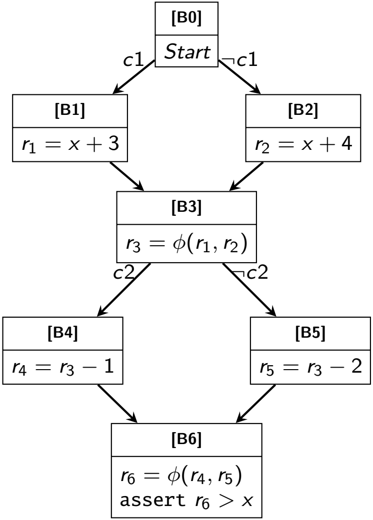


Path-based exploration

Vars: $c1$, $c2$, x, r_{1-6}

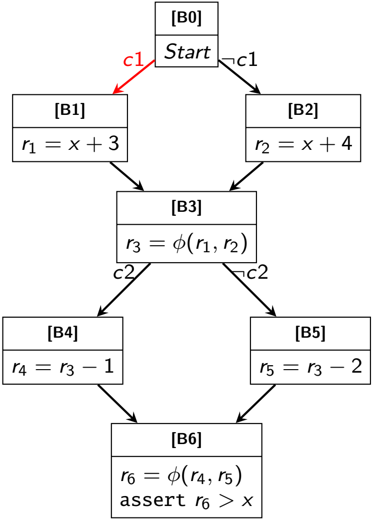
B0	Sym. repr. Path cond.	$\emptyset$ True



Path-based exploration

Vars: $c1$, $c2$, x, r_{1-6}

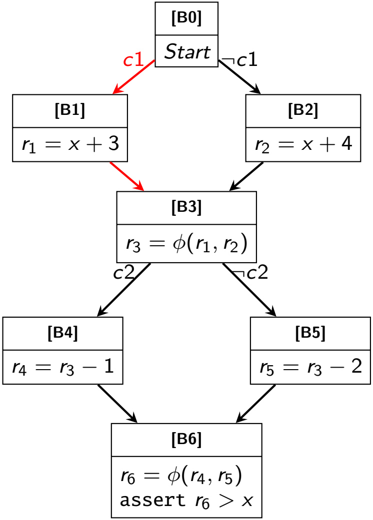
B0	Sym. repr. Path cond.	$\emptyset$ True
B1	Sym. repr. Path cond.	$r_1 = x + 3$ $c1$



Path-based exploration

Vars: $c1$, $c2$, x, r_{1-6}

B0	Sym. repr. Path cond.	$\emptyset$ True
B1	Sym. repr. Path cond.	$r_1 = x + 3$ $c1$
B3	Sym. repr. Path cond.	$r_1 = x + 3$ $r_3 = r_1$ $c1$



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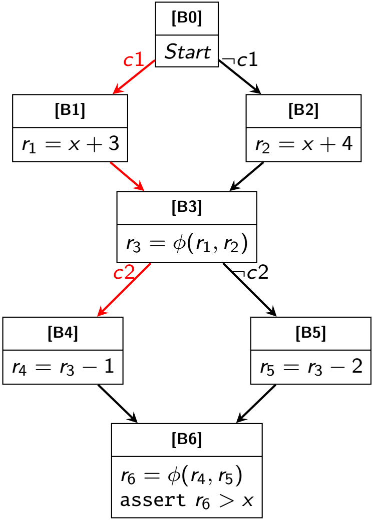

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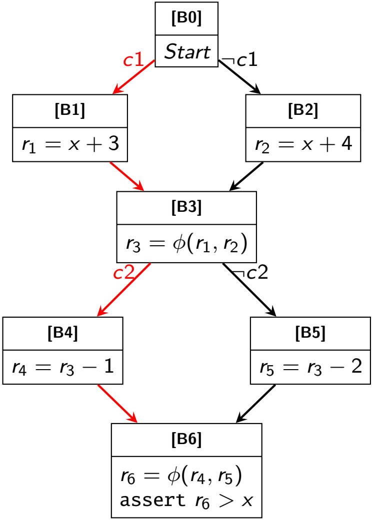
B6	Sym. repr. Path cond.	$r_1 = x + 3$ $r_3 = r_1$ $r_4 = r_3 - 1$ $r_6 = r_4$ $c_1 \wedge c_2$

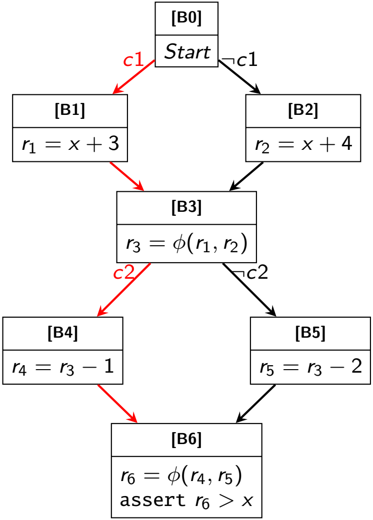


Proving procedure (per path)

Vars: $c1$, $c2$, x, r_{1-6}

B6	Sym. repr.	$r_1 = x + 3$ $r_3 = r_1$ $r_4 = r_3 - 1$ $r_6 = r_4$
	Path cond.	$c_1 \wedge c_2$

$\rightsquigarrow$



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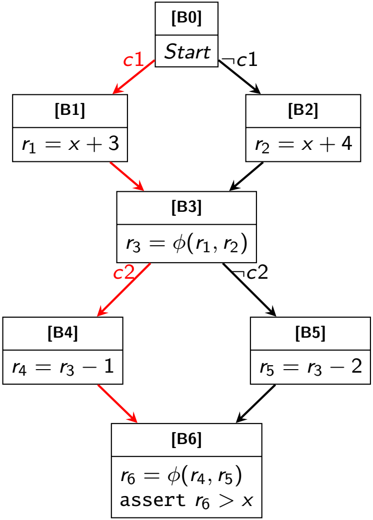
Prove that $\forall\ c1, c2, x, r_{1-6}$:

$$((c1 \wedge c2) \wedge (\\ (r_1 = x + 3) \\ (r_3 = r_1) \\ (r_4 = r_3 - 1) \\ (r_6 = r_4) \\ )) \Rightarrow (r_6 > x)$$



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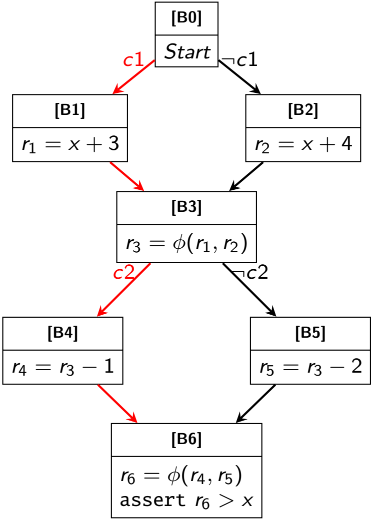
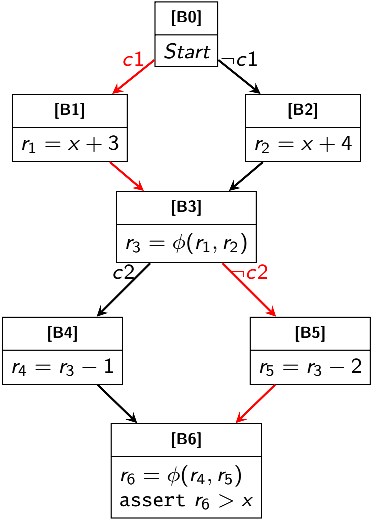


Proving procedure (all paths)

Prove that
$\forall\ c1, c2, x, r_{1-6}$:

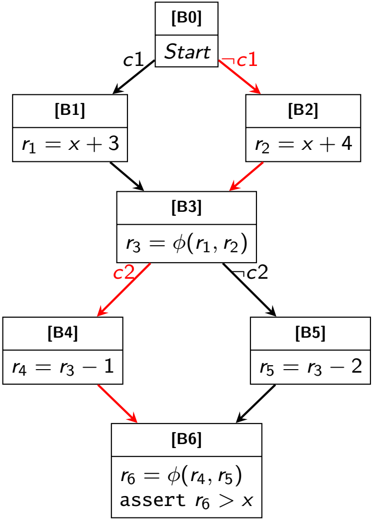
$$((c1 \land \neg c2) \land ($$
$$(r_1 = x + 3)$$
$$(r_3 = r_1)$$
$$(r_5 = r_3 - 2)$$
$$(r_6 = r_5)$$
$$)) \Rightarrow (r_6 > x)$$



Proving procedure (all paths)

Prove that
$\forall\ c1, c2, x, r_{1-6}$:

$$((\neg c1 \land c2) \land ($$
$$(r_2 = x + 4)$$
$$(r_3 = r_2)$$
$$(r_4 = r_3 - 1)$$
$$(r_6 = r_4)$$
$$)) \Rightarrow (r_6 > x)$$



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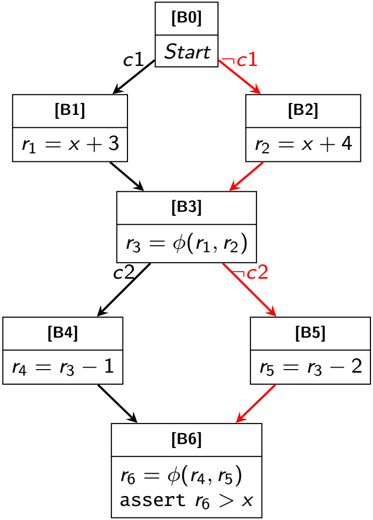
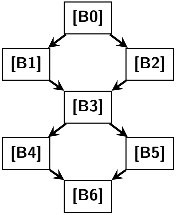


Path explosion

Path explosion

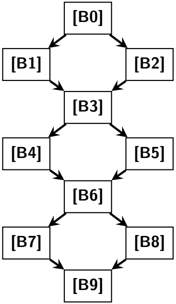
2^{2} paths



Path explosion

2^{2} paths

2^{3} paths



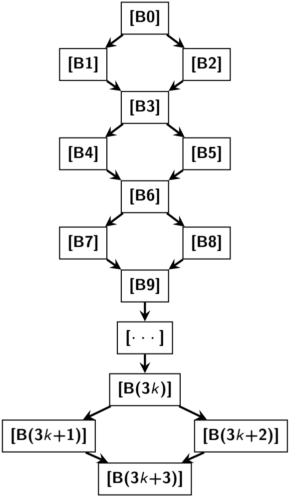
Path explosion

2^{2} paths

2^{3} paths

...

2^{k} paths



How about loops?

```
// a library function
fn sync(
  x: u64, y: u64, n: u64
) -> (u64, u64, u64) {
  let a = x, b = y, i = 0;
  while (a < n) {
    if (b > a) {
      a++;
    } else {
      b++;
    }
    i++;
  }
  return (a, b, i);
}
```

```
// core application logic
pub fn main() {
  let (x, y, n) = input();
  let (a, b, i) = sync(x, y, n);
  assert!(i == 0 || i < 2*n);
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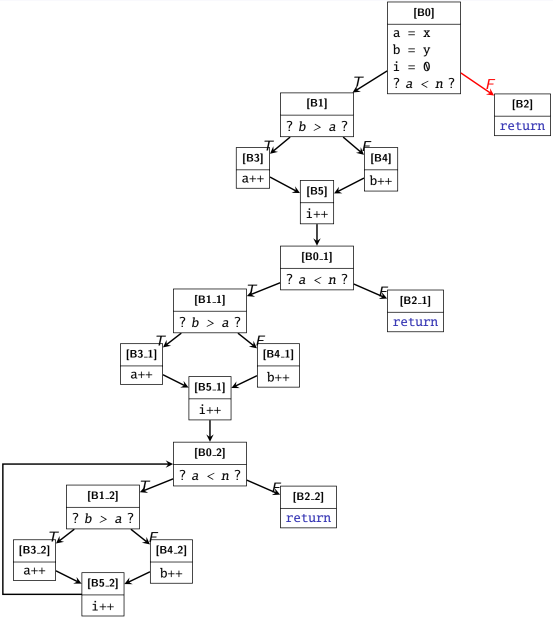
}
```

Conventional symbolic execution



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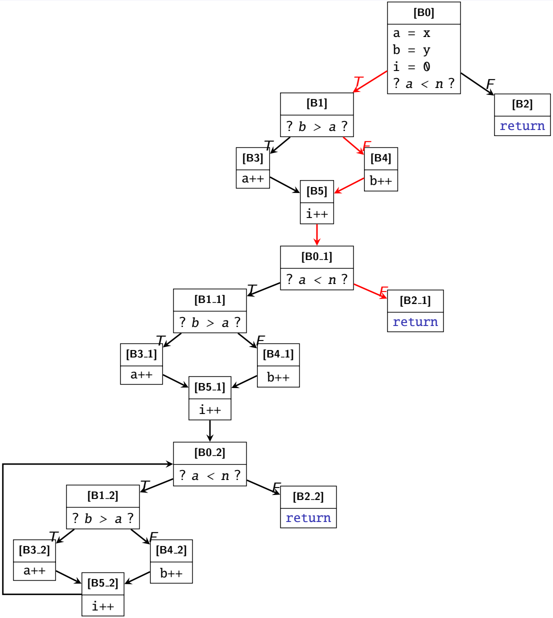

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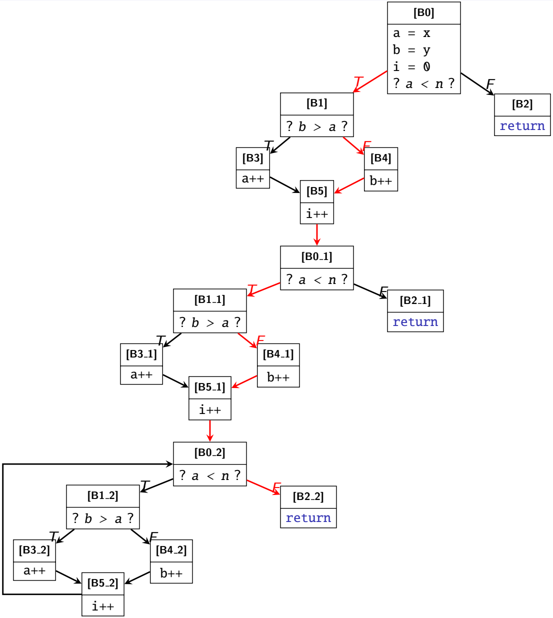

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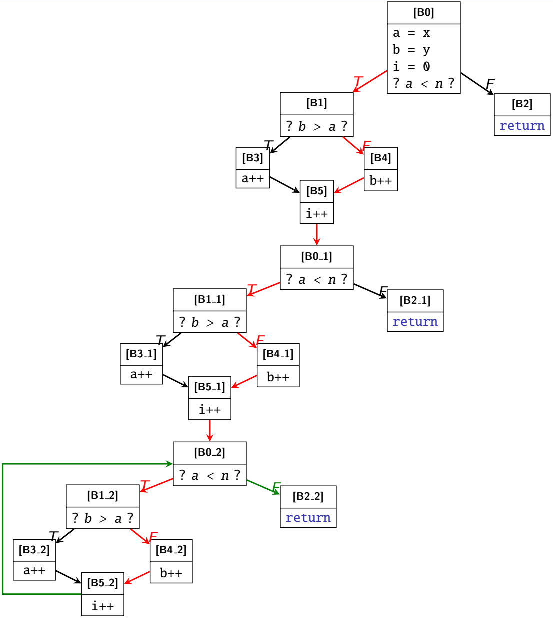

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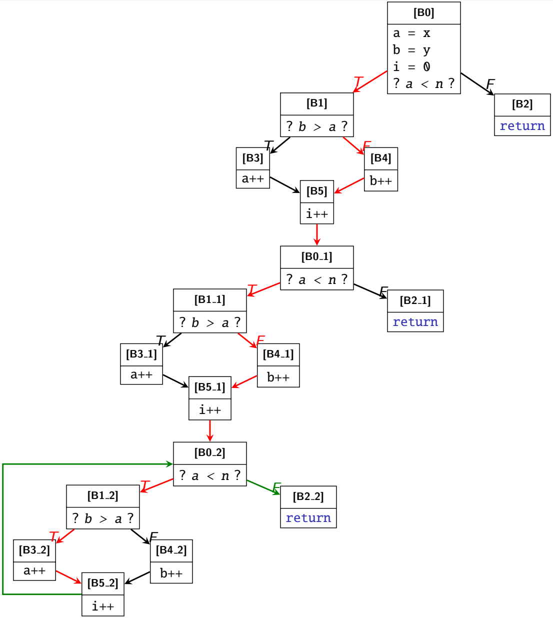

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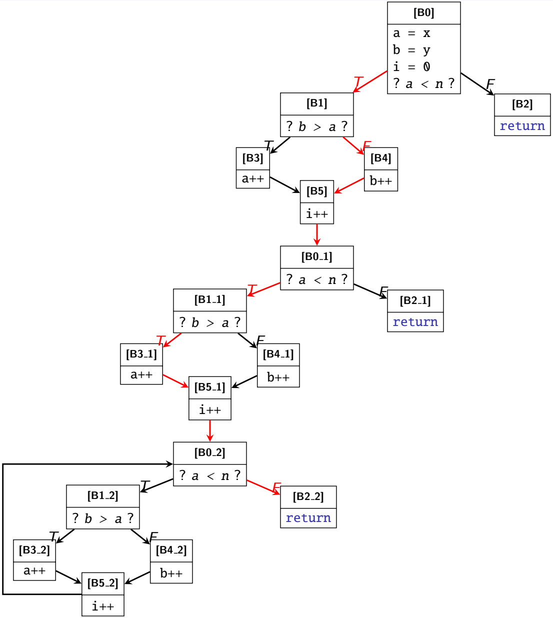
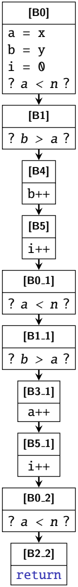


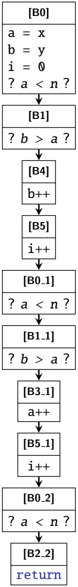
Encoding the path conditions



Find x, y, n such that

- $x < n$ (from [B0])
- $y \leq x$ (from [B1])
- $x < n$ (from [B0_1])
- $y + 1 > x$ (from [B1_1])
- $x + 1 \geq n$ (from [B0_2])
- $n \neq 0 \wedge i \geq 2n$ (from `assert!`)

Encoding the path conditions



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- $x + 1 \geq n$ (from [B0_2])
- $n \neq 0 \wedge i \geq 2n$ (from `assert!`)

Solving the predicates yield:
$\{\ x = 0,\ y = 0,\ n = 1\ \}$

Symbolic execution for bug finding

```
// a library function
fn sync(
  x: u64, y: u64, n: u64
) -> (u64, u64, u64) {
  let a = x, b = y, i = 0;
  while (a < n) {
    if (b > a) {
      a++;
    } else {
      b++;
    }
    i++;
  }
  return (a, b, i);
}
```

```
// core application logic
pub fn main() {
  let (x, y, n) = input();
  let (a, b, i) = sync(x, y, n);
  assert!(i == 0 || i < 2*n);
  \\\\\\\\\\\\\\\\\\\\\\\\\\\
}
```

- x=0, y=0, n=1 → a=1, b=1, i=2
- x=0, y=0, n=2 → a=2, b=2, i=4
- ......
- x=0, y=0, n=k → a=k, b=k, i=$2k$

Path explosion in symbolic execution

```
// a library function
fn sync(
  x: u64, y: u64, n: u64
) -> (u64, u64, u64) {
  let a = x, b = y, i = 0;
  while (a < n) {
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    } else {
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    i++;
  }
  return (a, b, i);
}
```

```
// core application logic
pub fn main() {
  let (x, y, n) = input();
  let (a, b, i) = sync(x, y, n);
  assert!(n-a-b+i != 42);
  \\\\\\\\\\\\\\\\\\\\\\\\
}
```

Q: What if a bug can only be triggered after exploring k branches?

Path explosion in symbolic execution

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// a library function
fn sync(
  x: u64, y: u64, n: u64
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  let (x, y, n) = input();
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}
```

Q: What if a bug can only be triggered after exploring *k* branches?

In fact, this bug can only be triggered after at least 42 levels of loop unrolling.

- x=0, y=0, n=42 → a=42, b=42, i=84
- x=9, y=5, n=56 → a=56, b=56, i=98

In the conventional way of symbolic execution, finding this bug requires an exhaustive search of 2^{42} paths.

Outline

Weakest precondition calculus

Move prover (Boogie actually) adopts a backward state exploration process, following the weakest precondition calculus.

The running example, once again

```
fn foo(
    c1: bool, c2: bool,
    x: u64
) -> u64 {
    let r = if (c1) {
        x + 3
    } else {
        x + 4
    };

    let r = if (c2) {
        r - 1
    } else {
        r - 2
    };

    r
}
spec foo {
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}
```

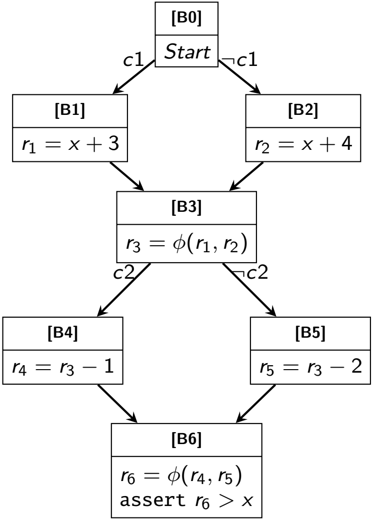

The passification process

Convert the program into a dynamic single assignment (DSA) form.

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DSA is extremely similar to static single assignment (SSA) with the ϕ-node eagerly uplifted.

The passification process

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fn foo(
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) -> u64 {
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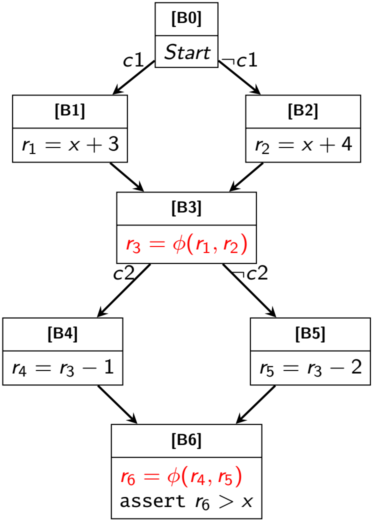

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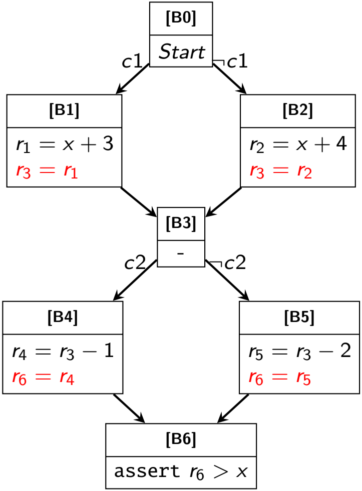

The walk-up process

Do a topological sort on the CFG and traverse backward.

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Do a topological sort on the CFG and traverse backward.

This ensures that for each block in the CFG, we visit it *once and only once* (assuming no loops).

The walk-up algorithm

Follow these rules for the intra-block walk-up process:

- $wp(\text{assert } c) = c$
- $wp(\text{assert } c, Q) = c \land Q$
- $wp(\text{assign } e, Q) = e \implies Q$
- $wp(s_1; s_2, Q) = wp(s_1, wp(s_2, Q))$

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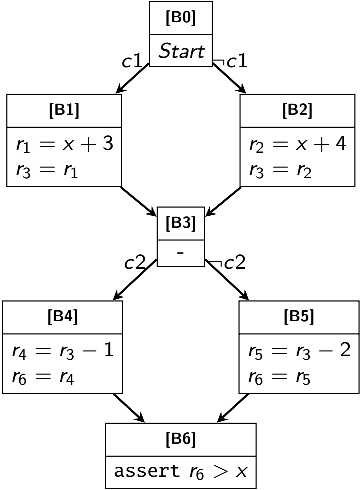
- $wp(\text{assign } e, Q) = e \implies Q$
- $wp(s_1; s_2, Q) = wp(s_1, wp(s_2, Q))$

The rule for inter-block walk-up is:

$$A \leftarrow wp(s_1; s_2; \ldots; s_n, \bigwedge_{B \in \text{Succ}(A)} B)$$

The walk-up process with an example

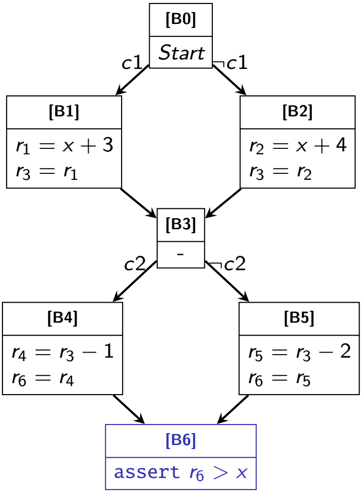
Vars: $c1$, $c2$, x, r_{1-6}, B_{0-6}



The walk-up process with an example

Vars: $c1$, $c2$, x, r_{1-6}, B_{0-6}

$B_6 \leftarrow r_6 > x$

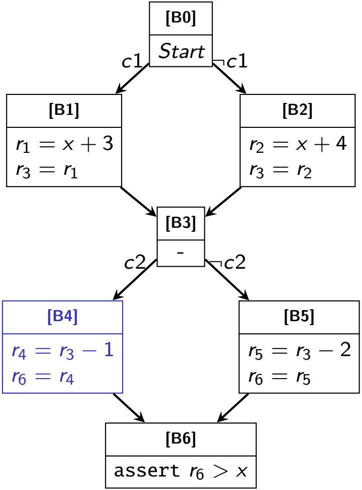


The walk-up process with an example

Vars: $c1$, $c2$, x, r_{1-6}, B_{0-6}

$B_6 \leftarrow r_6 > x$

$B_4 \leftarrow (c2) \Rightarrow ($
$\quad (r_4 = r_3 - 1) \Rightarrow ($
$\quad\quad (r_6 = r_4) \Rightarrow B_6))$



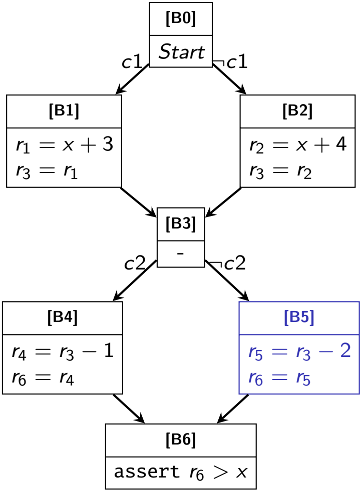
The walk-up process with an example

Vars: $c1,\ c2,\ x,\ r_{1-6},\ B_{0-6}$

$B_6 \leftarrow r_6 > x$

$$B_4 \leftarrow (c2) \Rightarrow ((r_4 = r_3 - 1) \Rightarrow ((r_6 = r_4) \Rightarrow B_6))$$

$$B_5 \leftarrow (\neg c2) \Rightarrow ((r_5 = r_3 - 2) \Rightarrow ((r_6 = r_5) \Rightarrow B_6))$$



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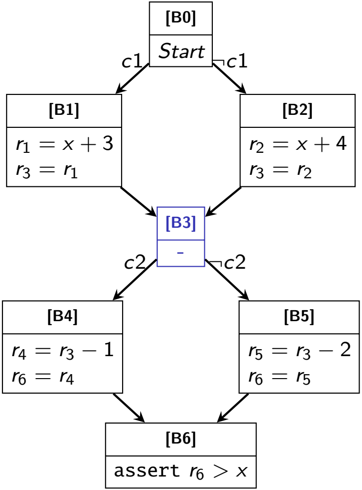
$B_3 \leftarrow B_4 \wedge B_5$



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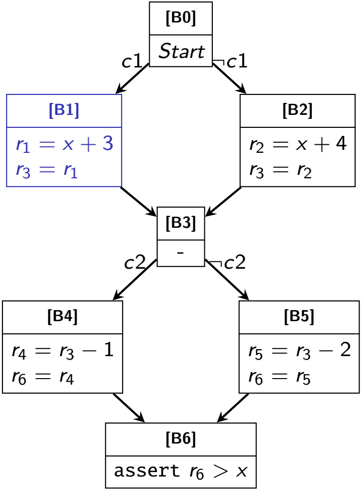
$$B_1 \leftarrow (c1) \Rightarrow ((r_1 = x + 3) \Rightarrow ((r_3 = r_1) \Rightarrow B_3))$$



The walk-up process with an example

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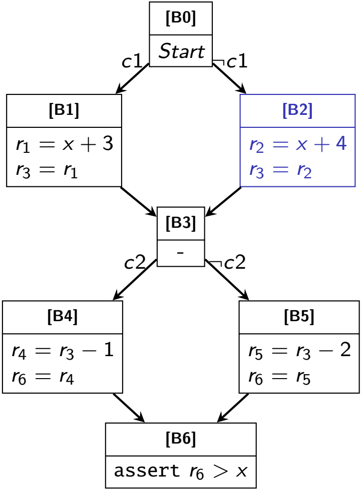
$B_3 \leftarrow B_4 \wedge B_5$

$$B_1 \leftarrow (c1) \Rightarrow ((r_1 = x + 3) \Rightarrow ((r_3 = r_1) \Rightarrow B_3))$$

$$B_2 \leftarrow (\neg c1) \Rightarrow ((r_2 = x + 4) \Rightarrow ((r_3 = r_2) \Rightarrow B_3))$$



The walk-up process with an example

Vars: $c1, c2, x, r_{1-6}, B_{0-6}$

$B_6 \leftarrow r_6 > x$

$B_4 \leftarrow (c2) \Rightarrow ($
$(r_4 = r_3 - 1) \Rightarrow ($
$(r_6 = r_4) \Rightarrow B_6))$

$B_5 \leftarrow (\neg c2) \Rightarrow ($
$(r_5 = r_3 - 2) \Rightarrow ($
$(r_6 = r_5) \Rightarrow B_6))$

$B_3 \leftarrow B_4 \wedge B_5$

$B_1 \leftarrow (c1) \Rightarrow ($
$(r_1 = x + 3) \Rightarrow ($
$(r_3 = r_1) \Rightarrow B_3))$

$B_2 \leftarrow (\neg c1) \Rightarrow ($
$(r_2 = x + 4) \Rightarrow ($
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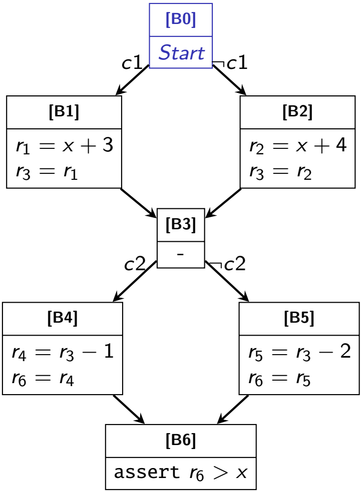
$B_0 \leftarrow B_1 \wedge B_2$

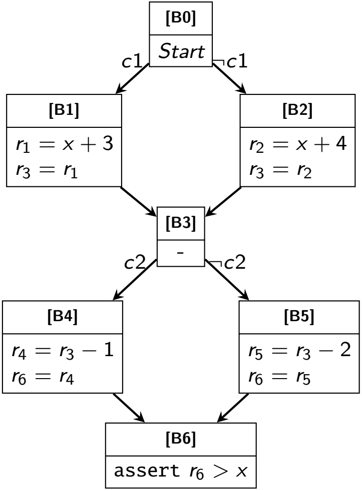


Proving procedure

Prove that
$\forall\ c1, c2, x, r_{1-6}, B_{0-6}$:

$$
\begin{aligned}
&B_6 \leftarrow r_6 > x\\
&B_4 \leftarrow (c2) \Rightarrow (\\
&\quad (r_4 = r_3 - 1) \Rightarrow (\\
&\qquad (r_6 = r_4) \Rightarrow B_6))\\
&B_5 \leftarrow (\neg c2) \Rightarrow (\\
&\quad (r_5 = r_3 - 2) \Rightarrow (\\
&\qquad (r_6 = r_5) \Rightarrow B_6))\\
&B_3 \leftarrow B_4 \wedge B_5\\
&B_1 \leftarrow (c1) \Rightarrow (\\
&\quad (r_1 = x + 3) \Rightarrow (\\
&\qquad (r_3 = r_1) \Rightarrow B_3))\\
&B_2 \leftarrow (\neg c1) \Rightarrow (\\
&\quad (r_2 = x + 4) \Rightarrow (\\
&\qquad (r_3 = r_2) \Rightarrow B_3))\\
&B_0 \leftarrow B_1 \wedge B_2
\end{aligned}
$$

$B_0 = \text{True}$



Comparison of forward and backward symbolic execution

Prove that $\forall\ c1, c2, x, r_{1-6}$:

$$
\begin{aligned}
&((c1 \wedge c2) \wedge (\\
&\quad (r_1 = x + 3) \\
&\quad (r_3 = r_1) \\
&\quad (r_4 = r_3 - 1) \\
&\quad (r_6 = r_4) \\
&)) \Rightarrow (r_6 > x)
\end{aligned}
$$

However, need to repeat this process multiple (worst case exponential) times.

Prove that
$\forall\ c1, c2, x, r_{1-6}, B_{0-6}$:

$$
\begin{aligned}
&B_6 \leftarrow r_6 > x \\
&B_4 \leftarrow (c2) \Rightarrow (\\
&\qquad (r_4 = r_3 - 1) \Rightarrow (\\
&\qquad\quad (r_6 = r_4) \Rightarrow B_6)) \\
&B_5 \leftarrow (\neg c2) \Rightarrow (\\
&\qquad (r_5 = r_3 - 2) \Rightarrow (\\
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&B_3 \leftarrow B_4 \wedge B_5 \\
&B_1 \leftarrow (c1) \Rightarrow (\\
&\qquad (r_1 = x + 3) \Rightarrow (\\
&\qquad\quad (r_3 = r_1) \Rightarrow B_3)) \\
&B_2 \leftarrow (\neg c1) \Rightarrow (\\
&\qquad (r_2 = x + 4) \Rightarrow (\\
&\qquad\quad (r_3 = r_2) \Rightarrow B_3)) \\
&B_0 \leftarrow B_1 \wedge B_2
\end{aligned}
$$

$B_0 = \text{True}$

Outline

Breaking cycles in the CFG

Loop invariants are keys to break cycles in the CFG

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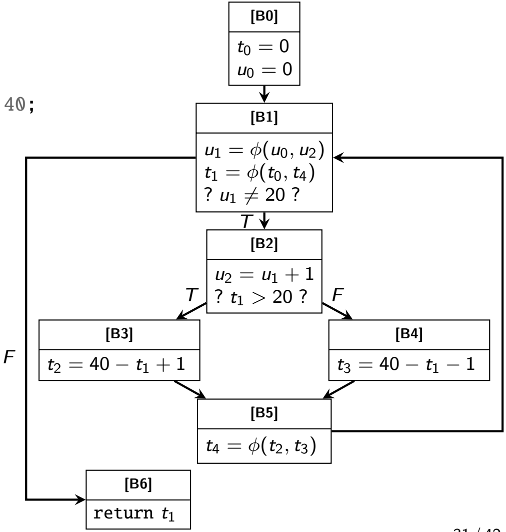
A loop invariant is transformed into statements that:

- Assert the invariant at the beginning of the loop
- Havoc (i.e., re-symbolize) the loop induction variables
- Assume the invariant to re-establish relations among the induction variables being havoc-ed
- Assert the invariant at the end of the loop body

A running example

```
fn bar(): u64 {
    t: u64 = 0;
    u: u64 = 0;
    while ({
spec {
    invariant t >= 20 ==> u + t == 40;
    invariant t <= 20 ==> u == t;
}
        (u != 20)
    }) {
        u = u + 1;
        if (t > 20) {
            t = 40 - t + 1;
        } else {
            t = 40 - t - 1;
        }
    }
    t
}
spec bar {
    ensures result == 20;
}
```



A running example

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fn bar(): u64 {
    t: u64 = 0;
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spec {
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}
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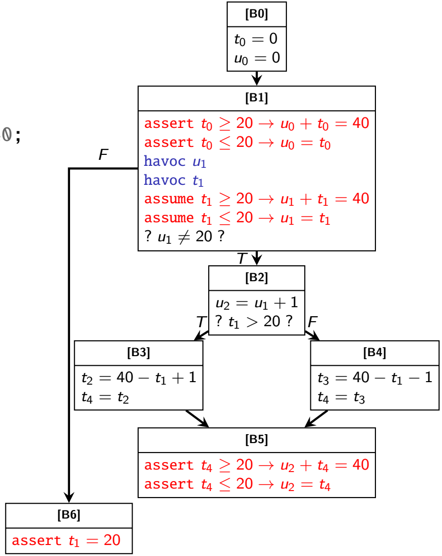
        u = u + 1;
        if (t > 20) {
            t = 40 - t + 1;
        } else {
            t = 40 - t - 1;
        }
    }
    t
}
spec bar {
    ensures result == 20;
}
```



A running example

$$B_6 \leftarrow (u_1 = 20) \Rightarrow ((t_1 = 20))$$

$$B_5 \leftarrow ((t_4 \leq 20 \rightarrow u_2 = t_4) \wedge (t_4 \geq 20 \rightarrow u_2 + t_4 = 40))$$

$$B_4 \leftarrow (t1 \leq 20) \Rightarrow ((t_3 = 40 - t_1 - 1) \Rightarrow ((t_4 = t_3) \Rightarrow B_5))$$

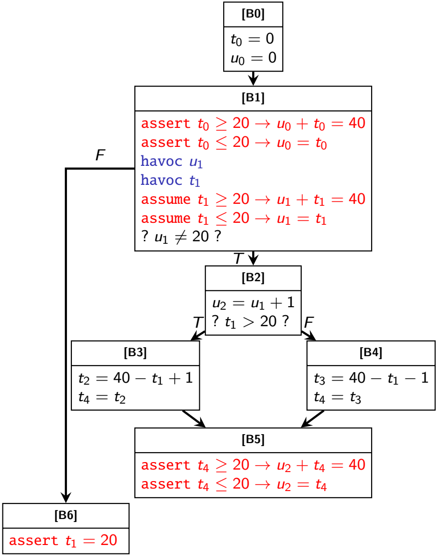
$$B_3 \leftarrow (t1 > 20) \Rightarrow ((t_2 = 40 - t_1 + 1) \Rightarrow ((t_4 = t_2) \Rightarrow B_5))$$

$$B_2 \leftarrow (u_1 \neq 20) \Rightarrow ((u_2 = u_1 + 1) \Rightarrow (B_3 \wedge B4))$$

$$B_1 \leftarrow ((t_0 \geq 20 \rightarrow u_0 + t_0 = 40) \wedge ((t_0 \leq 20 \rightarrow u_0 = t_0) \wedge ((t_1 \geq 20 \rightarrow u_1 + t_1 = 40) \Rightarrow ((t_1 \leq 20 \rightarrow u_1 = t_1) \Rightarrow (B_2 \wedge B6)))))$$

$$B_0 \leftarrow ((t_0 = 0) \Rightarrow ((u_0 = 0) \Rightarrow B_1))$$

Prove that: $B_0 = \text{True}$

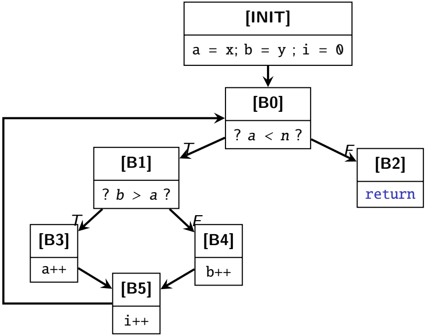


Back to the same symbolic loop unrolling example

```
// a library function
fn sync(
  x: u64, y: u64, n: u64
) -> (u64, u64, u64) {
  let a = x, b = y, i = 0;
  while (a < n) {
    if (b > a) {
      a++;
    } else {
      b++;
    }
    i++;
  }
  return (a, b, i);
}
```

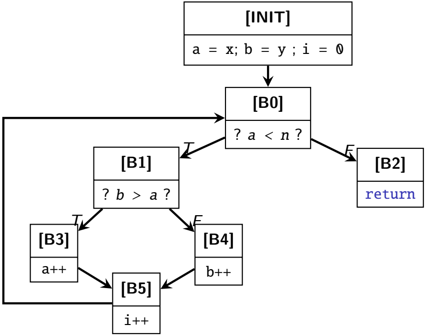
```
// core application logic
pub fn main() {
  let (x, y, n) = input();
  let (a, b, i) = sync(x, y, n);
  assert!(n-a-b+i != 42);

}
```



Constrained Horn Clause (CHC)

$$a = x \land b = y \land i = 0 \implies A(a, b, i)$$

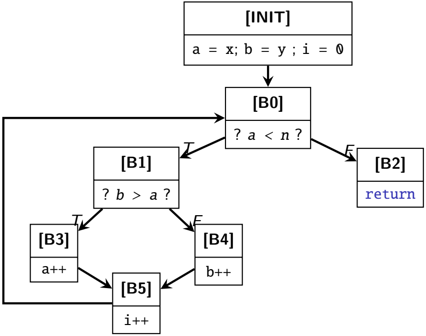


Constrained Horn Clause (CHC)

$$a = x \land b = y \land i = 0 \implies A(a, b, i)$$

$$A(a, b, i) \;\land\; a < n \land \\ b > a \;\land\; a' = a + 1 \;\land\; b' = b \land \\ i' = i + 1 \implies A(a', b', i')$$

$$A(a, b, i) \;\land\; a < n \land \\ b \leq a \;\land\; a' = a \;\land\; b' = b + 1 \land \\ i' = i + 1 \implies A(a', b', i')$$



Constrained Horn Clause (CHC)

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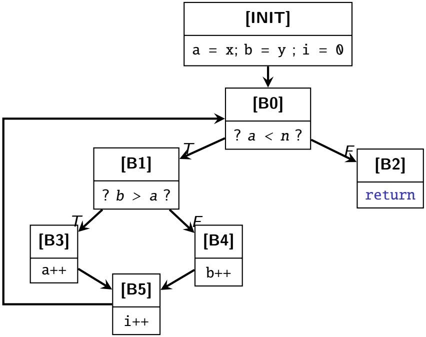
$$A(a, b, i) \wedge a < n \wedge b \leq a \wedge a' = a \wedge b' = b + 1 \wedge i' = i + 1 \implies A(a', b', i')$$

$$A(a, b, i) \wedge a \geq n \implies B(a, b, i)$$



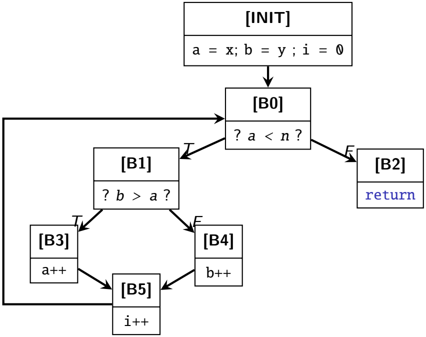
Constrained Horn Clause (CHC)

$$a = x \land b = y \land i = 0 \implies A(a, b, i)$$

$$\begin{aligned} &A(a, b, i) \ \land \ a < n \land \\ &\quad b > a \ \land \ a' = a + 1 \ \land \ b' = b \land \\ &\quad i' = i + 1 \implies A(a', b', i') \\ &A(a, b, i) \ \land \ a < n \land \\ &\quad b \leq a \ \land \ a' = a \ \land \ b' = b + 1 \land \\ &\quad i' = i + 1 \implies A(a', b', i') \end{aligned}$$

$$A(a, b, i) \ \land \ a \geq n \implies B(a, b, i)$$

solve $B(a, b, i) \ \land \ n - a - b + i = 42$



Benefits of the CHC encoding

Encoding the program in CHC allows solvers to infer invariants about the loop. In this particular case, an invariant can be inferred:

$$a + b - i = x + y$$

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Encoding the program in CHC allows solvers to infer invariants about the loop. In this particular case, an invariant can be inferred:

$$a + b - i = x + y$$

This helps to solve for a concrete test case that there exists x, y, and n such that $n = x + y + 42$.

Outline

Simple borrow

```
struct S {
    f1: u64,
    f2: u64,
}

fn foo(x: &mut S) {
    let p = &mut x.f1;
    *p = 1;
}
```

Simple borrow

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}

fn foo(x: &mut S) {
    let p = &mut x.f1;
    *p = 1;
}
```

```
fn _foo_(x: Mutation<S>) -> Mutation<S> {
    // p := borrow_field<S>.f1(x);
    let p = Mutation<u64> {
        root: x.root, // Param(0),
        paths: concat!(x.paths, Field(0)),
        value: x.value.f1,
    };

    // p2 := write_ref(p, 1);
    let p2 = update!(p, @value = 1);

    // x2 := write_back[x.f1](p2);
    let v = update!(x.value, @f1 = p2.value)
    let x2 = update!(x, @value = v)

    // return x2;
    x2
}
```

Mutations under borrow semantics

```rust
enum Root {
    Param(usize),
    Local(usize),
}

enum Path {
    Field(usize),
    Index(usize),
}

struct Mutation<T> {
    root: Root,
    paths: Vec<Path>,
    value: T,
}
```

Mutations under borrow semantics

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    root: Root,
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```

```rust
struct S {
    f1: u64,
    f2: u64,
}

fn foo(x: &mut S) {
    let p = &mut x.f1;
    *p = 1;
}
```

```rust
fn _foo_(x: Mutation<S>) -> Mutation<S> {
    Mutation<S> {
        root: x.root, // Root::Param(0)
        paths: x.paths, // vec[]
        value: S {
            f1: 1,
            f2: x.value.f2,
        }
    }
}
```

Outline

Definition of concolic execution

Background: **concolic**, as the name suggests, is the combination of two English words: *concrete* and *symbolic*, and the order matters!

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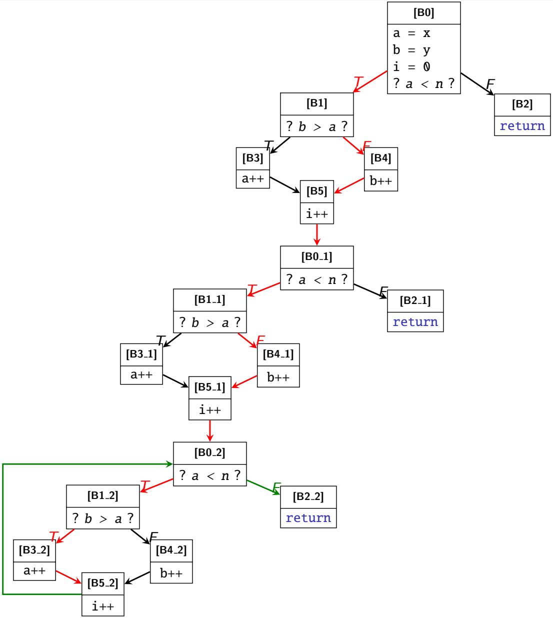
The basic idea of concolic execution is:

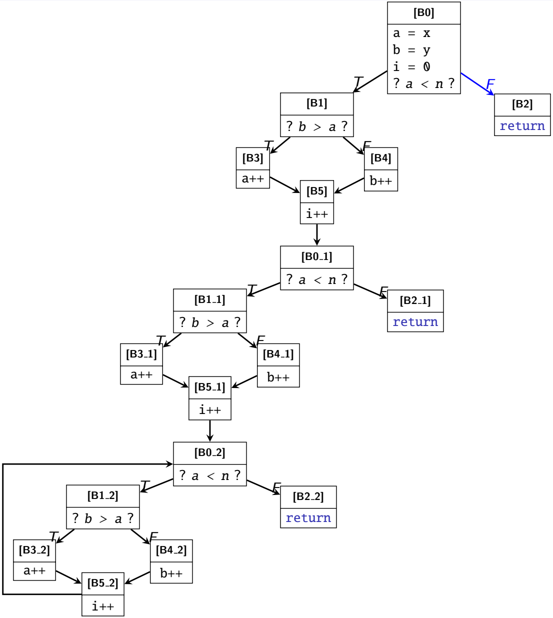
1. Execute a test case concretely
2. For each branch encountered in the test case, find another test case that toggles this branch symbolically.

Concolic execution with the running example



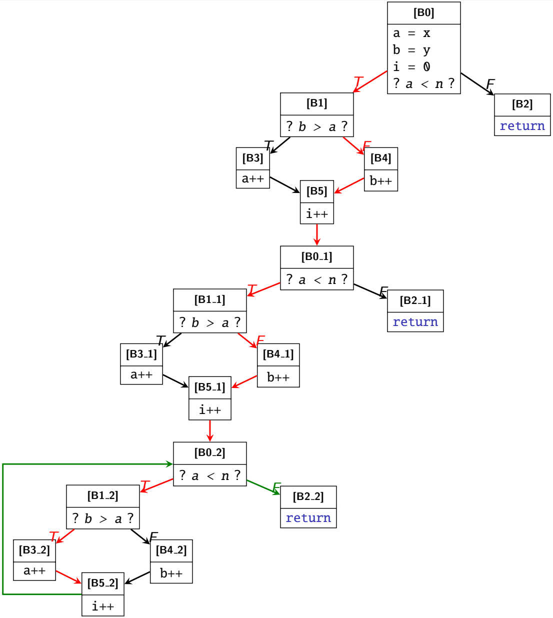
{x=1, y=0, n=2}

Concolic execution with the running example



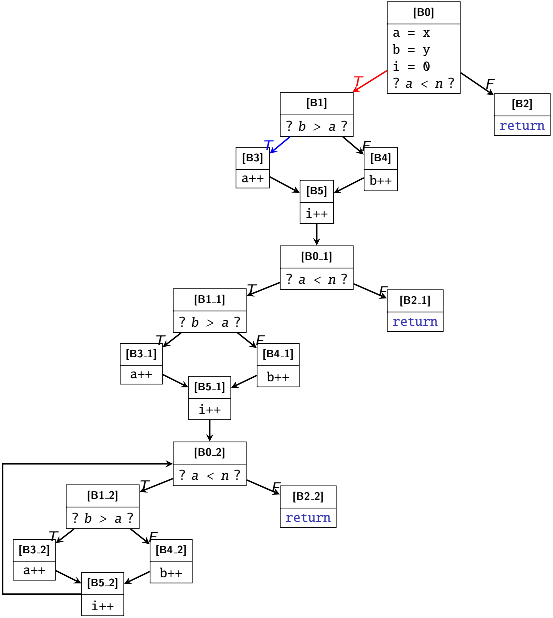
{x=9, y=2, n=6}

Concolic execution with the running example



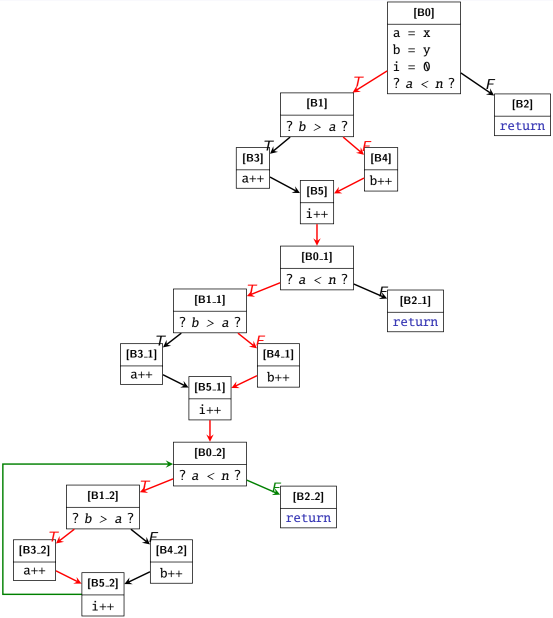
{x=1, y=0, n=2}

Concolic execution with the running example



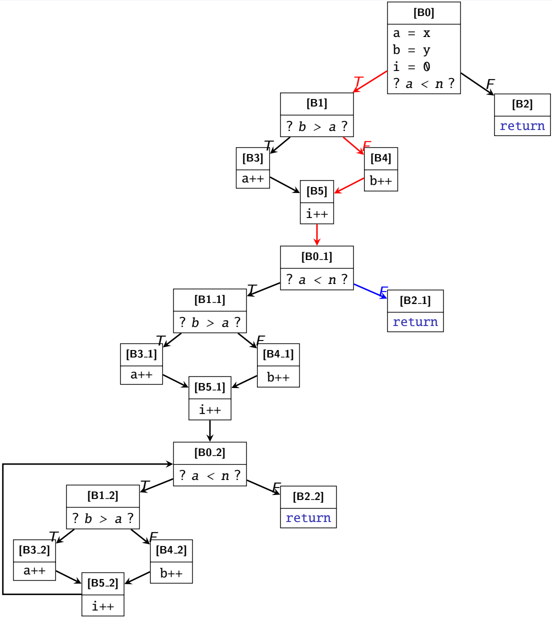
{x=3, y=4, n=5}

Concolic execution with the running example



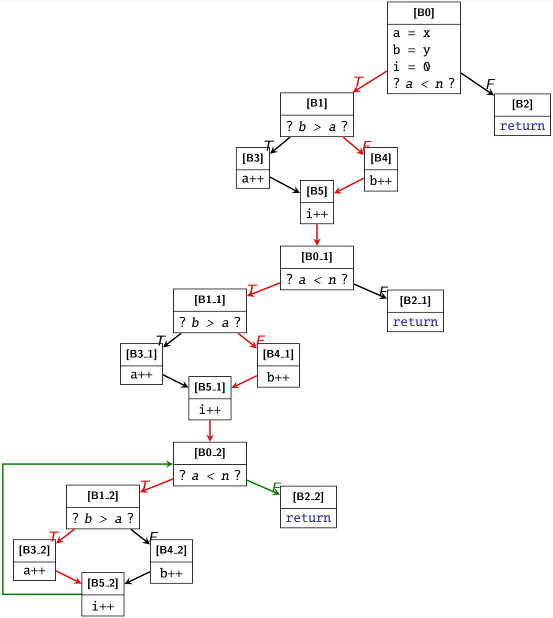
{x=1, y=0, n=2}

Concolic execution with the running example



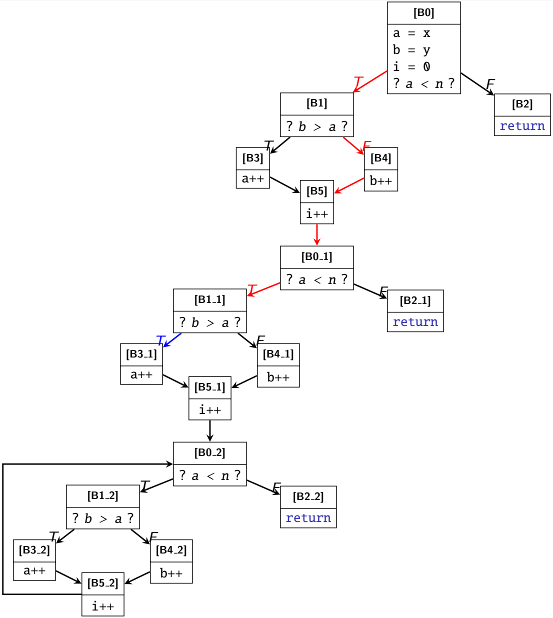
⟨*infeasible*⟩

Concolic execution with the running example



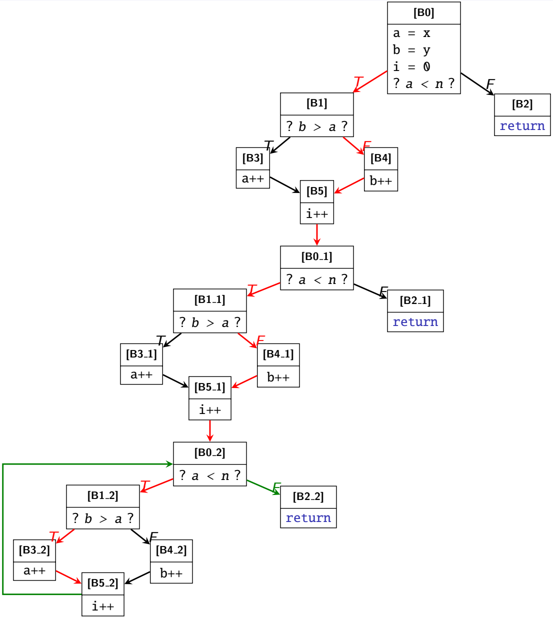
{x=1, y=0, n=2}

Concolic execution with the running example



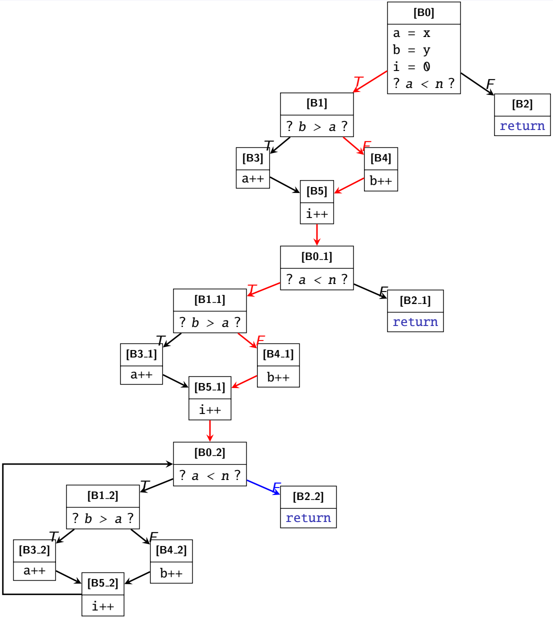
{x=5, y=5, n=8}

Concolic execution with the running example



{x=1, y=0, n=2}

Concolic execution with the running example



⟨*infeasible*⟩

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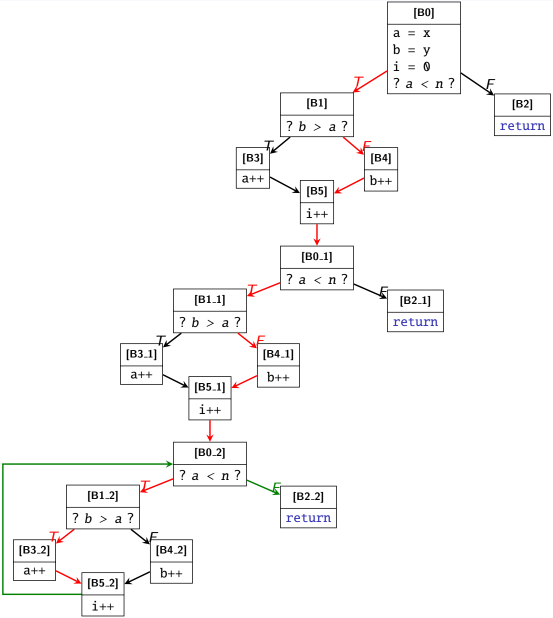
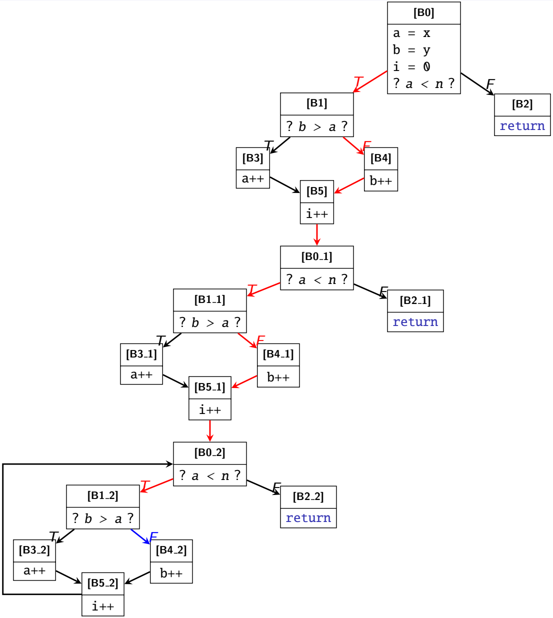


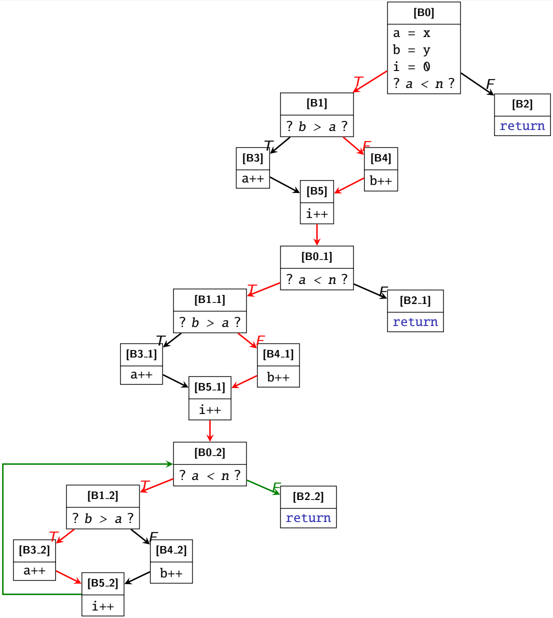
{x=1, y=0, n=2}

Concolic execution with the running example



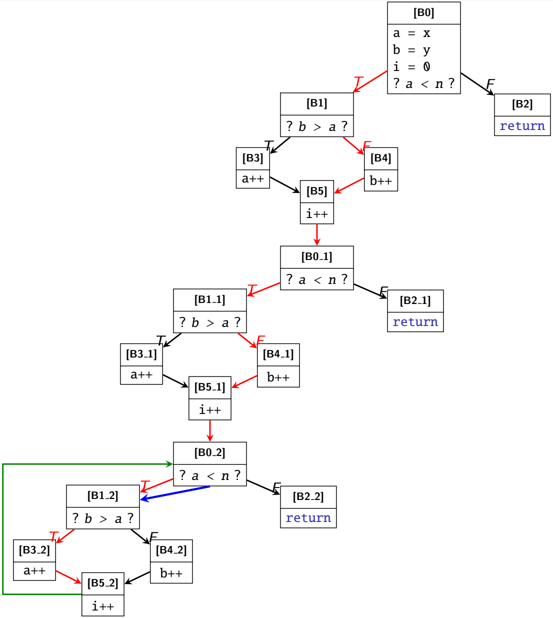
{x=7, y=3, n=9}

Concolic execution with the running example



{x=1, y=0, n=2}

Concolic execution with the running example



. . . endless loop . . .

$\langle$ **End** $\rangle$