

Hybrid Symbolic-Numeric Computing in Linear and Polynomial Algebra

Leili Rafiee Severyi¹, Robert M. Corless

[lrafiees@uwaterloo.ca]

David R. Cheriton School of Computer Science, University of Waterloo, Canada

We introduce hybrid symbolic-numeric methods which solve the *approximate GCD* problem for polynomials presented in Bernstein and Lagrange bases.

We adapt Victor Y. Pan’s root-based algorithm for finding approximate GCD to the case where the polynomials are expressed in Bernstein bases. We use the numerically stable companion pencil of Guðbjörn Jónsson to compute the roots, and the Hopcroft-Karp bipartite matching method to find the degree of the approximate GCD. We offer some refinements to improve the process.

We also introduce an algorithm with similar idea, which finds an approximate GCD for a pair of approximate polynomials given in a Lagrange basis. More precisely, we suppose that these polynomials are given by their approximate values at distinct known points. We first find each of their roots by using a Lagrange basis companion pencil for each polynomial. We introduce new clustering algorithms and use them to cluster the roots of each polynomial to identify multiple roots, and then *marry* the two polynomials using a *Maximum Weight Matching* algorithm, to find their GCD.

Keywords

Bernstein basis, Approximate GCD, Maximum matching, Root clustering, Lagrange basis, Maximum weight matching