1. **Bipartite Graphs**

   Use the Rayleigh quotient characterization of the first and the last eigenvalues. Compare them and use Perron-Frobenius theorem.

2. **Spanning Trees**

   Both parts are not difficult.

3. **Local Cheeger's Inequality**

   The easy direction can be proved by plugging in some specific vector.

   The hard direction can be reduced to Cheeger's rounding, for which you can just quote without doing the same proof again.

4. **Page Ranking**

   The approach is similar to the proof that there is a unique limiting distribution for random walks in a connected non-bipartite undirected graph in L23. Need to check that the conditions for the Perron-Frobenius theorem follow from our assumptions about the graph.

5. **Graph Partitioning by Random Walks**

   Use Q3 for part (a) and the fact that it is an eigenvector. You can assume that all entries of the eigenvector are positive, again by Perron-Frobenius theorem.

   For part (b), set $t = \Theta(\log|S|/\phi(S))$ and do some calculations similar to that in L24.

6. **Hitting Time**

   This is not difficult.

7. **Effective Resistances and Spanning Trees**

   Use Q2 for part (a), then you are naturally led to the equation about determinant update.

   For part (b), use the monotonicity principle in L25 to compare the effective resistances of two graphs. One is the original graph, and another is a graph corresponding to edge f being "fixed" in the spanning tree.