CS 370 - Assignment 4 Solutions   FALL 2008

Q1. (a)

$$A = \begin{bmatrix} 1.0 & 1.0 & 3.0 & 0.0 \\ 2.0 & 1.0 & -1.0 & 1.0 \\ -1.0 & 2.0 & 3.0 & -1.0 \\ 3.0 & -1.0 & -1.0 & 2.0 \end{bmatrix} ; \qquad \underline{b} = \begin{bmatrix} 4.0 \\ 1.0 \\ 4.0 \\ -3.0 \end{bmatrix}$$

Q1. (b)

Step $k=1$: Interchange rows 1 and 4.

$$\begin{bmatrix} 3.0 & -1.0 & -1.0 & 2.0 \\ 2.0 & 1.0 & -1.0 & 1.0 \\ -1.0 & 2.0 & 3.0 & -1.0 \\ 1.0 & 1.0 & 3.0 & 0.0 \end{bmatrix} \xrightarrow{k=1} \begin{bmatrix} 3.0 & -1.0 & -1.0 & 2.0 \\ 0.6666 & 1.667 & -0.3334 & -0.3330 \\ -0.3333 & 1.667 & 2.667 & -0.3334 \\ 0.3333 & 1.333 & 3.333 & -0.6666 \end{bmatrix}$$

row i ← row i $- m_{i1}$ row 1

for $i = 2, 3, 4$

multipliers $m_{21} = \frac{2.0}{3.0} = 0.6666$

$m_{31} = \frac{-1.0}{3.0} = -0.3333$

$m_{41} = \frac{1.0}{3.0} = 0.3333$

Step $k=2$: No row interchange is necessary.

$$\xrightarrow{k=2} \begin{bmatrix} 3.0 & -1.0 & -1.0 & 2.0 \\ 0.6666 & 1.667 & -0.3334 & -0.3330 \\ -0.3333 & 1.000 & 3.000 & 0.0 \\ 0.3333 & 0.7997 & 3.600 & -0.4003 \end{bmatrix}$$

row i ← row i $- m_{i2}$ row 2

for $i = 3, 4$

multipliers $m_{32} = \frac{1.667}{1.667} = 1.000$

$m_{42} = \frac{1.333}{1.667} = 0.7997$

Step $k=3$: Interchange rows 3 and 4.

$$\begin{bmatrix} 3.0 & -1.0 & -1.0 & 2.0 \\ 0.6666 & 1.667 & -0.3334 & -0.3330 \\ 0.3333 & 0.7997 & 3.600 & -0.4003 \\ -0.3333 & 1.000 & 3.000 & 0.0 \end{bmatrix} \xrightarrow{k=3} \begin{bmatrix} 3.0 & -1.0 & -1.0 & 2.0 \\ 0.6666 & 1.667 & -0.3334 & -0.3330 \\ 0.3333 & 0.7997 & 3.600 & -0.4003 \\ -0.3333 & 1.000 & 0.8334 & 0.3332 \end{bmatrix}$$

row 4 $\leftarrow$ row 4 $-\, m_{43}$ row 3

multiplier $m_{43} = \dfrac{3.000}{3.600} = 0.8334$

Matrices L and U are defined by the lower and upper portions of the final transformed matrix above, as follows:

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.6666 & 1 & 0 & 0 \\ 0.3333 & 0.7997 & 1 & 0 \\ -0.3333 & 1.000 & 0.8334 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 3.0 & -1.0 & -1.0 & 2.0 \\ 0 & 1.667 & -0.3334 & -0.3330 \\ 0 & 0 & 3.600 & -0.4003 \\ 0 & 0 & 0 & 0.3332 \end{bmatrix}$$

Then $PA = LU$

For P, start with I and apply row interchanges as above.

$$\underset{=I}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \xrightarrow{\text{Switch } 1 + 4} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{Switch } 3 + 4} \underset{=P}{\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}$$

<u>Q1. (c)</u> To solve $A\underline{x} = \underline{b}$ for $\underline{x}$, we have

$$PA\underline{x} = P\underline{b}, \quad \text{where } \underline{b} = \begin{bmatrix} 4.0 \\ 1.0 \\ 4.0 \\ -3.0 \end{bmatrix}.$$

I.e. $L\underbrace{U\underline{x}}_{\underline{y}} = P\underline{b}$.

<u>Forward substitution:</u>

Solve $L\underline{y} = P\underline{b}$ for $\underline{y}$.

I.e.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.6666 & 1 & 0 & 0 \\ 0.3333 & 0.7997 & 1 & 0 \\ -0.3333 & 1.000 & 0.8334 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \overset{P\underline{b}}{\begin{bmatrix} -3.0 \\ 1.0 \\ 4.0 \\ 4.0 \end{bmatrix}}$$

<u>Solution:</u> $y_1 = -3.0$; $y_2 = 3.000$; $y_3 = 2.601$; $y_4 = -2.168$.

<u>Back substitution:</u>

Solve $U\underline{x} = \underline{y}$ for $\underline{x}$.

I.e.

$$\begin{bmatrix} 3.0 & -1.0 & -1.0 & 2.0 \\ 0 & 1.667 & -0.3334 & -0.3330 \\ 0 & 0 & 3.600 & -0.4003 \\ 0 & 0 & 0 & 0.3332 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3.000 \\ 3.000 \\ 2.601 \\ -2.168 \end{bmatrix}$$

<u>Solution:</u> $x_4 = -6.507$; $x_3 = -0.001111$; $x_2 = 0.4997$; $x_1 = 3.503$.

<u>Final solution:</u> $\underline{x} = [3.503 \;\; 0.4997 \;\; -0.001111 \;\; -6.507]^T$.

Q1.(d) To solve $A\underline{x} = \underline{c}$ for $\underline{x}$

$$\Rightarrow PA\underline{x} = P\underline{c} \quad \text{where } \underline{c} = \begin{bmatrix} -1.8 \\ -5.1 \\ 6.3 \\ -8.4 \end{bmatrix}$$

I.e. $L\underbrace{U\underline{x}}_{\underline{y}} = P\underline{c}$.

Forward substitution:

Solve $L\underline{y} = P\underline{c}$ for $\underline{y}$.

I.e.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.6666 & 1 & 0 & 0 \\ 0.3333 & 0.7997 & 1 & 0 \\ -0.3333 & 1.000 & 0.8334 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \underbrace{\begin{bmatrix} -8.4 \\ -5.1 \\ -1.8 \\ 6.3 \end{bmatrix}}_{P\underline{c}}$$

Solution: $y_1 = -8.4$; $y_2 = 0.4990$; $y_3 = 0.6010$; $y_4 = 2.500$

Back substitution:

Solve $U\underline{x} = \underline{y}$ for $\underline{x}$.

I.e.

$$\begin{bmatrix} 3.0 & -1.0 & -1.0 & 2.0 \\ 0 & 1.667 & -0.3334 & -0.3330 \\ 0 & 0 & 3.600 & -0.4003 \\ 0 & 0 & 0 & 0.3332 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -8.400 \\ 0.4990 \\ 0.6010 \\ 2.500 \end{bmatrix}$$

Solution: $x_4 = 7.503$; $x_3 = 1.001$; $x_2 = 1.998$; $x_1 = -6.803$

Final solution: $\underline{x} = \begin{bmatrix} -6.803 \\ 1.998 \\ 1.001 \\ 7.503 \end{bmatrix}$.