CS 370 Fall 2008: Assignment 2 SOLUTIONS

Question 1 (presented as a Maple worksheet)

Let us use 5 significant digits for this computation.

> $Digits := 5$

$$Digits := 5 \quad (1)$$

For the function

> $f := x \rightarrow \sin(e^x - 2)$

$$f := x \rightarrow \sin(e^x - 2) \quad (2)$$

the given data points are (x_i, y_i) , $i = 1, 2, 3, 4$ where $y_i = f(x_i)$.

Specifically,

> $(x1, x2, x3, x4) := (0.6, 0.7, 0.8, 1.0)$

$$x1, x2, x3, x4 := 0.6, 0.7, 0.8, 1.0 \quad (3)$$

> $(y1, y2, y3, y4) := (f(x1), f(x2), f(x3), f(x4))$

$$y1, y2, y3, y4 := -0.17696, 0.013800, 0.22359, 0.65811 \quad (4)$$

>

Case n = 1

For degree $n = 1$ using the first two points:

> $(x1, y1)$

$$0.6, -0.17696 \quad (5)$$

> $(x2, y2)$

$$0.7, 0.013800 \quad (6)$$

the two relevant Lagrange polynomials are

> $\frac{(x - x2)}{(x1 - x2)}$

$$-10.\,x + 7. \quad (7)$$

> $L1 := unapply(\%, x)$

$$L1 := x \rightarrow -10.\,x + 7. \quad (8)$$

> $\frac{(x - x1)}{(x2 - x1)}$

$$10.\,x - 6. \quad (9)$$

> $L2 := unapply(\%, x)$

$$L2 := x \rightarrow 10.\,x - 6. \quad (10)$$

Then the desired interpolating polynomial of degree 1 is

> $y1\,L1(x) + y2\,L2(x)$

$$1.9076\,x - 1.3215 \tag{11}$$

> $p1 := unapply(\%, x)$

$$p1 := x \rightarrow 1.9076\,x - 1.3215 \tag{12}$$

>

Case n = 2

For degree $n = 2$ using the first three points:

> $(x1, y1)$

$$0.6,\ -0.17696 \tag{13}$$

> $(x2, y2)$

$$0.7,\ 0.013800 \tag{14}$$

> $(x3, y3)$

$$0.8,\ 0.22359 \tag{15}$$

the three relevant Lagrange polynomials are

> $\dfrac{(x - x2)\,(x - x3)}{(x1 - x2)\,(x1 - x3)}$

$$50.000\,(x - 0.7)\,(x - 0.8) \tag{16}$$

> $L1 := unapply(\%, x)$

$$L1 := x \rightarrow 50.000\,(x - 0.7)\,(x - 0.8) \tag{17}$$

> $\dfrac{(x - x1)\,(x - x3)}{(x2 - x1)\,(x2 - x3)}$

$$-100.00\,(x - 0.6)\,(x - 0.8) \tag{18}$$

> $L2 := unapply(\%, x)$

$$L2 := x \rightarrow -100.00\,(x - 0.6)\,(x - 0.8) \tag{19}$$

> $\dfrac{(x - x1)\,(x - x2)}{(x3 - x1)\,(x3 - x2)}$

$$50.000\,(x - 0.6)\,(x - 0.7) \tag{20}$$

> $L3 := unapply(\%, x)$

$$L3 := x \rightarrow 50.000\,(x - 0.6)\,(x - 0.7) \tag{21}$$

Then the desired interpolating polynomial of degree 2 is

> $y1\,L1(x) + y2\,L2(x) + y3\,L3(x)$

$$-8.8480\,(x - 0.7)\,(x - 0.8) - 1.3800\,(x - 0.6)\,(x - 0.8) + 11.180\,(x - 0.6)\,(x - 0.7) \tag{22}$$

> $p2 := unapply(\%, x)$

$$p2 := x \rightarrow -8.8480\,(x - 0.7)\,(x - 0.8) - 1.3800\,(x - 0.6)\,(x - 0.8) + 11.180\,(x - 0.6)\,(x - 0.7) \tag{23}$$

>

Case n = 3

For degree $n = 3$ using all four points:

> $(x1, y1)$

$$0.6, -0.17696 \tag{24}$$

> $(x2, y2)$

$$0.7, 0.013800 \tag{25}$$

> $(x3, y3)$

$$0.8, 0.22359 \tag{26}$$

> $(x4, y4)$

$$1.0, 0.65811 \tag{27}$$

the four relevant Lagrange polynomials are

> $\frac{(x - x2)\ (x - x3)\ (x - x4)}{(x1 - x2)\ (x1 - x3)\ (x1 - x4)}$

$$-125.00\ (x - 0.7)\ (x - 0.8)\ (x - 1.0) \tag{28}$$

> $L1 := unapply(\%, x)$

$$L1 := x \rightarrow -125.00\ (x - 0.7)\ (x - 0.8)\ (x - 1.0) \tag{29}$$

> $\frac{(x - x1)\ (x - x3)\ (x - x4)}{(x2 - x1)\ (x2 - x3)\ (x2 - x4)}$

$$333.33\ (x - 0.6)\ (x - 0.8)\ (x - 1.0) \tag{30}$$

> $L2 := unapply(\%, x)$

$$L2 := x \rightarrow 333.33\ (x - 0.6)\ (x - 0.8)\ (x - 1.0) \tag{31}$$

> $\frac{(x - x1)\ (x - x2)\ (x - x4)}{(x3 - x1)\ (x3 - x2)\ (x3 - x4)}$

$$-250.00\ (x - 0.6)\ (x - 0.7)\ (x - 1.0) \tag{32}$$

> $L3 := unapply(\%, x)$

$$L3 := x \rightarrow -250.00\ (x - 0.6)\ (x - 0.7)\ (x - 1.0) \tag{33}$$

> $\frac{(x - x1)\ (x - x2)\ (x - x3)}{(x4 - x1)\ (x4 - x2)\ (x4 - x3)}$

$$41.666\ (x - 0.6)\ (x - 0.7)\ (x - 0.8) \tag{34}$$

> $L4 := unapply(\%, x)$

$$L4 := x \rightarrow 41.666\ (x - 0.6)\ (x - 0.7)\ (x - 0.8) \tag{35}$$

Then the desired interpolating polynomial of degree 3 is

> $y1\ L1(x) + y2\ L2(x) + y3\ L3(x) + y4\ L4(x)$

$$22.120\ (x - 0.7)\ (x - 0.8)\ (x - 1.0) + 4.6000\ (x - 0.6)\ (x - 0.8)\ (x - 1.0) - 55.898\ (x - 0.6)\ (x - 0.7)\ (x - 1.0) + 27.421\ (x - 0.6)\ (x - 0.7)\ (x - 0.8) \tag{36}$$

> $p3 := unapply(\%, x)$

$$p3 := x \rightarrow 22.120\ (x - 0.7)\ (x - 0.8)\ (x - 1.0) + 4.6000\ (x - 0.6)\ (x - 0.8)\ (x - 1.0) - 55.898\ (x - 0.6)\ (x - 0.7)\ (x - 1.0) + 27.421\ (x - 0.6)\ (x - 0.7)\ (x - 0.8) \tag{37}$$

>

Double check p1, p2 and p3

Let us double check that $p1$, $p2$ and $p3$ satisfy the appropriate conditions of interpolation (up to some roundoff error).

> $p1(x1) = y1$

$$-0.1769 = -0.17696 \quad (38)$$

> $p1(x2) = y2$

$$0.0138 = 0.013800 \quad (39)$$

>

> $p2(x1) = y1$

$$-0.17696 = -0.17696 \quad (40)$$

> $p2(x2) = y2$

$$0.013800 = 0.013800 \quad (41)$$

> $p2(x3) = y3$

$$0.22360 = 0.22359 \quad (42)$$

>

> $p3(x1) = y1$

$$-0.17696 = -0.17696 \quad (43)$$

> $p3(x2) = y2$

$$0.013800 = 0.013800 \quad (44)$$

> $p3(x3) = y3$

$$0.22359 = 0.22359 \quad (45)$$

> $p3(x4) = y4$

$$0.65810 = 0.65811 \quad (46)$$

>

Estimate f(0.9) using p1, p2 and p3

> $xval := 0.9$

$$xval := 0.9 \quad (47)$$

> $p1(xval)$

$$0.3953 \quad (48)$$

> $err1 := |f(xval) - p1(xval)|$

$$err1 := 0.04829 \quad (49)$$

>

> $p2(xval)$

$$0.45244 \quad (50)$$

> $err2 := |f(xval) - p2(xval)|$

$$err2 := 0.00885 \quad (51)$$

>

> $p3(xval)$

$$0.44188 \quad (52)$$

> $err3 := |f(xval) - p3(xval)|$

$$err3 := 0.00171 \quad (53)$$

>

Note:

It is acceptable to choose a different pair of points for degree 1 interpolation rather than using $x1$, $x2$.
Similarly, it is acceptable to choose a different set of three points for degree 2 interpolation rather than using $x1$, $x2$, $x3$.

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