Multiagent Systems: Intro to Game Theory

CS 486/686: Introduction to Artificial Intelligence

Introduction

- So far almost everything we have looked at has been in a single-agent setting
    - Today - Multiagent Decision Making!
- For participants to act optimally, they must account for how others are going to act
- We want to
    - Understand the ways in which agents interact and behave
    - Design systems so that agents behave the way we would like them to

Hint for the midterm exam: MAS is my main research area. I like MAS problems. I even enjoy marking MAS questions. Two of the TAs do MAS research. They also like marking MAS questions. There ***will*** be a MAS question on the exam.

Self-Interest

- We will focus on ***self-interested*** MAS
- Self-interested does **not** necessarily mean
  - Agents want to harm others
  - Agents only care about things that benefit themselves
- Self-interested means
  - Agents have their own description of states of the world
  - Agents take actions based on these descriptions

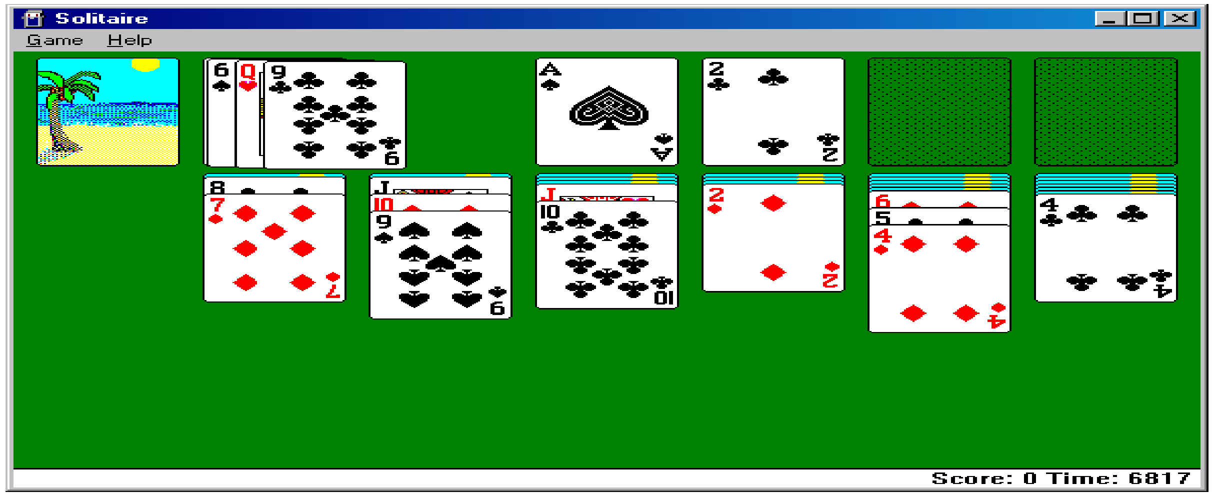
What is Game Theory?

- The study of games!
  - Bluffing in poker
  - What move to make in chess
  - How to play Rock-Paper-Scissors

Also auction design, strategic deterrence, election laws, coaching decisions, routing protocols,...

What is Game Theory?

- Game theory is a formal way to analyze **interactions** among a **group** of **rational** agents that behave **strategically**
  - **Group:** Must have more than 1 decision maker
    - Otherwise, you have a decision problem, not a game



Solitaire is not a game!

What is Game Theory?

- Game theory is a formal way to analyze **interactions** among a **group** of **rational** agents that behave **strategically**
  - **Interaction:** What one agent does directly affects at least one other
  - **Strategic**: Agents take into account that their actions influence the game
  - **Rational**: Agents chose their best actions

Example

- Decision Problem
  - Everyone pays their own bill
- Game
  - Before the meal, everyone decides to split the bill evenly

Strategic Game
(Matrix Game, Normal Form Game)

- Set of agents: $I=\{1,2,.,.,,N\}$
- Set of actions: $A_i=\{a_i^1,\ldots,a_i^m\}$
- Outcome of a game is defined by a profile $a=(a_1,\ldots,a_n)$
- Agents have preferences over outcomes
  - Utility functions u_i:A->**R**

Examples

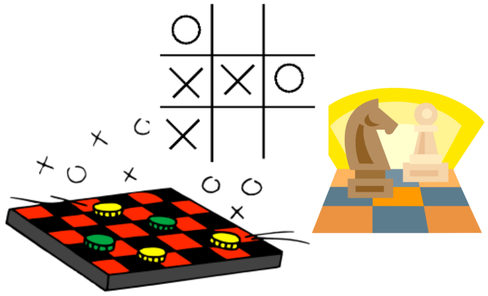
Agent 2

		One	Two
Agent 1	One	2,-2	-3,3
	Two	-3,3	4,-4

Zero-sum game.

$\sum_{i=1}^{n} u_i(o)=0$

I={1,2}
Ai={One,Two}
An outcome is (One, Two)
$U_1((One,Two))=-3$ and $U_2((One,Two))=3$



Examples

BoS

	B	S
B	2,1	0,0
S	0,0	1,2

Coordination Game

Chicken

	T	C
T	-1,-1	10,0
C	0,10	5,5

Anti-Coordination Game

Example: Prisoners' Dilemma

	Confess	Don't Confess
Confess	-5,-5	0,-10
Don't Confess	-10,0	-1,-1

Playing a Game

- Agents are rational
  - Let p_i be agent *i*'s belief about what its opponents will do
  - **Best response**: $a_i = \arg\max \sum_{a_{-i}} u_i(a_i, a_{-i}) p_i(a_{-i})$

Notation Break: $a_{-i} = (a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n)$

Dominated Strategies

- a'_i **strictly dominates** strategy a_i if

$$u_i(a_i', a_{-i}) > u_i(a_i, a_{-i}) \forall a_{-i}$$

- A rational agent will never play a dominated strategy!

Example

	Confess	Don't Confess
Confess	-5,-5	0,-10
Don't Confess	-10,0	-1,-1

Strict Dominance Does Not Capture the Whole Picture

	A	B	C
A	0,4	4,0	5,3
B	4,0	0,4	5,3
C	3,5	3,5	6,6

Nash Equilibrium

Key Insight: an agent's best-response depends on the actions of other agents

An action profile a* is a **Nash equilibrium** if no agent has incentive to change given that others do not change

$$\forall i u_i(a_i^*, a_{-i}^*) \geq u_i(a_i', a_{-i}^*) \forall a_i'$$

Nash Equilibrium

- Equivalently, a* is a N.E. iff

$$\forall i a_i^* = \arg\max_{a_i} u_i(a_i, a_{-i}^*)$$

	A	B	C
A	0,4	4,0	5,3
B	4,0	0,4	5,3
C	3,5	3,5	6,6

(C,C) is a N.E. because

$$u_1(C,C) = \max \begin{bmatrix} u_1(A,C) \\ u_1(B,C) \\ u_1(C,C) \end{bmatrix}$$

AND

$$u_2(C,C) = \max \begin{bmatrix} u_2(C,A) \\ u_2(C,B) \\ u_2(C,C) \end{bmatrix}$$

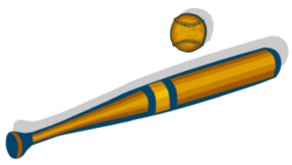
Nash Equilibrium

- If (a_1*,a_2*) is a N.E. then player 1 won't want to change its action given player 2 is playing a_2*
- If (a_1*,a_2*) is a N.E. then player 2 won't want to change its action given player 1 is playing a_1*

-5,-5	0,-10
-10,0	-1,-1

	A	B	C
A	0,4	4,0	5,3
B	4,0	0,4	5,3
C	3,5	3,5	6,6

Another Example



	B	S
B	2,1	0,0
S	0,0	1,2

2 Nash Equilibria

Coordination Game

Yet Another Example

		Agent 2: One	Agent 2: Two
Agent 1	One	2,-2	-3,3
	Two	-3,3	4,-4

(Mixed) Nash Equilibria

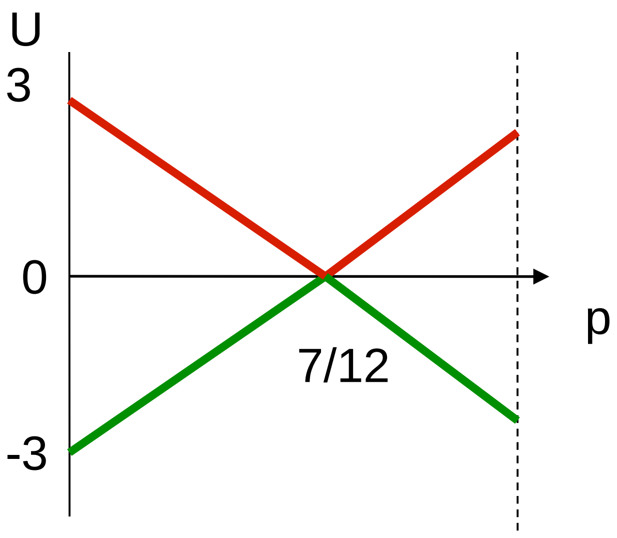
- **(Mixed) Strategy**: s_i is a probability distribution over A_i
- **Strategy profile**: $s=(s_1,\ldots,s_n)$
- **Expected utility**: $u_i(s)=\Sigma_a\Pi_j s(a_j)u_i(a)$
- **Nash equilibrium**: s* is a (mixed) Nash equilibrium if

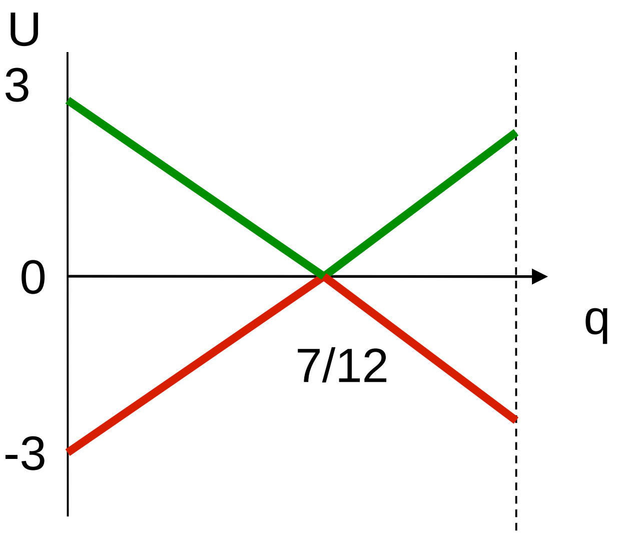
$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i', s_{-i}^*) \forall s_i'$$

Yet Another Example

		q One	Two
p	One	2,-2	-3,3
	Two	-3,3	4,-4

How do we determine p and q?





Yet Another Example

		q **One**	**Two**
p	**One**	2,-2	-3,3
	Two	-3,3	4,-4

How do we determine p and q?

Exercise

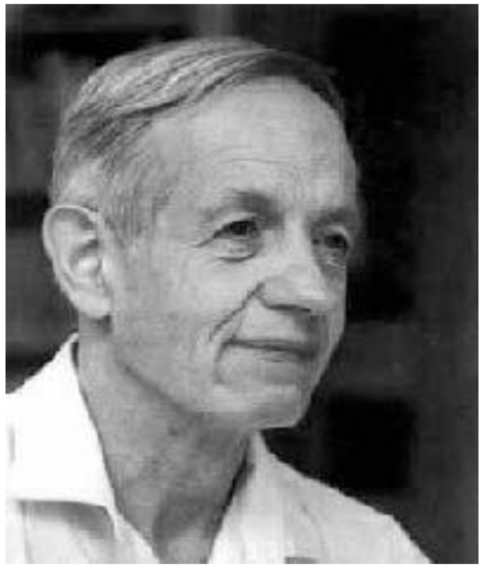
	B	S
B	2,1	0,0
S	0,0	1,2

This game has 3 Nash Equilibrium (2 pure strategy NE and 1 mixed strategy NE).

Mixed Nash Equilibrium

- Theorem (Nash 1950): Every game in which the action sets are finite, has a mixed strategy equilibrium.

John Nash
Nobel Prize in Economics (1994)



Finding NE

- Existence proof is non-constructive
- Finding equilibria?
  - 2 player zero-sum games can be represented as a linear program (Polynomial)
  - For arbitrary games, the problem is in PPAD
  - Finding equilibria with certain properties is often NP-hard

Repeated Games

Recall the Prisonner's Dilemma. What if the prisoners are habitual criminals?

-5,-5	0,-10
-10,0	-1,-1

-5,-5	0,-10
-10,0	-1,-1

-5,-5	0,-10
-10,0	-1,-1

…

How do we define payoffs?

What is the strategy space?

Repeated Games

Recall the Prisonner's Dilemma. What if the prisoners are habitual criminals?

-5,-5	0,-10
-10,0	-1,-1

-5,-5	0,-10
-10,0	-1,-1

-5,-5	0,-10
-10,0	-1,-1

…

How do we define payoffs?

- Average reward
- Discounted Awards
- …

Repeated Games

Recall the Prisonner's Dilemma. What if the prisoners are habitual criminals?

-5,-5	0,-10
-10,0	-1,-1

-5,-5	0,-10
-10,0	-1,-1

-5,-5	0,-10
-10,0	-1,-1

…

Strategy space becomes significantly larger!

S:H→A where H is the history of play so far

Can now reward and punish past behaviour, worry about reputation, establish trust,…

Repeated Games

Recall the Prisonner's Dilemma. What if the prisoners are habitual criminals?

-5,-5	0,-10
-10,0	-1,-1

-5,-5	0,-10
-10,0	-1,-1

-5,-5	0,-10
-10,0	-1,-1

…

Grim Strategy: In first step cooperate. If opponent defects at some point, then defect forever

Tit-for-Tat: In first step cooperate. Copy what ever opponent did in previous stage.

Extensive Form Games

- Normal form games assume agents are playing strategies simultaneously
  - What about when agents' take turns?
    - Checkers, chess,...

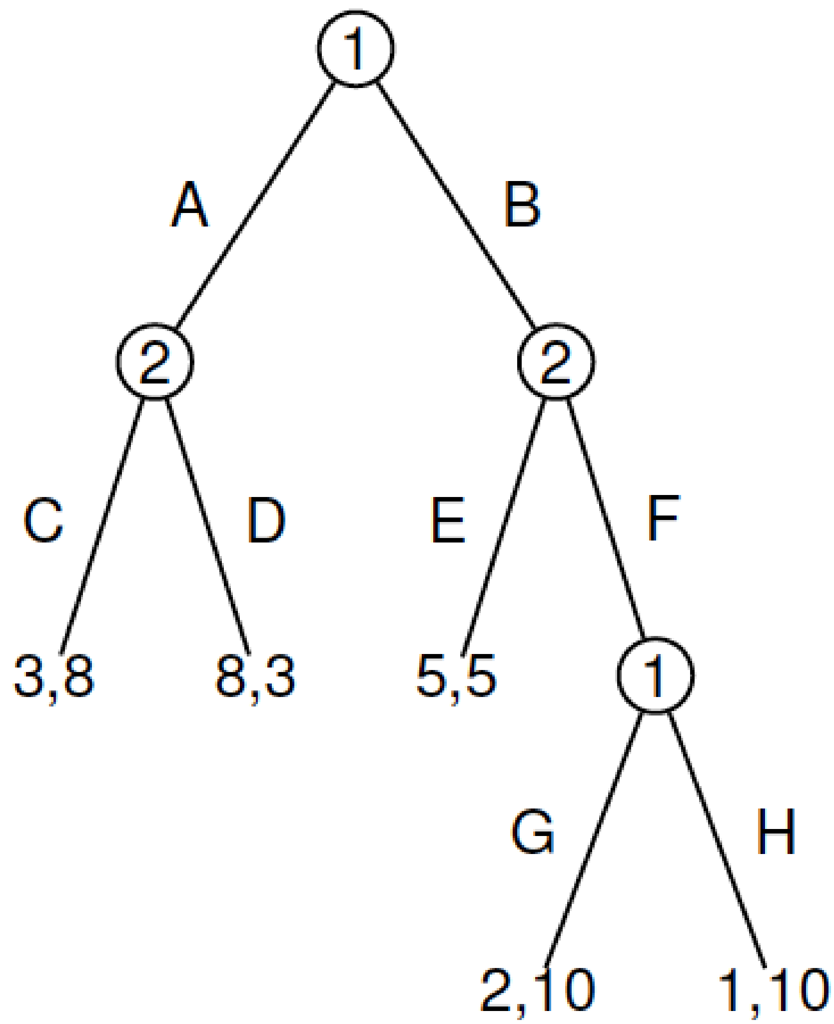
Extensive Form Games (with perfect information)

- $G=(I,A,H,Z,\alpha,\rho,\sigma,u)$
  - I: player set
  - A: action space
  - H: non-terminal choice nodes
  - Z: terminal nodes
  - α: action function $\alpha: H \rightarrow 2^A$
  - ρ: player function $\rho: H \rightarrow N$
  - σ: successor function $\sigma: H \times A \rightarrow H \cup Z$
  - $u=(u_1,...,u_n)$ where u_i is a utility function $u_i: Z \rightarrow R$

Extensive Form Games (with perfect information)

- The previous definition describes a tree

S



A strategy specifies an action to each non-terminal history at which the agent can move

$$S_1 = \{(A,G),(A,H),(B,G),(B,H)\}$$

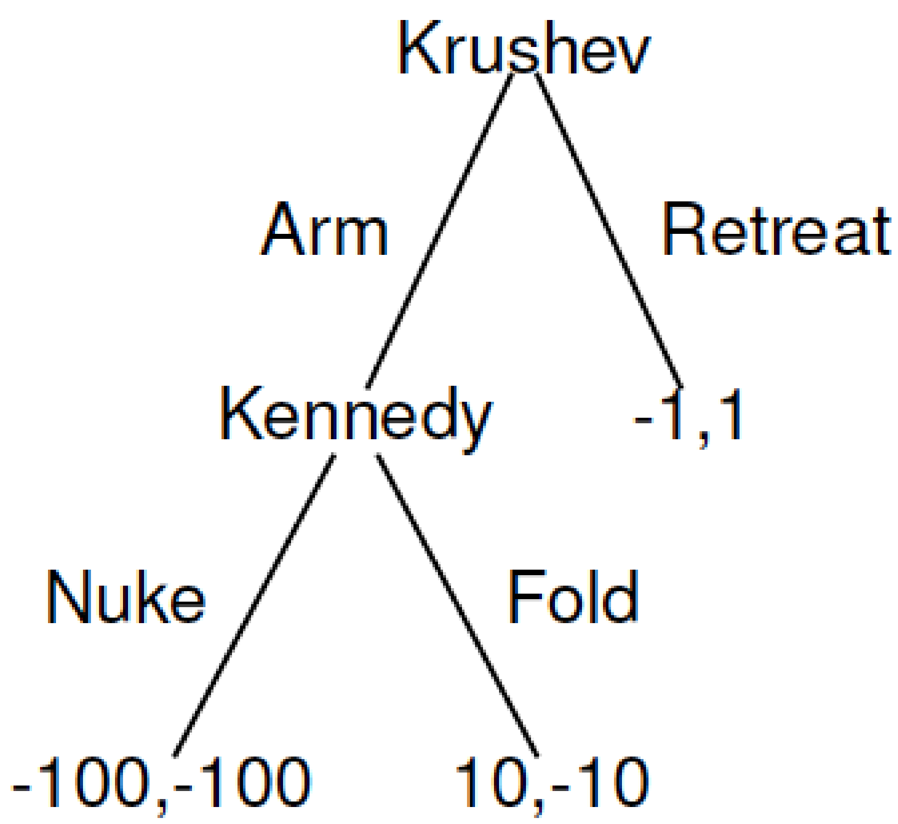
$$S_2 = \{(C,E),(C,F),(D,E),(D,F)\}$$

Nash Equilibria

We can transform an extensive form game into a normal form game.

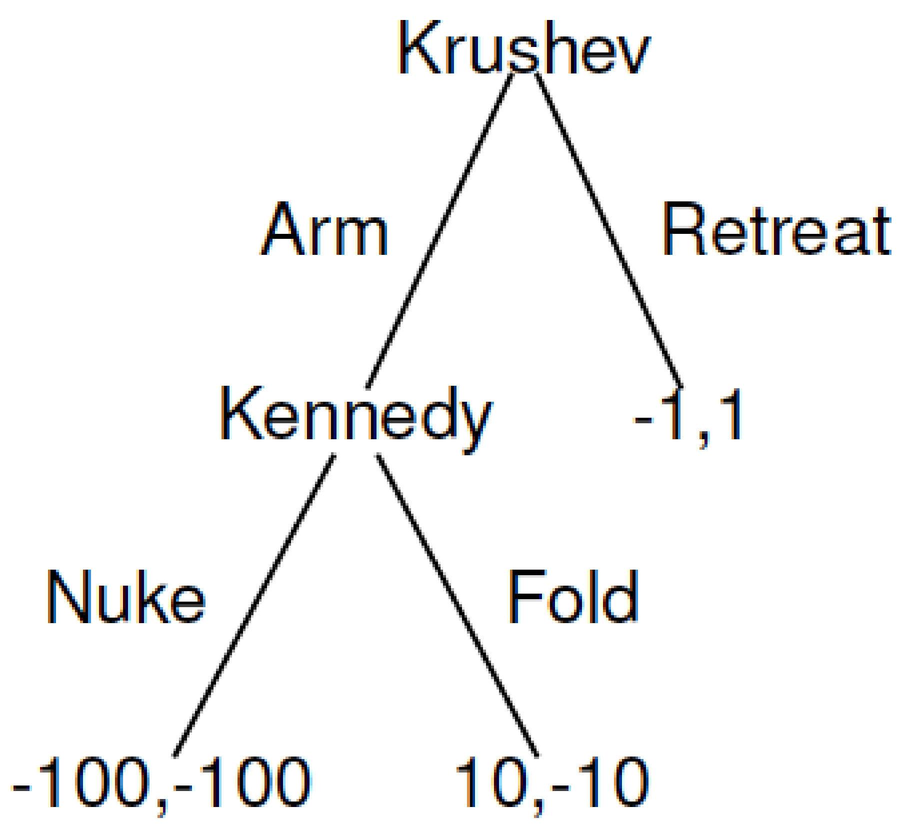
	(C,E)	(C,F)	(D,E)	(D,F)
(A,G)	3,8	3,8	8,3	8,3
(A,H)	3,8	3,8	8,3	8,3
(B,G)	5,5	2,10	5,5	2, 10
(B,H)	5,5	1,0	5,5	1,0

Subgame Perfect Equilibria



What are the NE?

Subgame Perfect Equilibria



Subgame Perfect Equilibria
s* must be a Nash equilibrium in all subgames

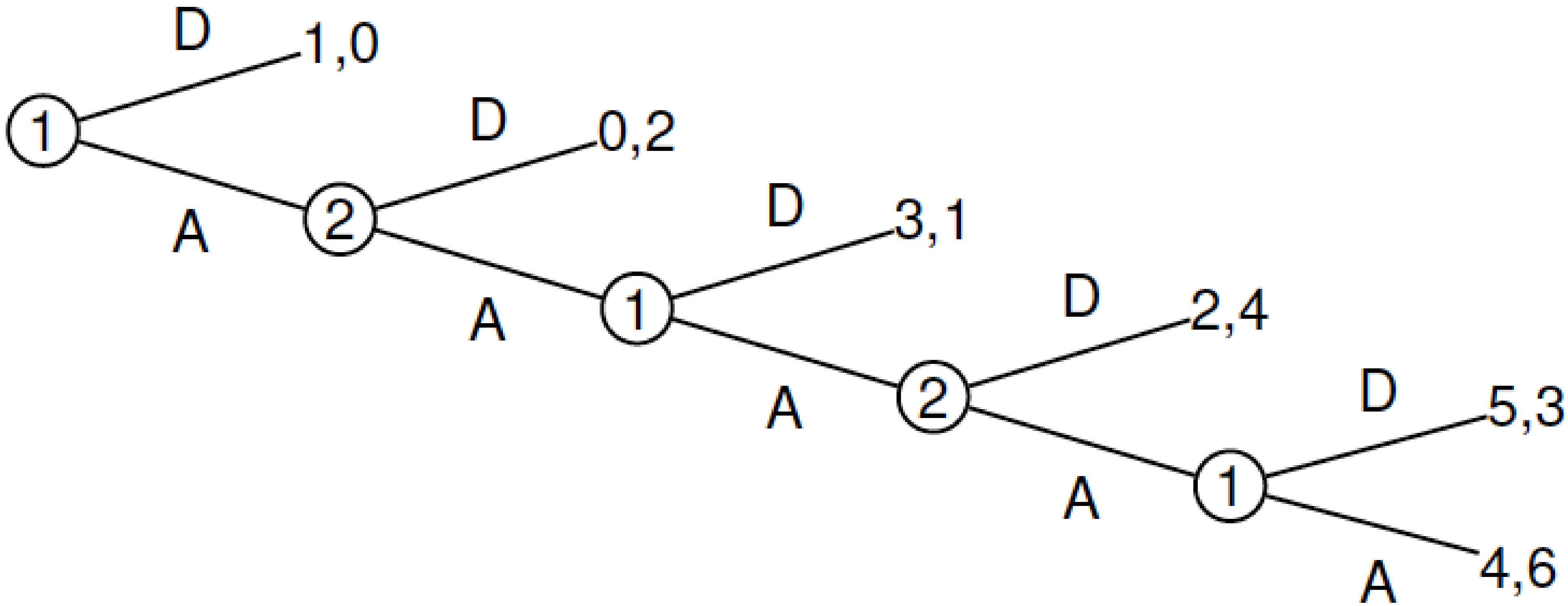
What are the SPE?

Existence of SPE

- **Theorem (Kuhn)**: Every finite extensive form game has an SPE.

- Compute the SPE using backward induction
  - Identify equilibria in the bottom most subtrees
  - Work upwards

Example: Centipede Game



Summary

- Definition of a Normal Form Game
- Dominant strategies
- Nash Equilibria
- Extensive Form Games with Perfect Information
- Subgame Perfect Equilibria