Local Search

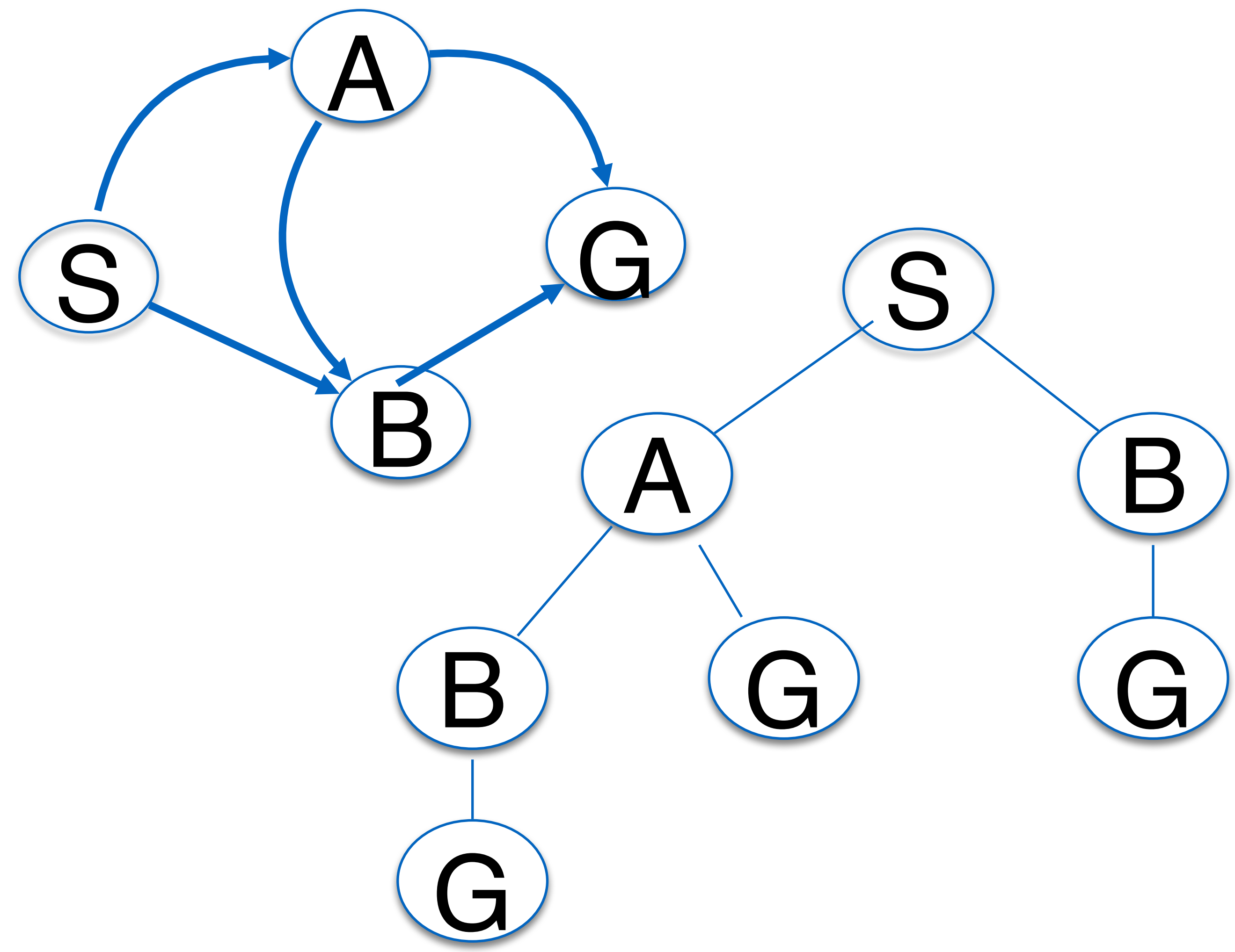
CS 486/686: Introduction to Artificial Intelligence

Overview

- Uninformed Search
  - Very general: assumes no knowledge about the problem
  - BFS, DFS, IDS
- Informed Search
  - Heuristics
  - A* search and variations
- **Search and Optimization**
  - What are the problem features?
  - Iterative improvement: hill climbing, simulated annealing
  - Genetic algorithms

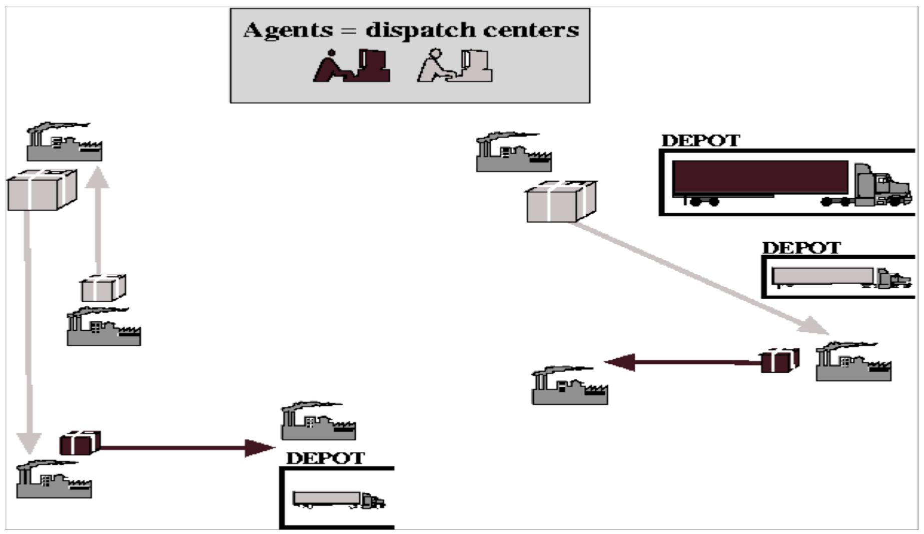
Introduction

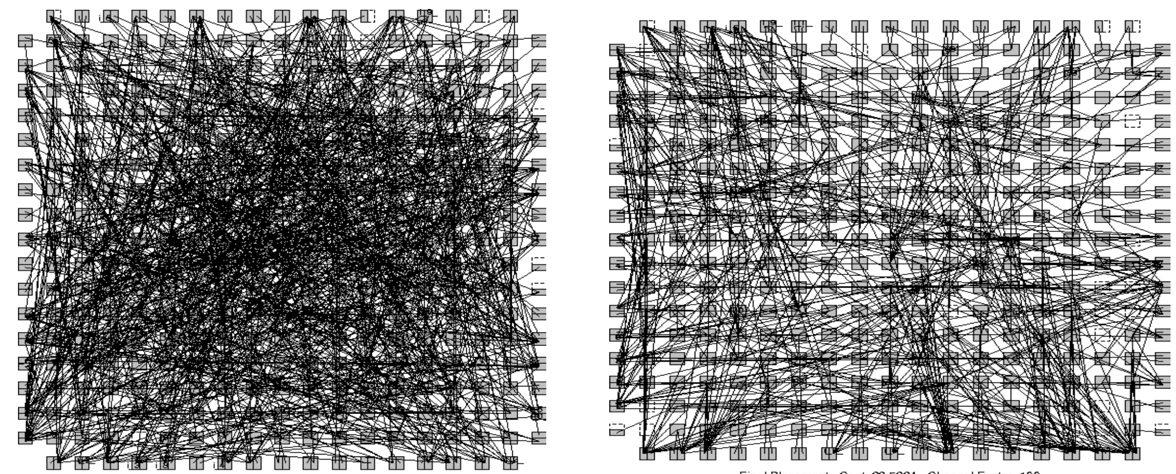
- Both uninformed and informed search systematically explore the search space
  - Keep 1 or more paths in memory
  - **Solution is a path to the goal**



For many problems the path is unimportant

Examples





Initial Placement. Cost: 74.5562. Channel Factor: 100

Final Placement. Cost: 28.5384. Channel Factor: 100

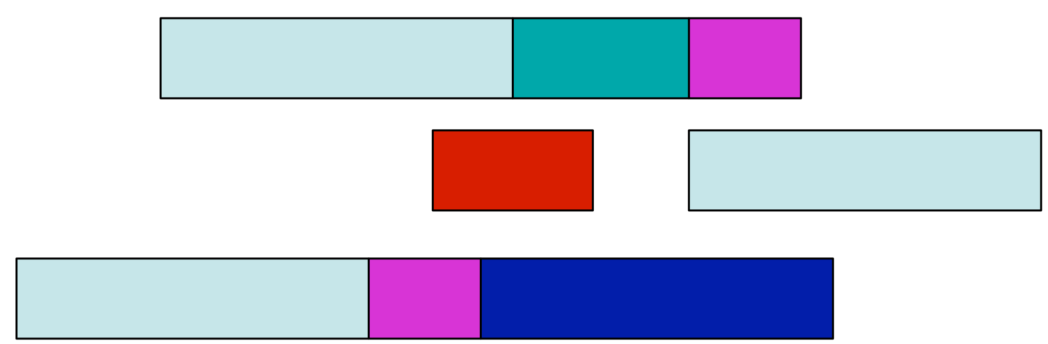
AV ~B V C

~A V C V D

B V D V ~E

~C V ~D V ~E

…

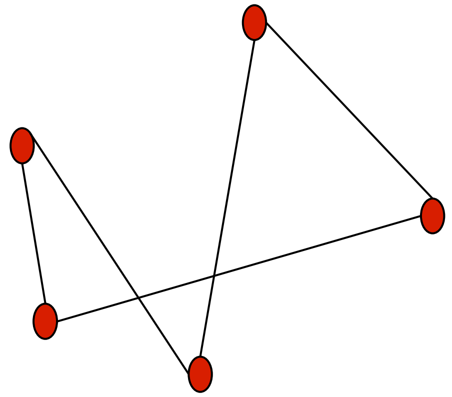


Informal Characterization

- Combinatorial structure being optimized
- Constraints have to be satisfied
- There is a cost function
  - We want to find a **good** solution
- Search all possible states is infeasible
  - Often easy to find **some solution** to the problem
  - Often provably **hard** (NP-complete) to find the **best** solution

Typical Example: TSP

Goal is to minimize the length of the route



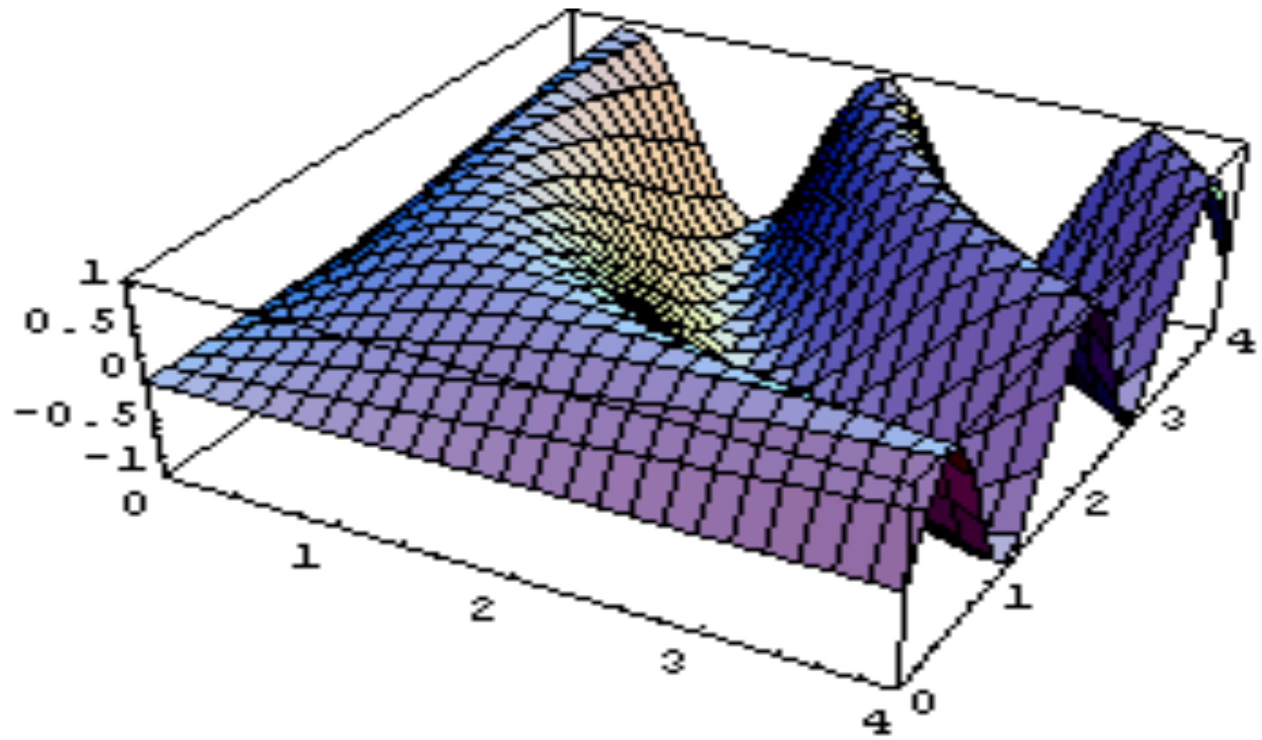
Constructive method: Start from scratch and build up a solution (using A* etc)

Iterative improvement method: Start with solution (may be suboptimal or broken) and improve it

Iterative Improvement Methods

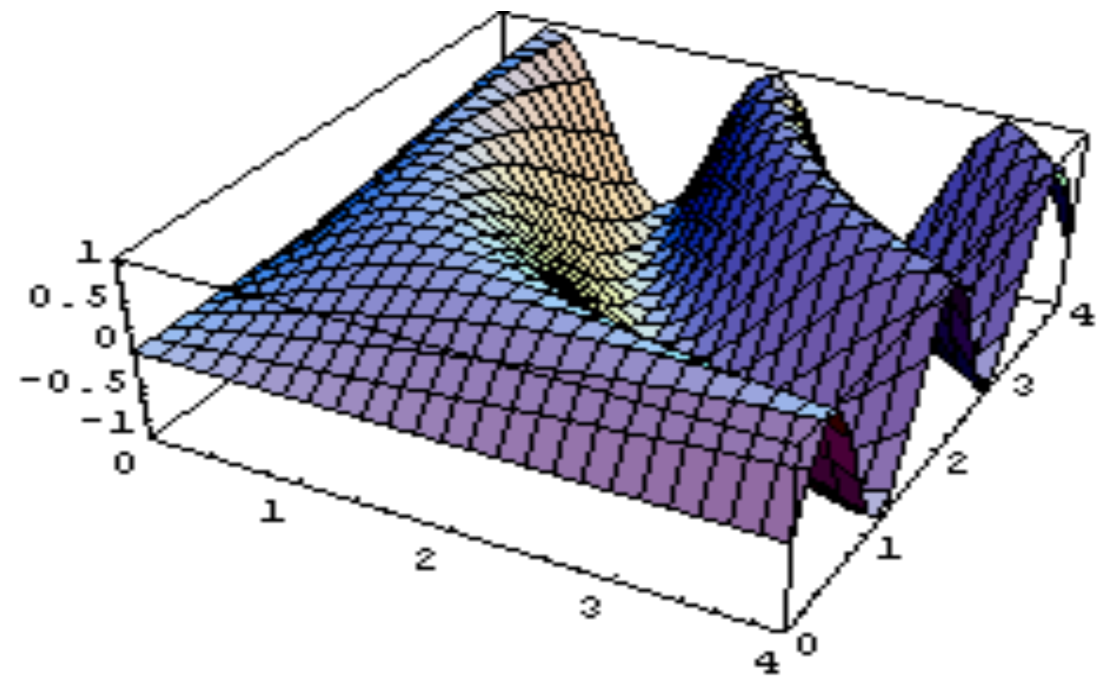
Idea: Imagine all possible solutions laid out on a landscape

Goal: find the highest (or lowest) point



Iterative Improvement Methods

- Start at some random point (potential solution)
- Generate all possible points to move to
- If the set is not empty, choose a point and move to it
- If you are stuck (set is empty), then restart



Iterative Improvement Methods

- What does it mean to “generate points to move to”
  - Generating the **moveset**
- Depends on the application

TSP

2-swap

Hill Climbing (Gradient Descent)

Main idea: Always take a step in the direction that improves the current solution value the most

Note: Variation of best-first search

Application: Very popular for learning algorithms

"…like trying to find the top of Mt Everest in a thick fog while suffering from amnesia", Russell and Norvig

Hill Climbing

1. Start with some initial configuration S, with value V(S)
2. Generate Moveset(S)= $\{S_1,\ldots,S_n\}$
3. S_{max}=argmax$_{Si}$ $V(S_i)$
4. If $V(S_{max})<V(S)$ return S (local optimium)
5. Let $S \leftarrow S_{max}$ Go to 2

Judging Hill Climbing

Good news

Easy to program!

Requires no memory of where we have been!

Judging Hill Climbing

Good news

Easy to program!

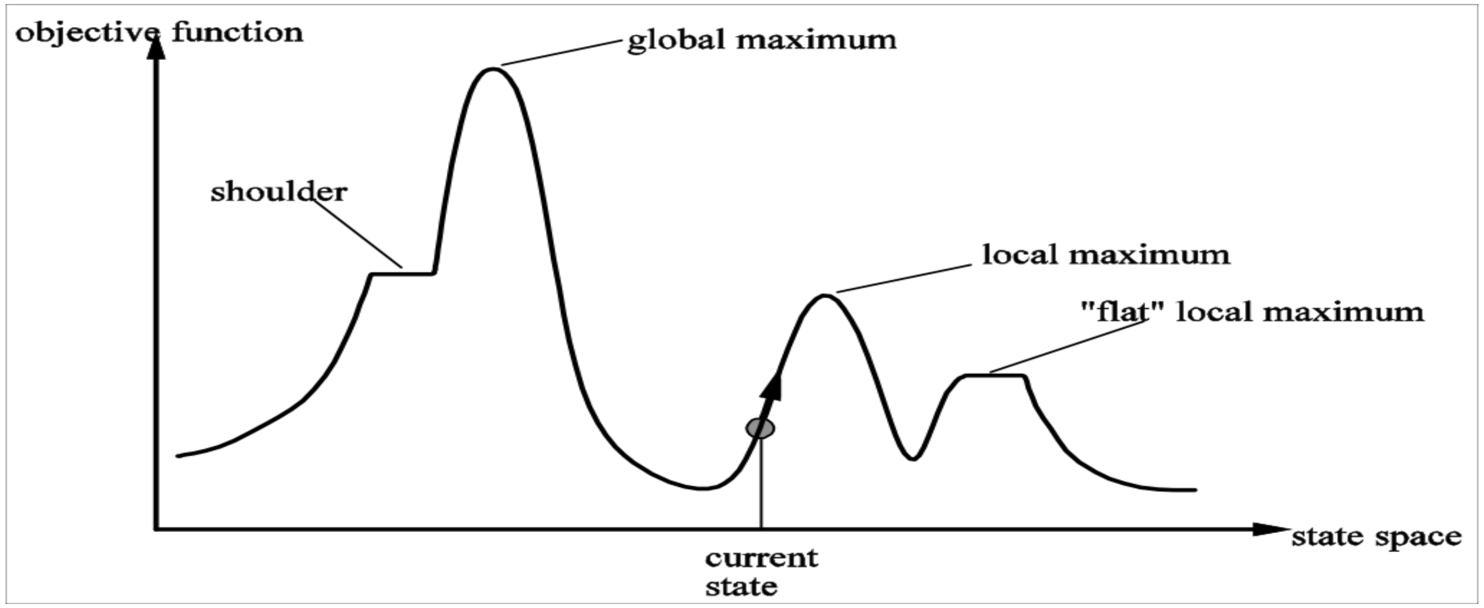
Requires no memory of where we have been!

Bad news

Not necessarily complete

Not optimal

It can get stuck in local optima/plateaus



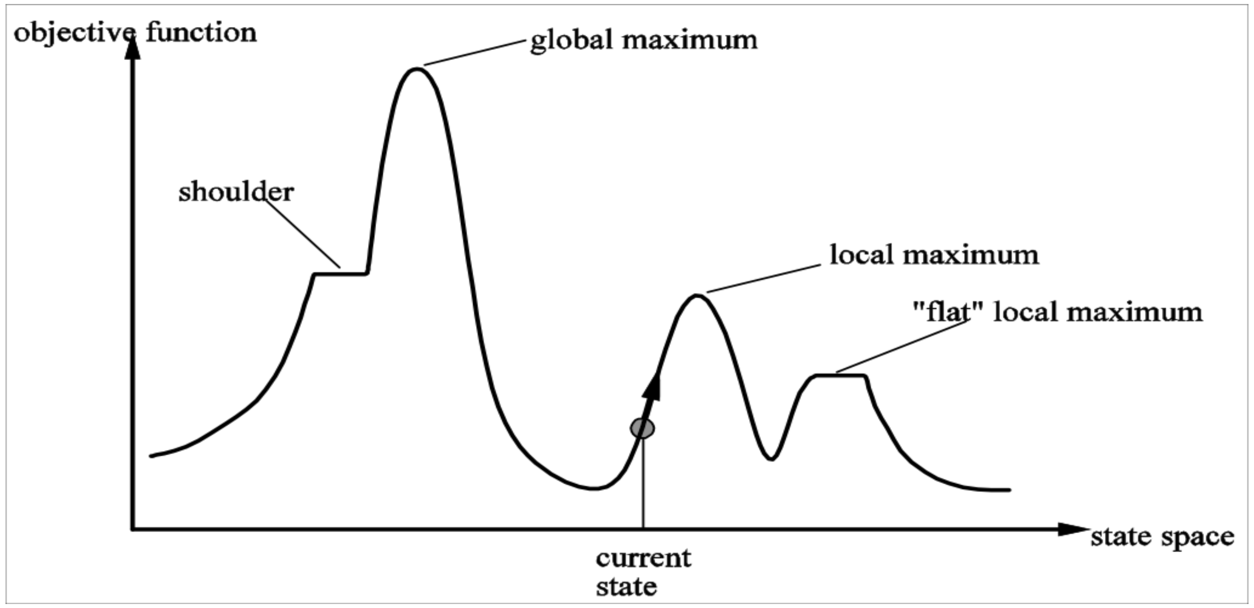
Improving Hill Climbing

Plateaus

- Allow for sideways moves
  - But be careful since might move sideways forever

Local Maxima

- **Random restarts:** *If at first you do not succeed, try, try again!*



Randomized Hill Climbing

Randomized hill climbing is like hill climbing except

- You choose a random state, S_i, from the Moveset
- Move to S_i if $V(S_i)>V(S)$

Even More Randomization!

- Hill climbing is incomplete
  - can get stuck at local optima
- A random walk is complete
  - but very inefficient

New Idea:
Allow the algorithm to make some "bad" moves in order to escape local optima

Example: GSAT

AV~BVC	1
~AVCVD	1
BVDV~E	0
~CV~DV~E	1
~AV~CVE	1

Configuration A=1, B=0, C=1, D=0, E=1

Goal is to maximize the number of satisfied clauses: Eval(config)=# satisfied clauses

GSAT Move_Set: Flip any 1 variable

WALKSAT (Randomized GSAT)

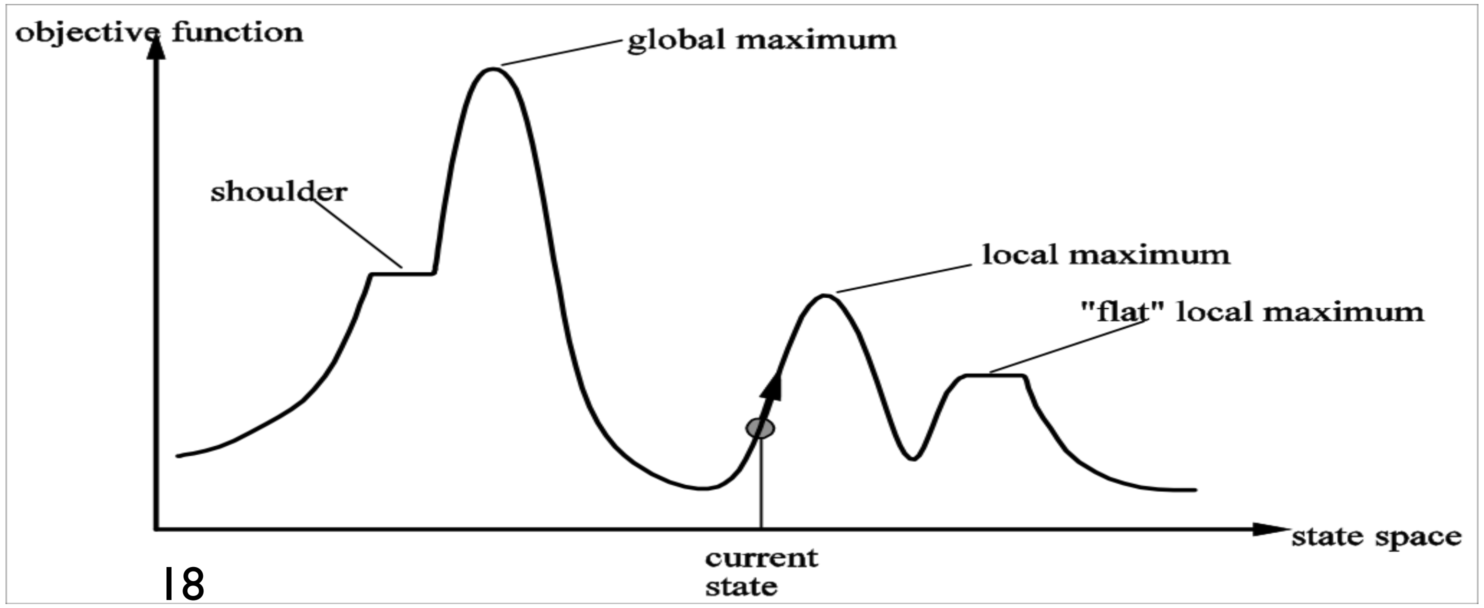
Pick a random unsatisfied clause;

Consider flipping each variable in the clause

- If any improve Eval, then accept the best
- If none improve Eval, then with prob p pick the move that is least bad; prob (1-p) pick a random one

Towards Simulated Annealing

1. Start with some initial configuration S, with value V(S)
2. Generate Moveset(S)= $\{S_1,\ldots,S_n\}$
3. Randomly choose S_i from Moveset(S)
4. Define $\Delta V=V(S_i)-V(S)$
5. If $\Delta V>0$ then $S \leftarrow S_i$ **else with probability p $S \leftarrow S_i$**
6. Go to 2



What About p?

Main Issue: How should we choose the probability of making a “bad” move?

Ideas:

p=0.1 (or some fixed value)?

Decrease p with time?

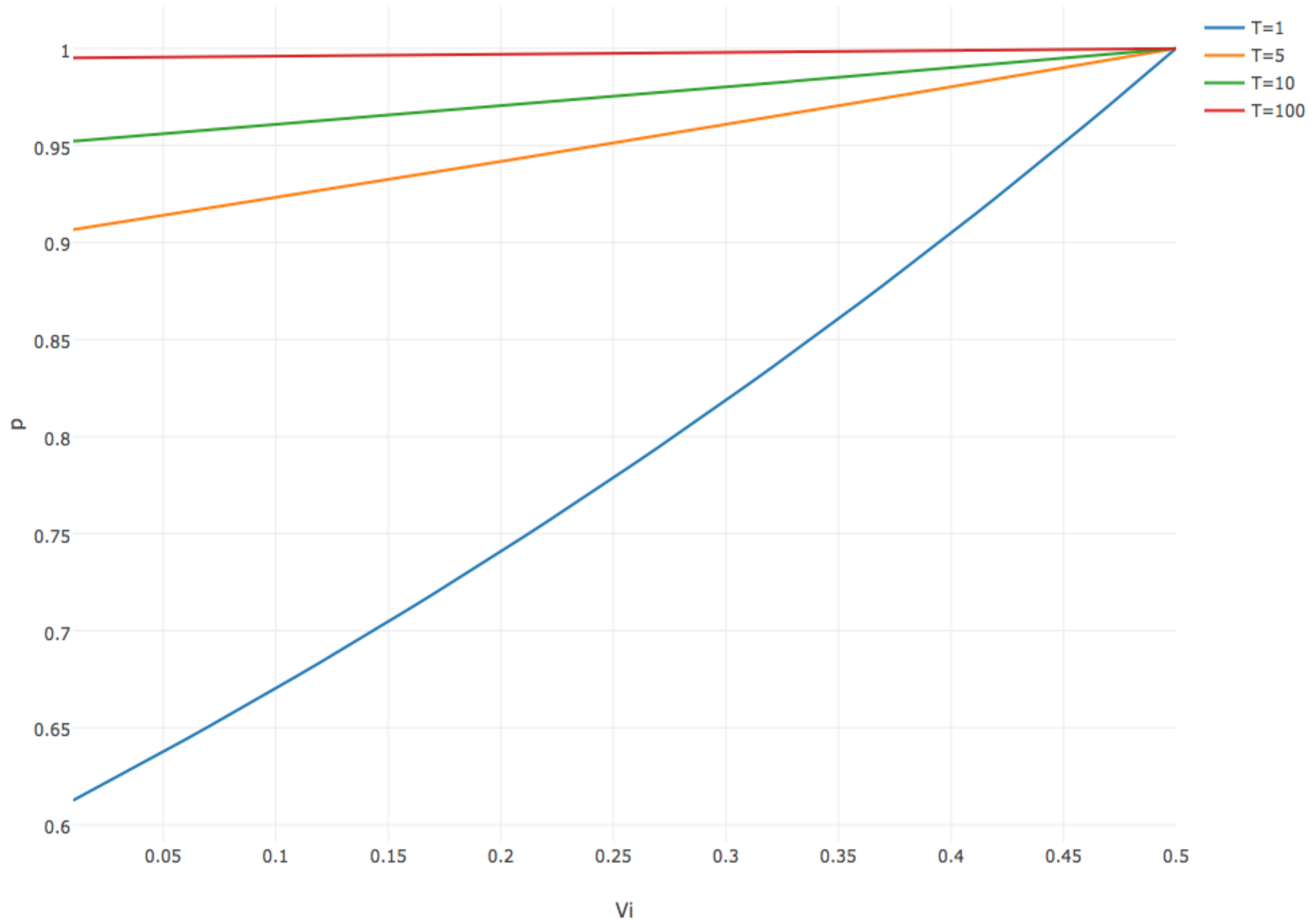
Make p a function of $|V-V_i|$?

…

Selecting Moves in Simulated Annealing

- If new value V_i is **better** than old value V then **definitely** move to new solution
- If new value V_i is **worse** than old value V then move to new solution with **probability**

$$e^{\frac{\Delta V}{T}}$$



<u>Boltzmann Distribution</u>: T>0 is a parameter called temperature. It starts high and decreases over time towards 0. If T is close to 0 then the prob. of making a bad move is almost 0.

Properties to Simulated Annealing

- When T is high:
  - **Exploratory phase**: even bad moveshave a chance of being picked (random walk)
- When T is low:
  - **Exploitation phase**: “bad” moves have low probability of being chosen (randomized hill climbing)
- If T is decreased slowly enough then simulated annealing is guaranteed to reach optimal solution

Genetic Algorithms

- Populations are encoded into a representation which allows certain operations to occur
- An encoded candidate solution is an **individual**
- Each individual has a **fitness**
  - Numerical value associated with its quality of solution
- A **population** is a set of individuals
- Populations change over **generations** by applying operators to them
  - Operations: selection, mutation, crossover

Typical Genetic Algorithm

- Initialize: Population P←N random individuals
- Evaluate: For each x in P, compute fitness(x)
- Loop
  - For i=1 to N
    - **Select** 2 parents each with probability proportional to fitness scores
    - **Crossove**r the 2 parents to prodice a new bitstring (child)
    - With some small probability **mutate** child
    - Add child to population
  - Until some child is fit enough or you get bored
- Return best child in the population according to fitness function

Selection

- Fitness proportionate selection: $$P(i) = \frac{\text{fitness}(i)}{\sum_j \text{fitness}(j)}$$
  - Can lead to overcrowding
- Tournament selection
  - Pick i, j at random with uniform probability
  - With probability p select fitter one
- Rank selection
  - Sort all by fitness
  - Probability of selection is proportional to rank
- Softmax (Boltzmann) selection: $$P(i) = \frac{e^{\text{fitness}(i)/T}}{\sum_j e^{\text{fitness}(j)/T}}$$

Crossover

- Combine parts of individuals to create new ones
- For each pair, choose a random crossover point
  - Cut the individuals there and swap the pieces

101|0101 011|1110

Cross over

011|0101 101|1110

Implementation: use a crossover mask m

Given two parents a and b the offspring are

(a^m)V(b^~m) and (a^~m)V (b^m)

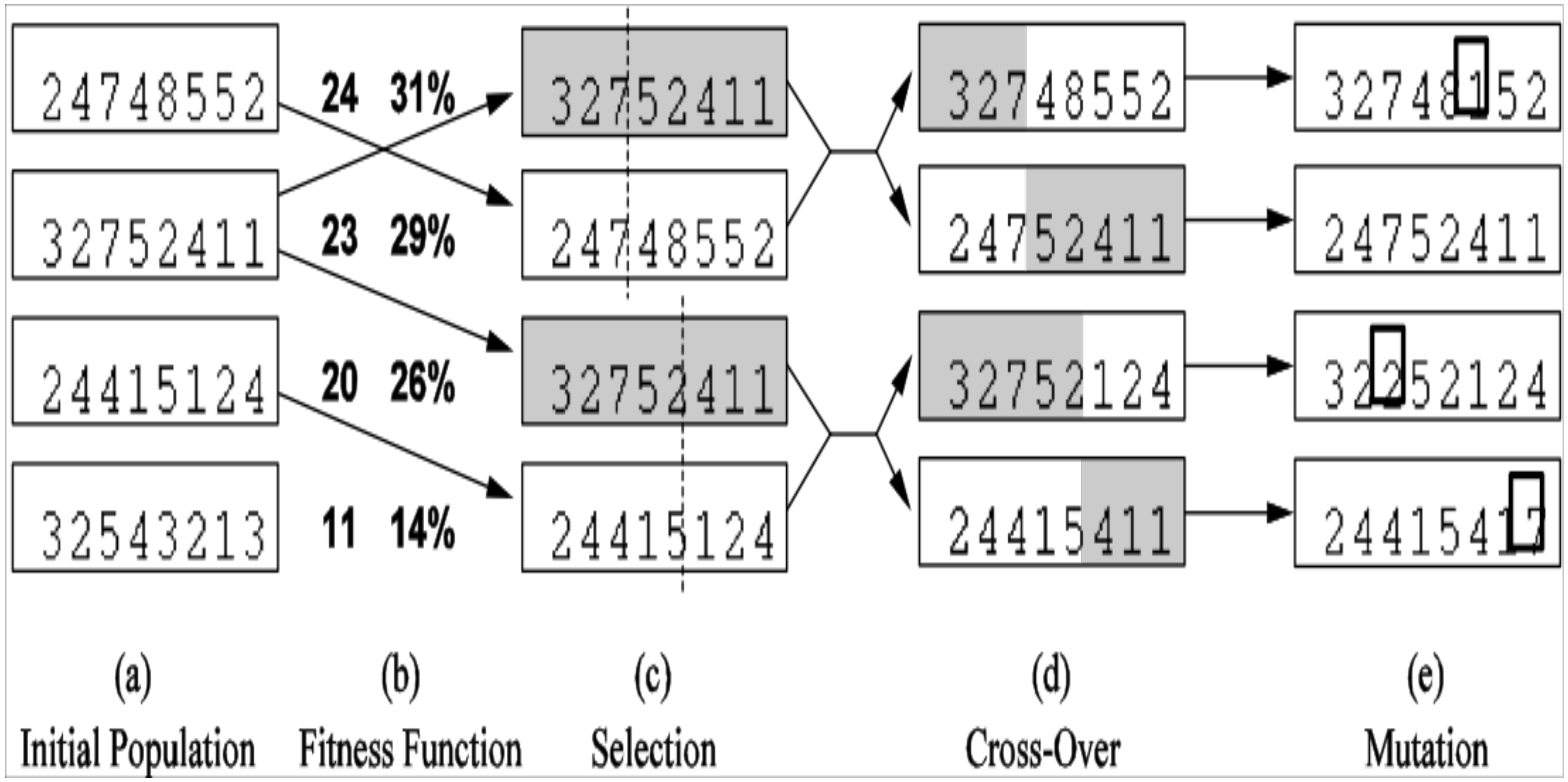
Mutation

- Mutation generates new features that are not present in original population
- Typically means flipping a bit in the string

  100111 mutates to 100101

- Can allow mutation in all individuals or just in new offspring

Example



24748552
32752411
24415124
32543213
24 31%
23 29%
20 26%
11 14%
32752411
24748552
32752411
24415124
32748552
24752411
32752124
24415411
32748152
24752411
32252124
24415417
(a)
Initial Population
(b)
Fitness Function
(c)
Selection
(d)
Cross-Over
(e)
Mutation

Summary

- Useful for optimization problems
- Often the second-best way to solve a problem
  - If you can, use A* or linear programming or ...
- Need to think about how to escape from local optima
  - Random restarts
  - Allowing for bad moves
  - ...