Adversarial Search

CS 486/686: Introduction to Artificial Intelligence
Introduction

• So far have only been concerned with single agents

• Today
  - Multiple agents planning against each other
    - Adversarial settings
Outline

- Games
- Minimax search
- Alpha-beta pruning
- Evaluation functions
- Coping with chance
- Game programs
Games

- Games are the oldest, most well-studied domain in AI
- Why?
  - They are fun
  - Easy to represent, rules are clear
  - State spaces can be very large
    - In chess, the search tree has $\sim 10^{154}$ nodes
  - Like the “real world” in that decisions have to be made and time is important
  - Easy to determine when a program is doing well
Types of Games

• **Perfect vs Imperfect Information**
  - Perfect information: You can see the entire state of the game
  - Imperfect information:

• **Deterministic vs Stochastic**
  - Deterministic: change in state is fully controlled by the players
  - Stochastic: change in state is partially determined by chance
Games as Search Problems

- **2-player perfect information game**
- **State**: board configuration plus the player who’s turn it is to move
- **Successor function**: given a state, returns a list of (move,state) pairs indicating legal move and resulting state
- **Terminal state**: states where there is a win/loss/draw
- **Utility function**: assigns a numerical value to terminal states
- **Solution**: a strategy (way of picking moves) that wins the game
Game Search Challenge

• What makes game search challenging?
  - There is an opponent
  - The opponent is malicious
    - it wants to win (by making you lose)
  - We need to take this into account when choosing moves

• Notation:
  - **MAX** player wants to maximize its utility
  - **MIN** player wants to minimize its utility
Example

MAX’s job is to use the search tree to determine the best move.
Optimal Strategies

- In standard search
  - Optimal solution is sequence of moves leading to winning terminal state

- **Strategy** (from MAX’s perspective)
  - Specify a move for the initial state
  - Specify a move for all possible states arising from MIN’s response
  - Then all possible responses to all of MIN’s responses to MAX’s previous moves
  - ...
Optimal Strategies

• **Goal**: Find optimal strategy

• What do we mean by optimal?
  - Strategy that leads to outcomes at least as good as any other strategy, *given that MIN is playing optimally*
    - Equilibrium (game theory)

• Today we focus mainly on **zero-sum games of perfect information**
  - Easy games according to game theory
Minimax Value

\[
\text{MINIMAX-VALUE}(n) = \begin{cases} 
\text{Utility}(n) & \text{if } n \text{ is a terminal state} \\
\max_s \in \text{Succ}(n) \text{ MINIMAX-VALUE}(s) & \text{if } n \text{ is a MAX node} \\
\min_s \in \text{Succ}(n) \text{ MINIMAX-VALUE}(s) & \text{if } n \text{ is a MIN node}
\end{cases}
\]
Properties of Minimax

• Complete if tree is finite
• Time complexity: $O(b^m)$
  - $m$ is depth of tree
• Space complexity: $O(bm)$
  - It is DFS
• Optimal against an *optimal opponent*
  - If opponent is not playing optimally, then may be better off doing something else
Minimax and Multi-Player Games

![Game Tree](image)
Question

- Can we now write a program that will play chess reasonably well?
Question

• Can we now write a program that will play chess reasonably well
  - For chess b~35 and m~100
Alpha-Beta Pruning

• If we are smart (and lucky) we can do pruning
  – Eliminate large parts of the tree from consideration

• Alpha-beta pruning applied to a minimax tree
  – Returns the same decision as minimax
  – Prunes branches that cannot influence final decision
Alpha-Beta Pruning

• Alpha:
  - Value of best (highest value) choice we have found so far on path for MAX

• Beta:
  - Value of best (lowest value) choice we have found so far on path for MIN

• Update alpha and beta as search continues

• Prune as soon as value of current node is known to be worse than current alpha or beta values for MAX or MIN
Example

MAX

MIN

≥ 3

3

3 12 8
Example

MAX

MIN

Prune remaining children

[3, ∞]

≥ 2

3

12

8

2
Example

\[
\begin{align*}
\text{MAX} & \quad [3, \infty] \\
\text{MIN} & \quad 3, 12, 8, 2, 14, \leq 2, \leq 14
\end{align*}
\]
Example

MAX

MIN

[3, \infty]

\leq 2

\leq 5
Example

MAX

MIN

$[3, \infty]$
Properties of Alpha-Beta

• Pruning does not affect the final result
  - Why?

• Move ordering is important

• Alpha-beta demonstrates the value of reasoning about which computations are important
Real-Time Decisions

• Alpha-Beta can be a huge improvement over minimax
  - Still not good enough
    - Need to search to terminal states for at least part of search space
    - Need to make decisions quickly

• Solution
  - Heuristic evaluation function + cutoff tests
Evaluation Functions

• Apply an evaluation function to a state
  - If terminal state, function returns actual utility
  - If non-terminal, function returns estimate of the expected utility

• Function must be fast to compute
Evaluation Functions

• How do we get evaluation functions?
  - Expert knowledge
  - Learned from experience

• Look for features of states
  - Weighted linear function $Eval(s) = \sum_i w_i f_i(s)$
Cutting Off Search

- Do we have to search to terminal states?
  - No! Cut search early and apply evaluation function

- When?
  - Arbitrarily (but deeper is better)
  - Quiescent states
    - States that are “stable”
  - Singular extensions
    - Searching deeper when you have a move that is “clearly better”
    - Can be used to avoid the horizon effect
Cutting Off Search

• How deep?
  - Novice player
    - 5-ply (minimax)
  - Master player
    - 10-ply (alpha-beta)
  - Grandmaster
    - 14-ply + fantastic evaluation function, opening and endgame databases,...
Stochastic Games
Stochastic Games

• Need to consider best/worst cases + probability they will occur

• Recall: Expected value of a random variable \( x \) \( \mathbb{E}[x] = \sum_{x \in X} P(x)x \)

• Expectiminimax: minimax but at chance nodes compute the expected value
Expectiminimax

![Game Tree]

- **MAX**
  - **CHANCE**: 3
    - 0.5
      - **MIN**: 2
        - 2
      - 0.5
        - **MIN**: 4
          - 4
      - 0.5
        - **MIN**: 7
          - 4
    - 0.5
      - **MIN**: 0
        - 6
      - 0.5
        - **MIN**: -2
          - -2
**WARNING:** exact values do matter! Order-preserving transformations of the evaluation function can change the choice of moves. Must have **positive linear transformations** only.
Summary

• Games pose lots of fascinating challenges for AI researchers

• Minimax search allows us to play optimally against an optimal opponent

• Alpha-beta pruning allows us to reduce the search space

• A good evaluation function is key to doing well

• Games are fun!
Some Game Programs
Checkers

Mr. Tinsley suffered his 4th and 5th losses ever against Chinook
Checkers

- Chinook (University of Alberta)
  - World Man-Machine Checkers Champion
  - Alpha-beta search
  - Opening database

- Secret weapon: **Endgame database**
  - Perfect knowledge into the search

- Checkers is now dominated by computers
  - Checkers is (weakly) solved
### Chess: Kasparov vs. Deep Blue

1997: Deep Blue wins by 3 wins, 1 loss, and 2 draws

<table>
<thead>
<tr>
<th>Kasparov</th>
<th>Deep Blue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height 5’10”</td>
<td>Height 6’5”</td>
</tr>
<tr>
<td>Weight 176 lbs</td>
<td>Weight 2,400 lbs</td>
</tr>
<tr>
<td>Age 34 years</td>
<td>Age 4 years</td>
</tr>
<tr>
<td>Computers 50 billion neurons</td>
<td>Computers 32 RISC processors + 256 VLSI chess engines</td>
</tr>
<tr>
<td>Speed 2 pos/sec</td>
<td>Speed 200,000,000 pos/sec</td>
</tr>
<tr>
<td>Knowledge Extensive</td>
<td>Knowledge Primitive</td>
</tr>
<tr>
<td>Power Source Electrical/chemical</td>
<td>Power Source Electrical</td>
</tr>
<tr>
<td>Ego Enormous</td>
<td>Ego None</td>
</tr>
</tbody>
</table>

Jonathan Schaeffer
• Its secret:
  - Specialized chess processor + special-purpose memory optimization
  - Very sophisticated evaluation function
    - Expert features and hand-tuned weights
  - Opening and closing books
  - Alpha-beta + improvements (searching up to 40 ply deep)
  - Searched over 200 million positions per second
Chess

• There are now apps that are on par with human champions

• Is Chess still a human game or have computers conquered it?
Backgammon

- TD-Gammon (Gerry Tesauro at IBM)
- One of the top players in the world
- Searches only two moves ahead!
- Its secret: **One amazing evaluation function**
  - Neural network trained with reinforcement learning during ~1 million games played against itself
  - Humans play backgammon differently now, based on what TD-Gammon learned about the game
  - Very cool AI 😊
Go

- Large branching factor makes Go too large to solve by classic search methods
  - pieces added to the board
  - evaluation function
  - ...

- Limited progress for decades
Go

• BUT computer Go has undergone a revolution in the past ~5 years
  - Close to perfection on 7x7 games
  - Reached top human level on 9x9 games
  - Still weaker than top humans on 19x9 boards
Go

- Monte-Carlo Tree Search (MCTS)
  - Build search tree according to outcomes of simulated plays

Upper Confidence Bounds for Trees (UCT): “Minimax search” using UCB
Card Games

• Focus has been on Bridge and Poker
  - Humans are still winning...
  - But machines are catching up!

• Issues
  - Stochastic and partially observable
    - Ideas discussed today don’t work well
    - New approaches are being developed