Below are a set of practice problems so that you gain experience working with game-theoretic concepts. These are optional and will not be graded. However, I am happy to mark your solutions or discuss any problems you encounter.

1. Solve the normal form game in Figure 1 by eliminating dominated strategies. Verify the resulting outcome is a Nash equilibrium of the game. Then, compute the minmax and maxmin values for each agent.

	N	C	J
N	73, 25	57, 42	66, 32
C	80, 26	35, 12	32, 54
J	28, 27	63, 31	54, 29

Figure 1:

2. *Game of Chicken* Two teenagers play the following risky game. They drive towards each other at stop speed in separate cars. Just before collision each one has the choice of continuing straight or avoiding collision by turning right. If both continue straight then they both die. If one continues straight while the other turns they both live, but the one who went straight gets boasting rights and the is humiliated. If both turn, then both survive and both are moderately humiliated. The game is represented in the table in Figure 2.

	straight	turn
straight	-3,-3	2,0
turn	0,2	1,1

Figure 2:

   (a) Does this game have pure strategy Nash equilibria? If so, what are they?
   (b) What are the mixed strategy Nash equilibria of this game?
   (c) In each equilibrium, what is the probability that the teenagers will die?

3. Throughout this question, you may restrict your analysis to pure strategies.
   (a) Draw the normal form game of the game tree in Figure 3.
   (b) Name the dominant strategy equilibria, if there are any.
   (c) Name the Nash equilibria of this game, if there are any.
   (d) Name the subgame perfect Nash equilibria in the game, if there are any.

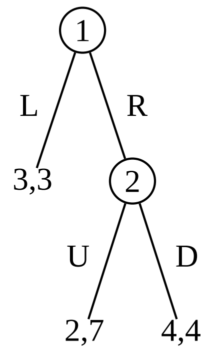

Figure 3:

(e) Name the Pareto efficient outcomes of this game, if there are any.

(f) Name the social welfare maximizing outcomes of this game, if there are any.

4. An agent's strategy is *strictly dominated* if that agent has another strategy that gives strictly better payoff to the agent no matter what strategies other agents do. An agent's strategy is *weakly dominated* if that agent has another strategy that gives at least equally high payoff to the agent no matter what other agents do, and strictly higher payoff to the agent for at least one choice of strategies of by the others. To solve a game, we can iteratively eliminate dominated strategies until all remaining strategies are undominated.

   (a) Prove that if strategies $s^* = (s_1^*, \ldots, s_n^*)$ are a Nash equilibrium in a normal form game, then they survive iterated elimination of strictly dominated strategies. (Hint: By contradiction, assume that one of the strategies in the Nash equilibrium is eliminated by iterated elimination of dominated strategies).

   (b) Prove that if the process of iterated elimination of strictly dominated strategies results in a unique strategy profile $s^* = (s_1^*, \ldots, s_n^*)$ then this is a Nash equilibrium of the game. (Hint: By contradiction, assume there exists some agent i for which $s_i \neq s_i^*$ is preferred over s_i^*, and show a contradiction with the fact that s_i was eliminated.)

5. The residents in Pleasantville live on Main Street which is the only road in town. Two residents decide to set up general stores. Each can locate at any point between the beginning of Main Street, which we will label 0, and the end, which we will label 1. Two two decide independently where to locate and they must remain there forever. Each store will attract the customers who are closest to it, and the stores will share equally customers who are equidistant between the two. Thus, for instance, if one store locates at point x and the second at point $y > x$ then the first will get a share of $x + (y - x)/2$ and the second will get a share $(1 - y) + (y - x)/2$ of the customers each day. Each customer contributes $\$1.00$ in profits each day to the general store it visits.

(a) Define the actions, strategies, and daily payoffs to this game. Find the unique pure strategy Nash equilibrium.

(b) Suppose there are three General Stores, each independently choosing a location point along the road. Show that there is no pure strategy Nash equilibrium.

6. In a certain market there are two firms, which we label a and b. If the firms produce output q_a and q_b, then the price they receive for their goods is given by $p = \alpha - \beta(q_a + q_b)$ or zero if p would otherwise be negative. Each firm has marginal cost $c > 0$ and no fixed costs. Suppose $\alpha > 3c$.

   (a) Suppose the firms choose the quantities q_a and q_b independently in each period (ignoring past behaviour), and each maximizes profits $\pi_a = (p - c)q_a$ and $\pi_b = (p - c)q_b$. Find the unique pure strategy Nash equilibrium to this game. This is called the *Cournot duopoly* model. (Recall that in equilibrium each choice of q_a and q_b are a best response to each other. You might find a graphical approach useful.)

   (b) Suppose the two firms collude by agreeing that each will produce an amount $q^* = q_a = q_b$, and they have some way to enforce this agreement. What should q^* be? What are the profits of the two firms? This is called the *monopoly* or *cartel* model.

   (c) Suppose that firm a reneges on its promise in the previous question but b does not. What should a chose for q_a? What are the profits for both firms?

   (d) Suppose that b finds out what a is planning on doing in the previous question, and then choses q_b so as to maximize its profits given that a is going to renege. What is q_b now? What do you think will happen if they go back and forth in that way forever?

   (e) Suppose form a gets to choose it output first and only afterwards does firm b get to choose its output. Find the equilibrium choices of q_a and q_b. This is called the *Stackelberg duopoly model*.

7. Adam is sitting at the best seat in the sports bar, minding his own business, when Chris walks into the bar. Chris would like to **B**ully Adam to get the seat, but only if Adam is a **W**imp. If Adam is **T**ough then Chris will **D**efer to Adam. Chris does not know if Adam is **T**ough or a **W**imp, but he knows that with probability $\frac{1}{3}$ Adam is **T**ough. If Adam is a **W**imp and Chris **B**ullies him, Adam will move, but if Adam is **T**ough than Adam will fight for his place at the bar. Adam gets 2 points if Chris **D**efers to him, plus 1 point for avoiding consuming something he does not like. Chris gets 1 point for guessing correctly.

   Suppose Adam can signal whether he is **T**ough or a **W**imp by choosing to consume either Apple **J**uice or **M**ilk. Assume also that **T**ough men don't like **M**ilk and **W**imps do not like **J**uice. Find all Nash equilibrium to this game. The game is shown in Figure 4. (Recall the definition of a strategy!)

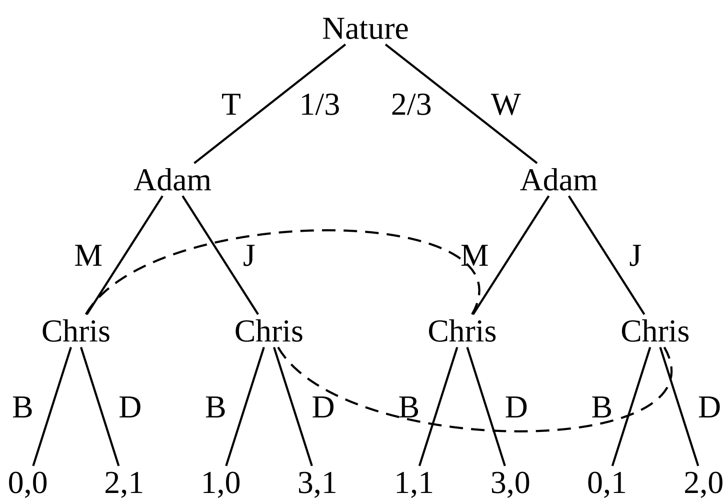
Nature
T
1/3
2/3
W
Adam
Adam
M
J
M
J
Chris
Chris
Chris
Chris
B
D
B
D
B
D
B
D
0,0
2,1
1,0
3,1
1,1
3,0
0,1
2,0

Figure 4: