CS 798: Multiagent Systems

Introduction to Social Choice

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Outline

Properties for Voting Protocols

Property (Universality)

A voting protocol should work with any set of preferences.

Property (Transitivity)

A voting protocol should produce an ordered list of alternatives (social welfare function).

Property (Pareto efficiency)

If all agents prefer X to Y, then in the outcome X should be prefered to Y. That is, SWF f is pareto efficient if for any $o_1, o_2 \in O, \forall i \in N, o_1 >_i o_2$ *then* $o_1 >_f o_2$.

More Properties

Property (Independence of Irrelevant Alternatives (IIA))

Comparison of two alternatives depends only on their standings among agents' preferences, and not on the ranking of other alternatives. That is, SWF f is IIA if for any $o_1, o_2 \in O$

Property (No Dictators)

A SWF f has no dictator if $\neg \exists i \forall o_1, o_2 \in O,\ o_1 >_i o_2 \Rightarrow o_1 >_f o_2$

Arrows Theorem

Theorem (Arrow, 1951)

Let $|O| \geq 3$. Then any social welfare function f that is Pareto efficient and independent of irrelevant alternatives (IIA) is dictatorial.

Definition

A set $S \subseteq A$ is decisive for x over y whenever

- $x >_i y$ *for all* $i \in S$
- $x <_i y$ *for all* $i \in A \setminus S$

then $x > y$

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Single-Peaked Preferences

Definition

A binary relation $\geq$ on a set of alternatives O is a linear order on O if it is reflexive, transitive, and total.

Definition

A preference relation $\succeq$ is single-peaked with respect to the linear order $\geq$ on O if there is an alternative $x \in O$ with the property that $\succeq$ is increasing with respect to $\geq$ on $\{y \in O | x \geq y\}$ and decreasing with respect to $\geq$ on $\{y \in O | y \geq x\}$. That is

$$\text{If } x \geq z > y \text{ then } z \succ y$$

and

$$\text{If } y > z \geq x \text{ then } z \succ y.$$

Single-Peaked Preferences

Definition

Let x_i denote agent $i \in A$'s "peak". Agent $h \in A$ is a median agent if

$$|\{a_i \in A | x_i \geq x_h\}| \geq \frac{|A|}{2} \text{ and } |\{a_i \in A | x_h \geq x_i\}| \geq \frac{|A|}{2}.$$

Single-Peaked Preferences

Theorem

If agents all have single-peaked preferences with respect to the same linear order, then a Condorcet winner always exists. The Condorcet winner is x_h where h is the median agent and x_h is its "peak".