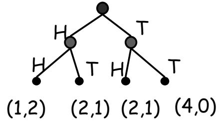
# Mechanism Design

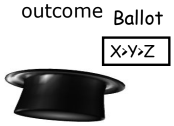
CS 886
Multiagent Systems
University of Waterloo

# Introduction

So far we have looked at

- Game Theory
  - Given a game we are able to analyze the strategies agents will follow
- Social Choice Theory
  - Given a set of agents' preferences we can choose some outcome





# Introduction

- Today, Mechanism Design
  - Game Theory + Social Choice
- Goal of Mechanism Design is to
  - Obtain some outcome
  - But agents are rational
- "Solution":
  - Define the rules of a game so that in equilibrium the agents do what we want
- CS Spin
  - Defining protocols for distributed systems

# Example: London Bus System

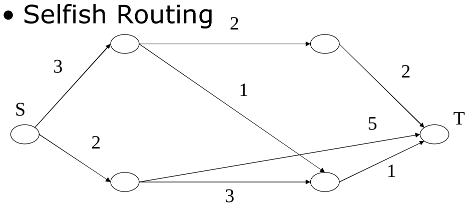
(as of April 2004)

- 5 million passengers each day
- 7500 buses
- 700 routes

- The system has been privatized since 1997 by using competitive tendering
- Idea: Run an auction (mechanism) to allocate routes to companies

# Example

- Selfish Routing



Want to find the least-cost route from S to T.
You do not know costs.
You do know that each links wants to maximize revenue.

How do you use this information to extract information needed to find least-cost path?

# Fundamentals

- Set of possible outcomes, O
- Agents i∈N, |N|=n, each agent i has type $\theta_i \in \Theta_i$
  - Type captures all private information that is relevant to agent's decision making
- Utility $u_i(o, \theta_i)$, over outcome o∈O
- Recall: goal is to implement some system-wide solution
  - Captured by a social choice function

$$\mathbf{f}:\Theta_1 \times \ldots \times \Theta_n \rightarrow \mathcal{O}$$

$\mathbf{f}(\theta_1,\ldots\theta_n)=\mathbf{o}$ is a collective choice

# Examples of social choice functions

- Voting: choose a candidate among a group
- Public project: decide whether to build a swimming pool whose cost must be funded by the agents themselves
- Allocation: allocate a single, indivisible item to one agent in a group

# Mechanisms

- Recall: We want to implement a social choice function
  - Need to know agents' preferences
  - They may not reveal them to us truthfully
- Example:
  - 1 item to allocate, and want to give it to the agent who values it the most
  - If we just ask agents to tell us their preferences, they may lie

# Mechanism Design Problem

- By having agents interact through an institution we might be able to solve the problem
- Mechanism:

$$M=(S_1,\ldots,S_n,\ g(\cdot))$$

Strategy spaces of agents

Outcome function

$$g: S_1\times\ldots\times S_n \rightarrow O$$

# Implementation

- A mechanism $M=(S_1,\ldots,S_n,g())$ implements social choice function $f(\theta)$ if there is an equilibrium strategy profile $s^*=(s_1^*,\ldots,s_n^*)$ of the game induced by M such that
  - $g(s_1^*(\theta_1),\ldots,s_n^*(\theta_n))=f(\theta_1,\ldots,\theta_n)$ $\forall\ (\theta_1,\ldots,\theta_n)\in\ \Theta_1\times\ldots\times\Theta_n$

# Implementation

- We did not specify the type of equilibrium in the definition
- Nash

$$u_i(s_i^*(\theta_i),s^*_{-i}(\theta_{-i}),\theta_i)\geq u_i(s_i'(\theta_i),s^*_{-i}(\theta_{-i}),\theta_i),\ \forall i,\ \forall \theta,\ \forall s_i' \neq s_i^*$$

- Bayes-Nash

$$E[u_i(s_i^*(\theta_i),s^*_{-i}(\theta_{-i}),\theta_i)]\geq E[u_i(s_i'(\theta_i),s^*_{-i}(\theta_{-i}),\theta_i)],\ \forall i,\ \forall \theta,\ \forall s_i' \neq s_i^*$$

- Dominant

$$u_i(s_i^*(\theta_i),s_{-i}(\theta_i),\theta_i)\geq u_i(s_i'(\theta_i),s_{-i}(\theta_{-i}),\theta_i),\ \forall i,\ \forall \theta,\ \forall s_i'\neq s_i^*,\ \forall s_{-i}$$

# Direct Mechanisms

- Recall that a mechanism specifies the strategy sets of the agents
  - These sets can contain complex strategies
- **Direct mechanisms:**
  - Mechanism in which $S_i=\Theta_i$ for all i, and $g(\theta)=f(\theta)$ for all $\theta\in\Theta_1\times\ldots\times\Theta_n$
- **Incentive compatible:**
  - A direct mechanism is incentive compatible if it has an equilibrium $s^*$ where $s^*_i(\theta_i)=\theta_i$ for all $\theta_i\in\Theta_i$ and all i
    - truth telling by all agents is an equilibrium
  - Strategy-proof if dominant-strategy equilibrium

## Dominant Strategy Implementation

- Is a certain social choice function implementable in dominant strategies?
  - In principle we would need to consider all possible mechanisms
- **Revelation Principle**
  - Suppose there exists a mechanism $M=(S_1,\ldots,S_n,g(\cdot))$ that implements social choice function f() in dominant strategies.
  - Then there is a direct strategy-proof mechanism, M′, which also implements f().

## Revelation Principle

> "*the computations that go on within the mind of any bidder in the nondirect mechanism are shifted to become part of the mechanism in the direct mechanism*" [McAfee&McMillian 87]

- Vickrey auction and English auction

## Revelation Principle: Proof

- $M=(S_1,\ldots,S_n,g())$ implements SCF f() in dom str.
  - Construct direct mechanism $M'=(\Theta^n,f(\theta))$
  - By contradiction, assume

  $\exists\ \theta_i'\neq\theta_i$ s.t. $u_i(f(\theta_i',\theta_{-i}),\theta_i)>u_i(f(\theta_i,\theta_{-i}),\theta_i)$

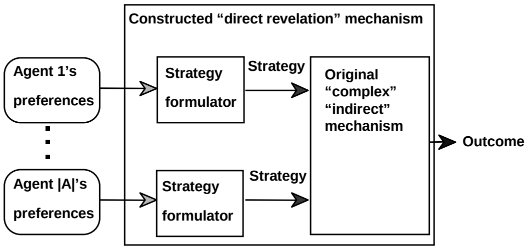
  for some $\theta_i'\neq\theta_i$, some $\theta_{-i}$.

  - But, because $f(\theta)=g(s^*(\theta))$, this implies

  $u_i(g(s_i^*(\theta_i'),s_{-i}^*(\theta_{-i})),\theta_i)>u_i(g(s^*(\theta_i),s^*(\theta_{-i})),\theta_i)$

  Which contradicts the strategy proofness of $s^*$ in M

## Revelation Principle: Intuition



## Theoretical Implications

- Literal interpretation: Need only study direct mechanisms
    - This is a smaller space of mechanisms
  - Negative results
    - If no direct mechanism can implement SCF f() then no mechanism can do it
  - Analysis tool:
    - Best direct mechanism gives us an upper bound on what we can achieve with an indirect mechanism
    - Analyze all direct mechanisms and choose the best one

## Practical Implications

- Incentive-compatibility is "free" from an implementation perspective
- **BUT!!!**
  - A lot of mechanisms used in practice are not direct and incentive-compatible
    - Maybe there are some issues that are being ignored here

# Quick review

- We now know
  - What a mechanism is
  - What is means for a SCF to be dominant strategy implementable
  - Implementable in dominant strategies ⇒ implementable by a direct incentive-compatible mechanism
- We do not know
  - What types of SCF are dominant-strategy implementable

# Gibbard-Satterthwaite Thm

- Assume
  - $\mathcal{O}$ is finite and $|\mathcal{O}| \geq 3$
  - Each $o \in \mathcal{O}$ can be achieved by social choice function f() for some $\theta$

Then:

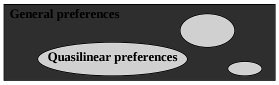
f() is truthfully implementable in dominant strategies ⟷f() is dictatorial

# Circumventing G-S

- Use a weaker equilibrium concept
  - Nash, Bayes-Nash
- Design mechanisms where computing a beneficial manipulation is hard
  - Many voting mechanisms are NP-hard to manipulate (or can be made NP-hard with small "tweaks) [Bartholdi, Tovey, Trick 89] [Conitzer, Sandholm 03]
- Randomization

Almost need this much

- Agents' preferences have special structure

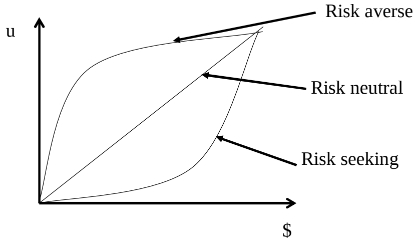


# Quasi-Linear Preferences

- Outcome: $o=(x,t_1,\ldots,t_n)$
  - x is a "project choice"
  - $t_i$ is a "monetary" transfer
- Utility of agent i:
  - $U_i(o,\theta_i)=u_i(x,\theta_i)-f(t_i)$
  - Preference of x is independent from the payment
  - Can choose to reward or punish by a monetary amount

# Quasi-linear preferences

- $U_i(o,\theta_i)=u_i(x,\theta_i)-f_i(t_i)$
- $f_i()$ gives i's risk attitude



# SCF and quasi-linear settings

- $f:\Theta \rightarrow (x(\Theta),t(\Theta))$
- SCF f is efficient if for all types $\theta=(\theta_1,\ldots,\theta_n)$
  - $\sum_{i=1}^{n} u_i(x(\theta),\theta_i) \geq \sum_{i=1}^{n} u_i(x'(\theta),\theta_i) \;\; \forall\, x'(\theta)$
  - Aka social welfare maximizing
- SCF f is budget-balanced if $\sum^{n}_{i=1} t_i(\theta)=0$
- SCF f is weakly budget-balanced if $\sum^{n}_{i=1} t_i(\theta) \geq 0$

# Mechanisms and quasi-linear utilities

- $M=(S_1,\ldots,S_n,(x(S),t(S))$
- Valuation for choice x $v_i(x)=u_i(x,\theta_i)$
- Agents reveal their valuation functions in a direct mechanism
  - $v'_i$ denotes the valuation that agent i declares to the mechanism (may be different from true valuation $v_i$)
  - $v=(v'_1,\ldots,v'_n)$

# Properties of mechanisms

- **Truthful**: $\forall$ i $\forall$ $v_i$, the equilibrium strategy for agent i is to adopt $v_i'=v_i$
- **Efficient**: Mechanism selects choice x such that $\forall$ I $\forall$ $v_i$ $\forall$ x′ $\sum_i v_i(x) \geq \sum_i v_i(x')$
- **Budget balanced**: $\forall$ v′ $\sum_i t_i(v')=0$
- **Individually rational**: $v_i(s^*(v))-t_i(s(v)) \geq 0$ where $S^*$ is the equilibrium

# Groves Mechanisms

[Groves 1973]

- A **Groves mechanism**, $M=(S_1,\ldots,S_n,(x,t_1,\ldots,t_n))$ is defined by
  - Choice rule $x^*(\theta')=\mathrm{argmax}_x \sum_i v_i(x,\theta_i')$
  - Transfer rules
    - $t_i(\theta')=h_i(\theta_{-i}')-\sum_{j\neq i} v_j(x^*(\theta'),\theta'_j)$

  where $h_i(\cdot)$ is an (arbitrary) function that **does not depend** on the reported type $\theta_i'$ of agent i

# Groves Mechanisms

- **Thm:** Groves mechanisms are strategy-proof and efficient (We have gotten around Gibbard-Satterthwaite!)
- Proof: Agent i's utility for strategy $\theta_i'$, given $\theta_{-i}'$ from agents j≠i is

  $U_i(\theta_i')=v_i(x^*(\theta'),\theta_i)-t_i(\theta')$

  $=v_i(x^*(\theta'),\theta_i)+\sum_{j\neq i} v_j(x^*(\theta'),\theta'_j)-h_i(\theta'_{-i})$

  Ignore $h_i(\theta_{-i})$. Notice that

  $x^*(\theta')=\mathrm{argmax} \sum_i v_i(x,\theta'_i)$

  i.e. it maximizes the sum of reported values.

  Therefore, agent i should announce $\theta_i'=\theta_i$ to maximize its own payoff

Thm: Groves mechanisms are unique (up to $h_i(\theta_{-i})$)

# VCG Mechanism

(aka Clarke mechanism aka Pivotal mechanism)

- Def: Implement efficient outcome,

  $$x^*=\max_x \sum_i v_i(x,\theta_i')$$

  Compute transfers

  $$t_i(\theta')=\sum_{j\neq i} v_j(x^{-i},\theta'_j) - \sum_{j\neq i} v_j(x^*, \theta_i')$$

  Where $x^{-i}=\max_x \sum_{j\neq i} v_j(x,\theta_j')$

*VCG are efficient and strategy-proof*

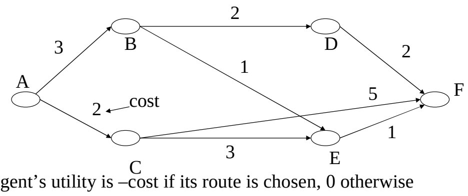
*Agent's equilibrium utility is:*

$u_i(x^*,t_i,\theta_i)=v_i(x^*,\theta_i)-[\sum_{j\neq i} v_j(x^{-i},\theta_j) - \sum_{j\neq i} v_j(x^*,\theta_j)]$

$= \sum_j v_j(x^*,\theta_j) - \sum_{j\neq i} v_j(x^*,\theta_j)$

*= marginal contribution to the welfare of the system*

# Example: Selfish Routing



Agent's utility is –cost if its route is chosen, 0 otherwise

$x(v)=\mathrm{argmax} \sum_i v_i(x) = ABEF$

Payments:
$T_{AC}=5-5=0$
$T_{CE}=5-5=0$
$T_{BD}=5-5=0$
$T_{DF}=5-5=0$
$T_{DF}=5-5=0$

Payments (Pivotal Agents):
$T_{AB}=2-6=-4$ (paid 4 for its contribution)
$T_{BE}=4-6=-2$ (paid 2 for its contribution)
$T_{EF}=4-7=-3$ (paid 3 for its contribution)

**"Market Power"**

# Example: Building a pool

- The cost of building the pool is \$300
- If together all agents value the pool more than \$300 then it will be built
- Clarke Mechanism:
  - Each agent announces their value, $v_i$
  - If $\sum v_i \geq 300$ then it is built and each pays 100
  - Payments $t_i(\theta_\iota') = \sum_{j\neq i} v_j(x^{-i}, \theta'_j) - \sum_{j\neq i} v_j(x^*, \theta_i')$ if built, 0 otherwise

v1=50, v2=50, v3=250

After each pay 100

Pool should be built

$t_1=(250+50)-(250+50)=0$
$t_2=(250+50)-(250+50)=0$
$t_3=(0)-(100)=-100$

Not budget balanced

# Example: Vickrey Auction

- Highest bidder gets item, and pays second highest amount

- Also a VCG mechanism
  - Allocation rule: get item if $b_i = \max_i[b_j]$
  - Every agent pays

$$t_i(\theta_\iota') = \sum_{j\neq i} v_j(x^{-i}, \theta'_j) - \sum_{j\neq i} v_j(x^*, \theta_i')$$

$\max_{j\neq i}[b_j]$

$\max_{j^1\, i}[b_j]$ if i is not the highest bidder, 0 if it is