Auctions

CS 886 – Multiagent Systems

Auctions

- Methods for allocating goods, tasks, resources...
- Participants: auctioneer, bidders
- Enforced agreement between auctioneer & winning bidder(s)
- Easily implementable e.g. over the Internet
  - Many existing Internet auction sites
- Auction (selling item(s)): One seller, multiple buyers
  - E.g. selling a bull on eBay

- Reverse auction (buying item(s)): One buyer, multiple sellers
  - E.g. procurement
- We will discuss the theory in the context of auctions, but same theory applies to reverse auctions
  - at least in 1-item settings

Auction settings

- *Private value* : value of the good depends only on the agent's own preferences
  - E.g. cake which is not resold or showed off
- *Common value* : agent's value of an item determined entirely by others' values
  - E.g. treasury bills
- *Correlated value* : agent's value of an item depends partly on its own preferences & partly on others' values for it
  - E.g. auctioning a transportation task when bidders can handle it or reauction it to others

Auction protocols: *All-pay*

- Protocol: Each bidder is free to raise his bid. When no bidder is willing to raise, the auction ends, and the highest bidder wins the item. All bidders have to pay their last bid
- Strategy: Series of bids as a function of agent's private value, his prior estimates of others' valuations, and past bids
- Best strategy: ?
- In private value settings it can be computed (low bids)
- Potentially long bidding process
- Variations
  - Each agent pays only part of his highest bid
  - Each agent's payment is a function of the highest bid of all agents
- E.g. CS application: tool reallocation [Lenting&Braspenning ECAI-94]

The 4 common auctions

- English auction
- First price sealed bid
- Dutch auction
- Second price, sealed bid (Vickrey)

Auction protocols: *English (first-price open-cry = ascending)*

- Protocol: Each bidder is free to raise his bid. When no bidder is willing to raise, the auction ends, and the highest bidder wins the item at the price of his bid
- Strategy: Series of bids as a function of agent's private value, his prior estimates of others' valuations, and past bids
- Best strategy: In *private value auctions*, bidder's dominant strategy is to always bid a small amount more than current highest bid, and stop when his private value price is reached
  - No counterspeculation, but long bidding process
- Variations
  - In correlated value auctions, auctioneer often increases price at a constant rate or as he thinks is appropriate
  - Open-exit: Bidder has to openly declare exit without re-entering possibility => More info to other bidders about the agent's valuation

Auction protocols: *First-price sealed-bid*

- Protocol: Each bidder submits one bid without knowing others' bids, highest bidder wins the item at the price of his bid
  - Single round of bidding
- Strategy: Bid as a function of agent's private value and his prior estimates of others' valuations
- Best strategy: No dominant strategy in general
  - Strategic underbidding & counterspeculation
  - Can determine Nash equilibrium strategies via common knowledge assumptions about the probability distributions from which valuations are drawn

Example: 1st price sealed-bid auction

2 agents (1 and 2) with values v_1,v_2 drawn uniformly from [0,1].

Utility of agent i if it bids b_i and wins the item is $u_i=v_i-b_i$.

Assume agent 2's bidding strategy is $b_2(v_2)=v_2/2$

How should 1 bid? (i.e. what is $b_1(v_1)=z$?)

$$\boxed{U_1=\int_{z=0}^{2z}(v_1-z)dz = (v_1-z)2z=2zv_1-2z^2}$$

Note: given z=b2(v2)=v2/2, 1 only wins if v2<2z

Therefore, $Max_z[2zv_1-2z^2]$ when $z=b_1(v_1)=v_1/2$

Similar argument for agent 2, assuming $b_1(v_1)=v_1/2$.
We have an equilibrium

Strategic underbidding in first-price sealed-bid auction…

- Example 2
  - 2 risk-neutral bidders: A and B
  - A knows that B's value is 0 or 100 with equal probability
  - A's value of 400 is common knowledge
  - In Nash equilibrium, B bids either 0 or 100, and A bids 100 + ε (winning more important than low price)

Auction protocols: *Dutch (descending)*

- Protocol: Auctioneer continuously lowers the price until a bidder takes the item at the current price
  - Strategically equivalent to first-price sealed-bid protocol in all auction settings
- Strategy: Bid as a function of agent's private value and his prior estimates of others' valuations
- Best strategy: No dominant strategy in general
  - Lying (down-biasing bids) & counterspeculation
  - Possible to determine Nash equilibrium strategies via common knowledge assumptions regarding the probability distributions of others' values
  - Requires multiple rounds of posting current price
- Dutch flower market, Ontario tobacco auction, Filene's basement, Waldenbooks

Dutch (Aalsmeer) flower auction

Auction protocols: *Vickrey (= second-price sealed bid)*

- Protocol: Each bidder submits one bid without knowing (!) others' bids. Highest bidder wins item at *2nd highest price*
- Strategy: Bid as a function of agent's private value & his prior estimates of others' valuations
- Best strategy: Truthful bidding (dominant strategy)
  - Strategically equivalent to English auction (private values)
  - No counterspeculation
  - Independent of others' bidding plans, operating environments, capabilities...
  - Single round of bidding
- Widely advocated for computational multiagent systems
- Old [Vickrey 1961], but not widely used among humans
- Revelation principle --- proxy bidder agents on www.ebay.com, www.webauction.com, www.onsale.com

Vickrey auction is a special case of Clarke tax mechanism

- Who pays?
  - The bidder who takes the item away from the others (makes the others worse off)
  - Others pay nothing
- How much does the winner pay?
  - The declared value that the good would have had for the others had the winner stayed home = second highest bid

Results for *private value* auctions

- Dutch strategically equivalent to first-price sealed-bid
- Risk neutral agents => Vickrey strategically equivalent to English
- All four protocols allocate item efficiently
  - (assuming no reservation price for the auctioneer)
- English & Vickrey have dominant strategies => no effort wasted in counterspeculation

- Which of the four auction mechanisms gives highest expected revenue to the seller?
  - Assuming valuations are drawn independently & agents are risk-neutral
- The four mechanisms have equal expected revenue!

Revenue equivalence ceases to hold if agents are not risk-neutral

- **Risk averse bidders:**
  - **Dutch, first-price sealed-bid ≥ Vickrey, English**
- **Risk averse auctioneer:**
  - **Dutch, first-price sealed-bid ≤ Vickrey, English**

Optimal Auctions

(Myerson)

Optimal auctions (risk-neutral, asymmetric bidders)

- Private-value auction with 2 risk-neutral bidders
  - A's valuation is uniformly distributed on [0,1]
  - B's valuation is uniformly distributed on [1,4]
- What revenue do the 4 basic auction types give?
- Can the seller get higher expected revenue?
  - Is the allocation Pareto efficient?
  - What is the worst-case revenue for the seller?
  - For the revenue-maximizing auction, see Wolfstetter's survey on class web page

Common Value Auctions

- In a common value auction, the item has some unknown value, each agent has partial information about the value

  - Examples: Art auctions and resale, construction companies effected by common events (eg weather), oil drilling

Common Value Auctions

- At time of bidding, the common value is unknown
- Bidders may have imperfect estimates about the value
- True value is only observed after the auction takes place

Winner's Curse

- No agent knows for sure the true value of the item
- The winner is the agent who made the highest guess
- If bidders all had "reasonable" information about the value of the item, then the average of all guesses should be correct
  - i.e the winner has overbid! (the curse)
- Agents should shade their bids downward (even in English and Vickrey auctions)

Results for *non-private value* auctions

- Dutch strategically equivalent to first-price sealed-bid
- Vickrey not strategically equivalent to English
- All four protocols allocate item efficiently
- Thrm (*revenue non-equivalence*). With more than 2 bidders, the expected revenues are not the same: English ≥ Vickrey ≥ Dutch = first-price sealed bid

Results for *non-private value* auctions...

- Common knowledge that auctioneer has private info
  - Q: What info should the auctioneer release ?
- A: auctioneer is best off releasing all of it
  - "No news is worst news"
  - Mitigates the winner's curse

Results for *non-private value* auctions...

- Asymmetric info among bidders
  - E.g. 1: auctioning pennies in class
  - E.g. 2: first-price sealed-bid common value auction with bidders A, B, C, D
    - A & B have same good info. C has this & extra signal. D has poor but independent info
    - A & B should not bid; D should sometimes
- => "Bid less if more bidders or your info is worse"
    - Most important in sealed-bid auctions & Dutch

Vulnerabilities in Auctions

Vulnerability to bidder collusion

[even in private-value auctions]

- v_1 = 20, v_i = 18 for others
- Collusive agreement for English: e.g. 1 bids 6, others bid 5. Self-enforcing
- Collusive agreement for Vickrey: e.g. 1 bids 20, others bid 5. Self-enforcing
- In first-price sealed-bid or Dutch, if 1 bids below 18, others are motivated to break the collusion agreement
- Need to identify coalition parties

Vulnerability to shills

- Only a problem in non-private-value settings
- English & all-pay auction protocols are vulnerable
  - Classic analyses ignore the possibility of shills
- Vickrey, first-price sealed-bid, and Dutch are not vulnerable

Vulnerability to a lying auctioneer

- Truthful auctioneer classically assumed
- In Vickrey auction, auctioneer can overstate 2nd highest bid to the winning bidder in order to increase revenue

  - Bid verification mechanisms, e.g. cryptographic signatures
  - Trusted 3rd party auction servers (reveal highest bid to seller after closing)
- In English, first-price sealed-bid, Dutch, and all-pay, auctioneer cannot lie because bids are public

Auctioneer's other possibilities

- **Bidding**
  - **Seller may bid more than his reservation price because truth-telling is not dominant for the seller even in the English or Vickrey protocol (because his bid may be 2nd highest & determine the price) => seller may inefficiently get the item**
    - In an expected revenue maximizing auction, seller sets a reservation price strategically like this [Myerson 81]
      - Auctions are not Pareto efficient (not surprising in light of Myerson-Satterthwaite theorem)
- **Setting a minimum price**
- **Refusing to sell after the auction has ended**

Undesirable private information revelation

- Agents strategic marginal cost information revealed because truthful bidding is a dominant strategy in Vickrey (and English)
  - Observed problems with subcontractors
- First-price sealed-bid & Dutch may not reveal this info as accurately
  - Lying
  - No dominant strategy
  - Bidding decisions depend on beliefs about others

Sniping

= bidding very late in the auction in the hopes that other bidders do not have time to respond

Especially an issue in electronic auctions with network lag and lossy communication links

	Hypotheses	Predicted contribution to late bidding
Strategic hypotheses	• *Rational response to naïve English auction behavior or to shill bidders:* bidders bid late to avoid bidding wars with incremental bidders. • *Collusive equilibrium:* bidders bid late to avoid bidding wars with other like-minded bidders. • *Informed bidders protecting their information*: e.g. late bidding by experts/dealers.	All three strategic hypotheses suggest more late bidding on eBay than on Amazon, with a bigger effect for more experienced bidders. Plus (via the third point) more late bidding in categories in which expertise is important than in categories in which it is not.
Non-strategic hypotheses	Bidders bid late because … • of procrastination; • search engines present soon-to-expire auctions first; • of a desire to retain flexibility to bid on other auctions offering the same item; • they remain unaware of the proxy bidding system; • of an increase in the willingness to pay over time caused by, e.g., an endowment effect; or because • bidders don't like to leave bids "hanging."	No difference between eBay and Amazon.

Table 1–Hypotheses about the causes of late bidding

[from Roth & Ockenfels]

Sniping…

Amazon auctions give automatic extensions, eBay does not

Antiques auctions have experts

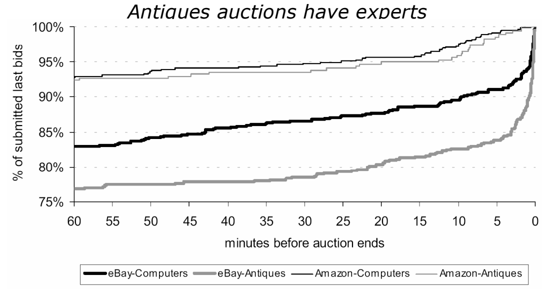


Figure 1a–Cumulative distributions over time of bidders' last bids

[from Roth & Ockenfels]

Sniping…

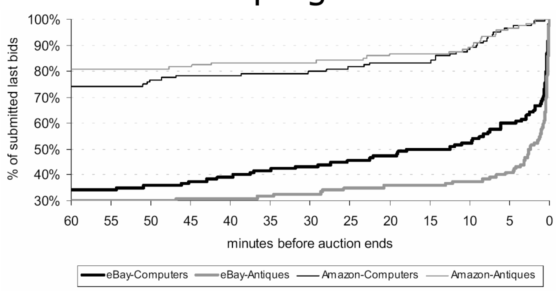


Figure 1b–Cumulative distributions over time of auctions' last bids

[from Roth & Ockenfels]

Sniping…

- Can make sense to both bid through a regular insecure channel and to snipe
- Might end up sniping oneself

Conclusions on 1-item auctions

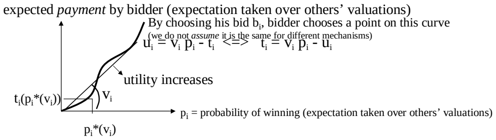
- Nontrivial, but often analyzable with reasonable effort
  - Important to understand merits & limitations
  - Unintuitive protocols may have better properties
    - Vickrey auction induces truth-telling & avoids counterspeculation (in limited settings)
- Choice of a good auction protocol depends on the setting in which the protocol is used

Revenue equivalence theorem

- Even more generally: Thrm.
  - Assume risk-neutral bidders, valuations drawn independently from potentially different distributions with no gaps
  - Consider two Bayes-Nash equilibria of *any* two auction mechanisms
  - Assume allocation probabilities $y_i(v_1, \ldots v_{|A|})$ are same in both equilibria
    - Here $v_1, \ldots v_{|A|}$ are true types, not revelations
    - E.g., if the equilibrium is efficient, then $y_i = 1$ for bidder with highest v_i
  - Assume that if any agent i draws his lowest possible valuation $\underline{v}_i$, his expected payoff is same in both equilibria
    - E.g., may want a bidder to lose & pay nothing if bidders' valuations are drawn from same distribution, and the bidder draws the lowest possible valuation
  - Then, the two equilibria give the same expected payoffs to the bidders (& thus to the seller)

Revenue equivalence theorem

Proof sketch. We show that expected payment by an arbitrary bidder i is the same in both equilibria. By revelation principle, can restrict to Bayes-Nash incentive-compatible direct revelation mechanisms. So, others' bids are identical to others' valuations.



$dt_i(p_i^*(v_i)) / dp_i^*(v_i) = v_i$ Integrate both sides from $p_i^*(\underline{v}_i)$ to $p_i^*(v_i)$:

$$t_i(p_i^*(v_i)) - t_i(p_i^*(\underline{v}_i)) = \int_{p_i^*(\underline{v}_i)}^{p_i^*(v_i)} v_i(q)\, dq = \int_{\underline{v}_i}^{v_i} s\, dp_i^*(s)$$

Since the two equilibria have the same allocation probabilities $y_i(v_1, \ldots v_{|A|})$ and every bidder reveals his type truthfully, for any realization v_i, $p_i^*(v_i)$ has to be the same in the equilibria. Thus the RHS is the same. Now, since $t_i(p_i^*(\underline{v}_i))$ is same by assumption, $t_i(p_i^*(v_i))$ is the same. QED