CS 486/686: Introduction to Artificial Intelligence

Neural Networks

Introduction

Machine learning algorithms can be viewed as approximations of functions that describe the data

In practice, the relationships between input and output can be extremely complex.

We want to:

- Design methods for learning arbitrary relationships
- Ensure that our methods are efficient and do not overfit the data

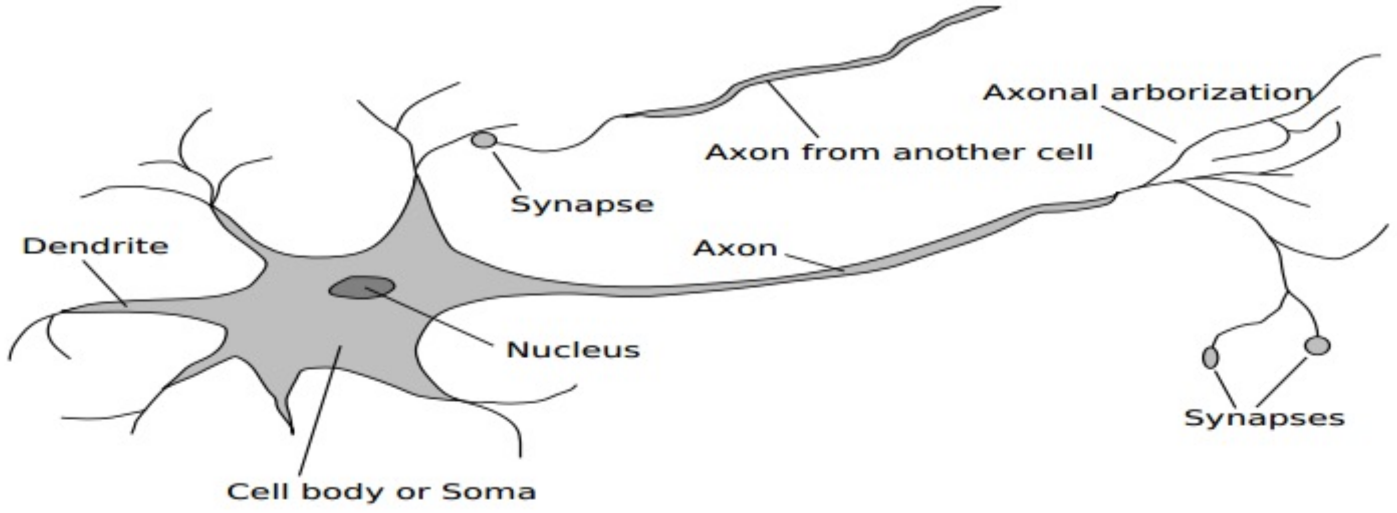
Artificial Neural Nets

Idea: The humans can often learn complex relationships very well.

Maybe we can simulate human learning?

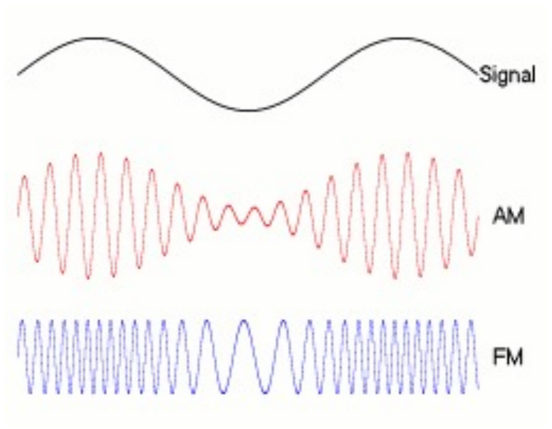
Human Brains

- A brain is a set of densely connected neurons.
- A neuron has several parts:
  - Dendrites: Receive inputs from other cells
  - Soma: Controls activity of the neuron
  - Axon: Sends output to other cells
  - Synapse: Links between neurons



Human Brains

- Neurons have two states
  - Firing, not firing
- All firings are the same



- Rate of firing communicates information (FM)
- Activation passed via chemical signals at the synapse between firing neuron's axon and receiving neuron's dendrite
- Learning causes changes in how efficiently signals transfer across specific synaptic junctions.

Artificial Brains?

Artificial Neural Networks are based on very early models of the neuron.

Better models exist today, but are usually used theoretical neuroscience, not machine learning

Artificial Brains?

An artificial Neuron (McCulloch and Pitts 1943)

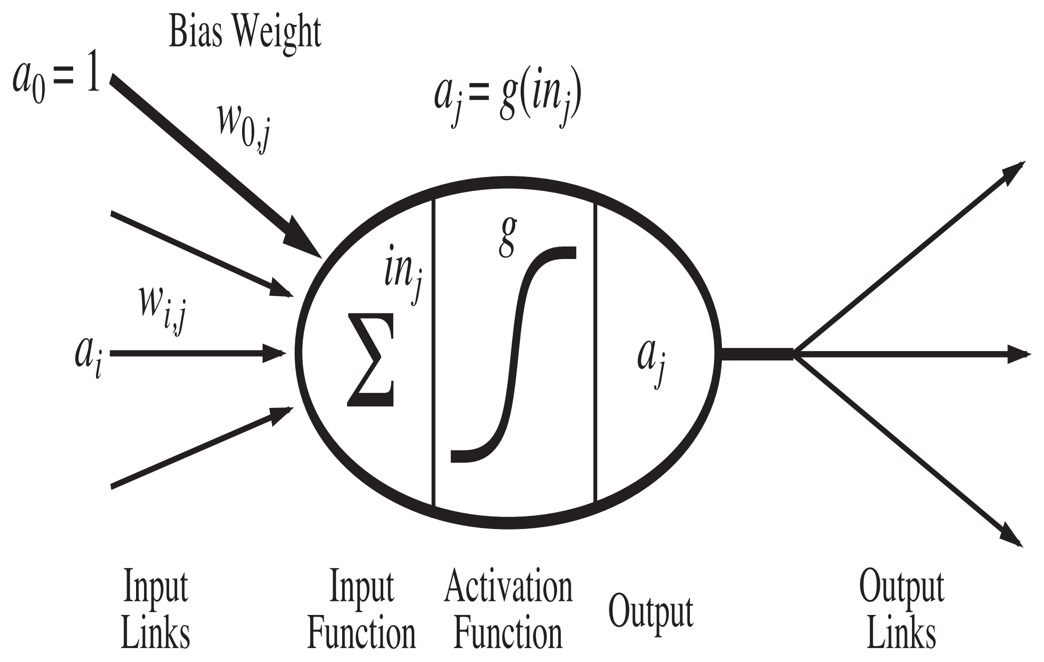
Link~ Synapse

Weight ~ Efficiency

Input Fun.~ Dendrite

Activation Fun.~ Soma

Output = Fire or not



Artificial Neural Nets

- **Collection** of simple artificial neurons.
- Weights $W_{i,j}$ denote strength of connection from i to j
- Input function: $$in_i = \sum_j W_{i,j} \times a_j$$
- Activation Function: $$a_i = g(in_i)$$

Activation Function

Activation Function: $a_i = g(in_i)$

Should be non-linear (otherwise, we just have a linear equation)

Should mimic firing in real neurons

- Active ($a_i \sim 1$) when the "right" neighbors fire the right amounts
- Inactive ($a_i \sim 0$) when fed "wrong" inputs

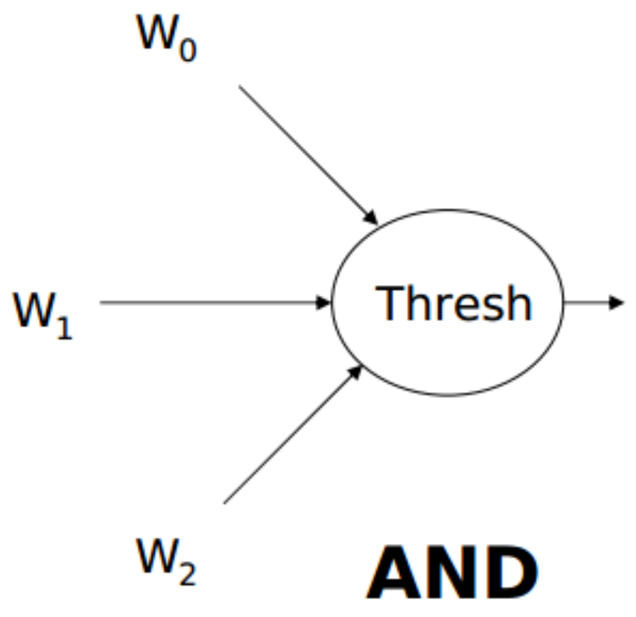
Common Activation Functions

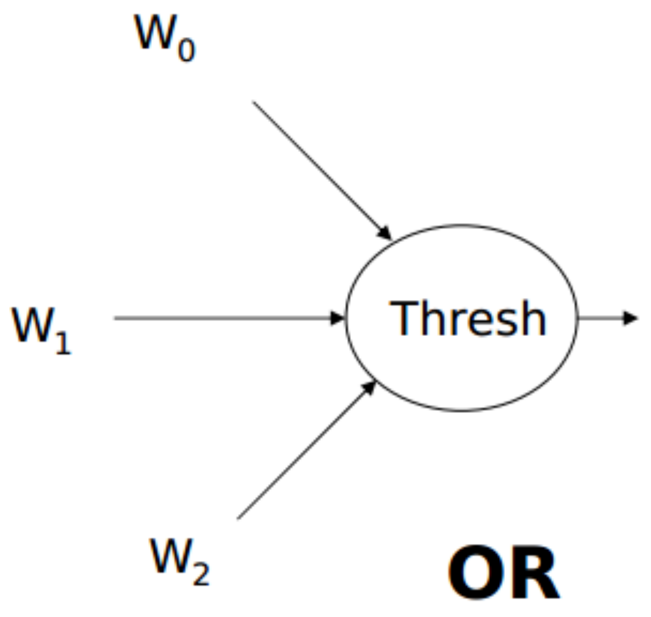
- Rectified Linear Unit (ReLU): $g(x)=\max\{0,x\}$
- Sigmoid Functions: $g(x)=1/(1+e^{x})$
- Hyperbolic Tangent: $g(x)=\tanh(x)=(e^{2x}-1)/(e^{2x}+1)$

- Threshold Function: $g(x)=1$ if $x \geq b$, 0 otherwise
  - (not really used in practice often but useful to explain concepts)

Logic Gates

It is possible to construct a universal set of logic gates using the neurons described (McCulloch and Pitts 1943)





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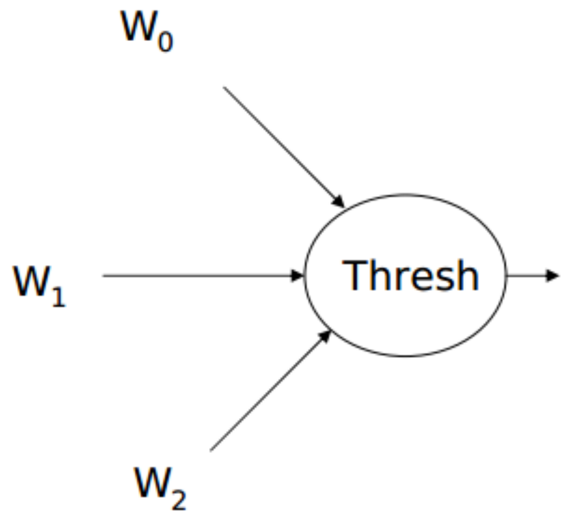


NOT

Network Structure

Feed-forward ANN

Direct acyclic graph

No internal state: maps inputs to outputs.

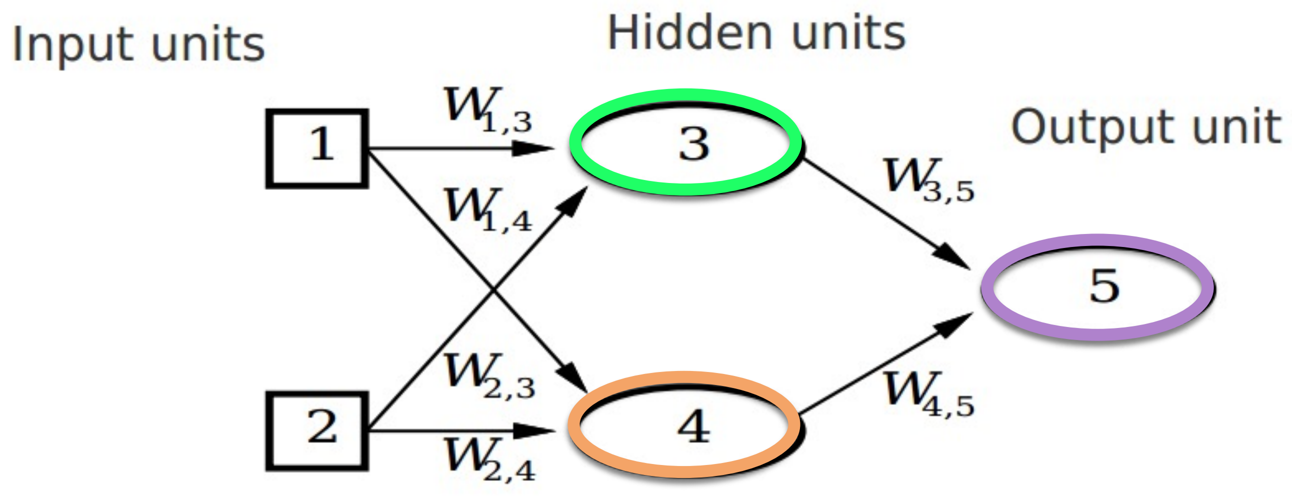
Recurrant ANN

Directed cyclic graph

Dynamical system with an internal state

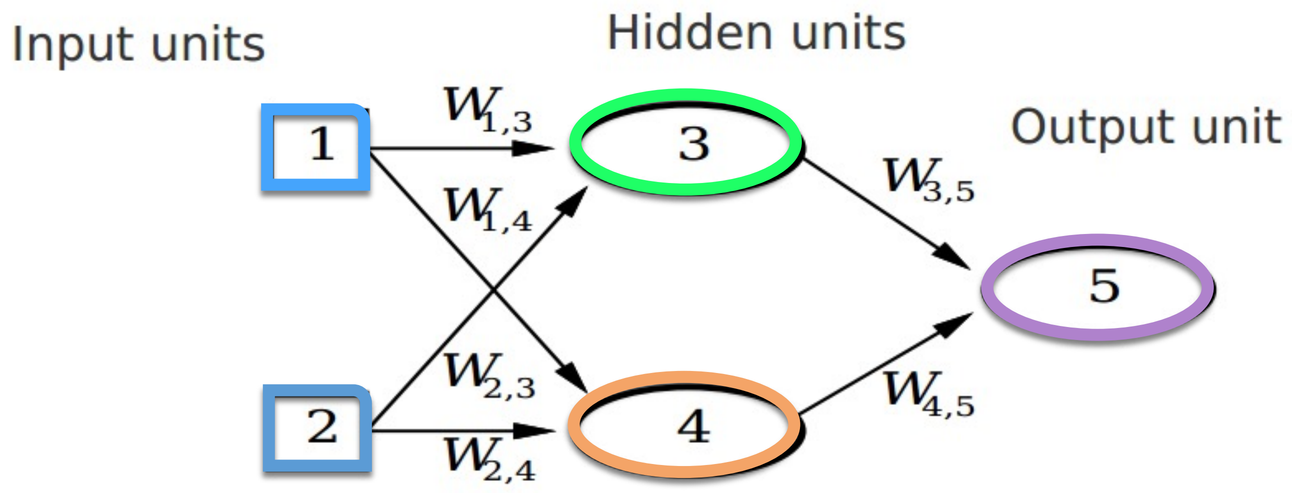
Can remember information for future use

Example



$$a_5 = g(W_{3,5} \cdot a_5 + W_{4,5} \cdot a_4)$$

Example

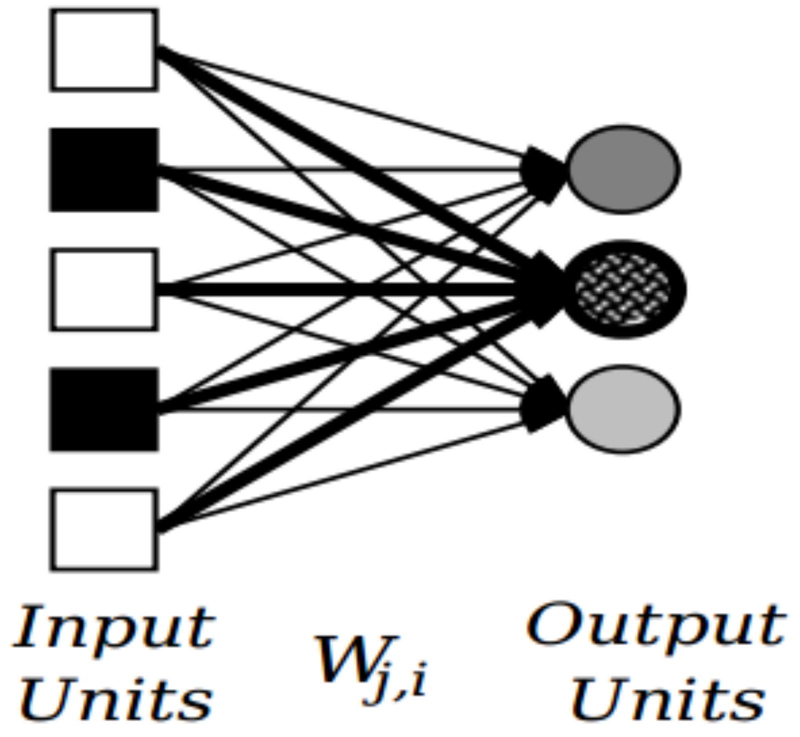


$$a_5 = g(W_{3,5} \cdot a_5 + W_{4,5} \cdot a_4)$$

$$a_5 = g(W_{3,5} \cdot g(W_{1,3} \cdot a_1 + W_{2,3} \cdot a_2) + W_{4,5} \cdot g(W_{1,4} \cdot a_1 + W_{2,4} \cdot a_2))$$

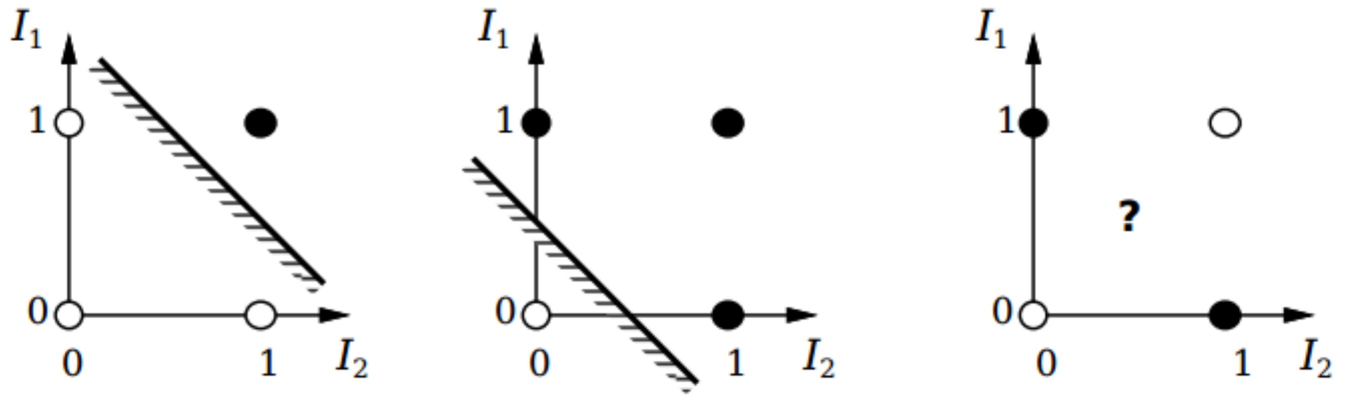
Perceptrons

Single layer feed-forward network



Perceptrons

Can learn only linear separators



(a) I_1 **and** I_2 (b) I_1 **or** I_2 (c) I_1 **xor** I_2

Training Perceptrons

Learning means adjusting the weights

- Goal: minimize loss of fidelity in our approximation of a function

How do we measure loss of fidelity?

- Often: Half the sum of squared errors of each data point

$$E = \sum_k \frac{1}{2}(y_k - (h_W(x))_k)^2$$

Learning Algorithm: Gradient Descent

Repeat for "some time"

For each example i:

$$
\begin{aligned}
I &\leftarrow \mathbf{w} \cdot \mathbf{x_i} \\
E &\leftarrow y_i - g(I) \\
W_j &\leftarrow W_j + \alpha(E \cdot g'(I) \cdot x_{i,j}) \forall j
\end{aligned}
$$

Multilayer Networks

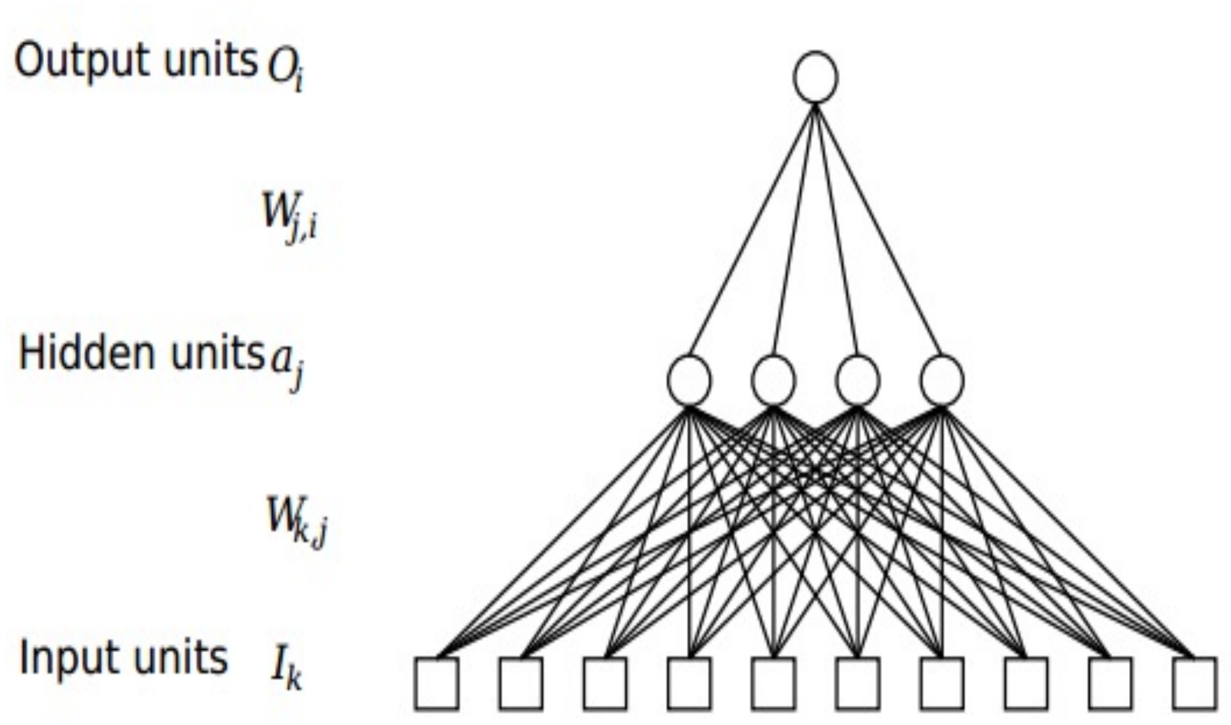
Minsky's 1969 book *Perceptrons* showed perceptrons could not learn XOR.

At the time, no one knew how to train deeper networks.

Most ANN research abandoned.

Multilayer Networks

Any continuous function can be learned by an ANN with just one hidden layer (if the layer is large enough).



XOR

Training Multilayer Nets

For weights from hidden to output layer, just use Gradient Descent, as before.

$$\Delta_i = E \cdot g'(I)$$
$$W_{j,i} = W_{j,i} + \alpha \Delta_i a_j$$

For weights from input to hidden layer, we have a problem: What is y?

$$Err = \sum_i 0.5(y_i - h_W(x_i))^2$$

Back Propagation

Idea: Each hidden layer caused *some* of the error in the output layer.

Amount of error caused should be proportionate to the connection strength.

$$\Delta_i = E \cdot g'(I)$$
$$W_{j,i} = W_{j,i} + \alpha \Delta_i a_j$$

$$\Delta_j = g'(I) \cdot \sum_i W_{j,i} \Delta_i$$
$$W_{k,j} = W_{k,j} + \alpha \Delta_j x_k$$

Back Propagation

```
Repeat for "some time":
Repeat for each example:
    Compute Deltas and weight change for output layer, and update the weights .
    Repeat until all hidden layers updated:
        Compute Deltas and weight change for the deepest hidden layer not yet updated, and update it.
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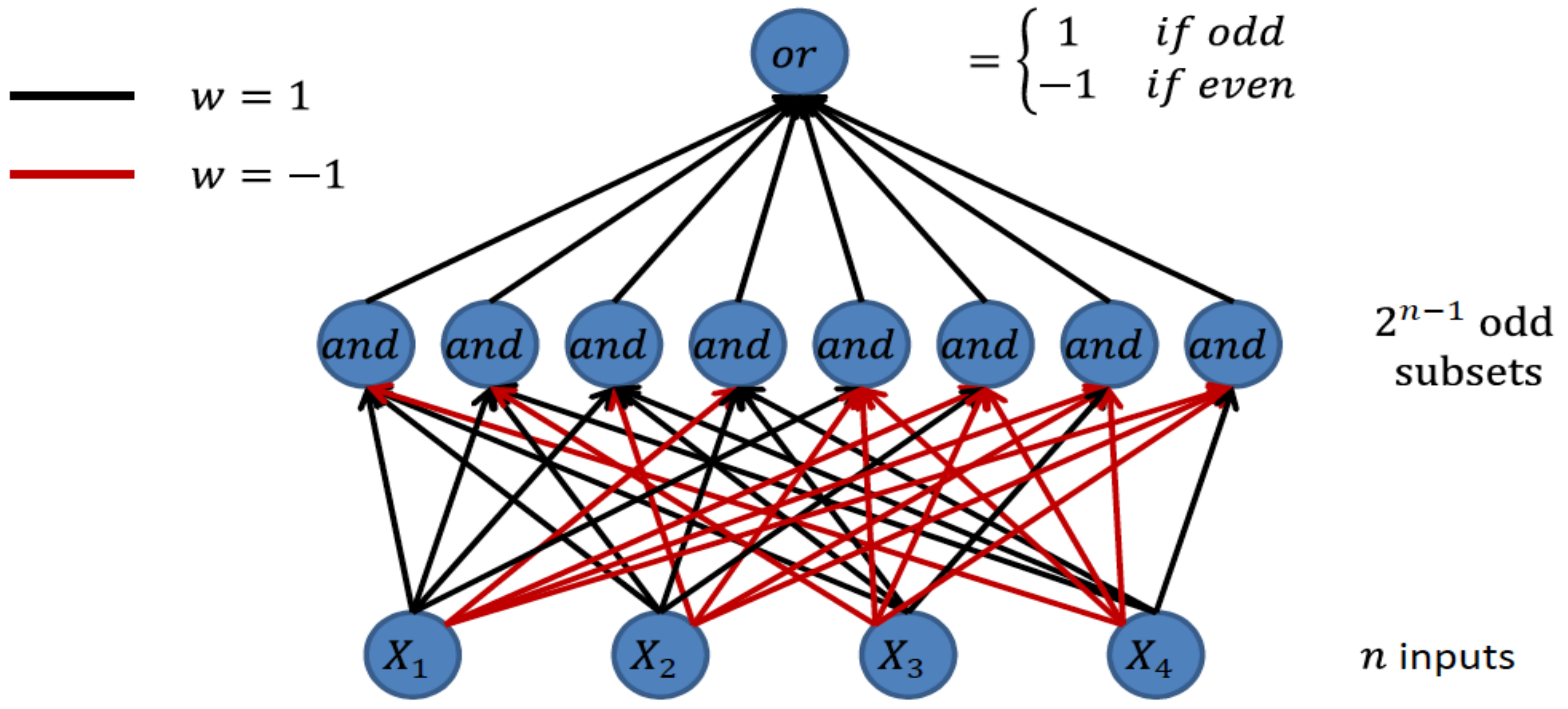
Deep Learning

Roughly “deep learning” refers to neural networks with more than one hidden layer

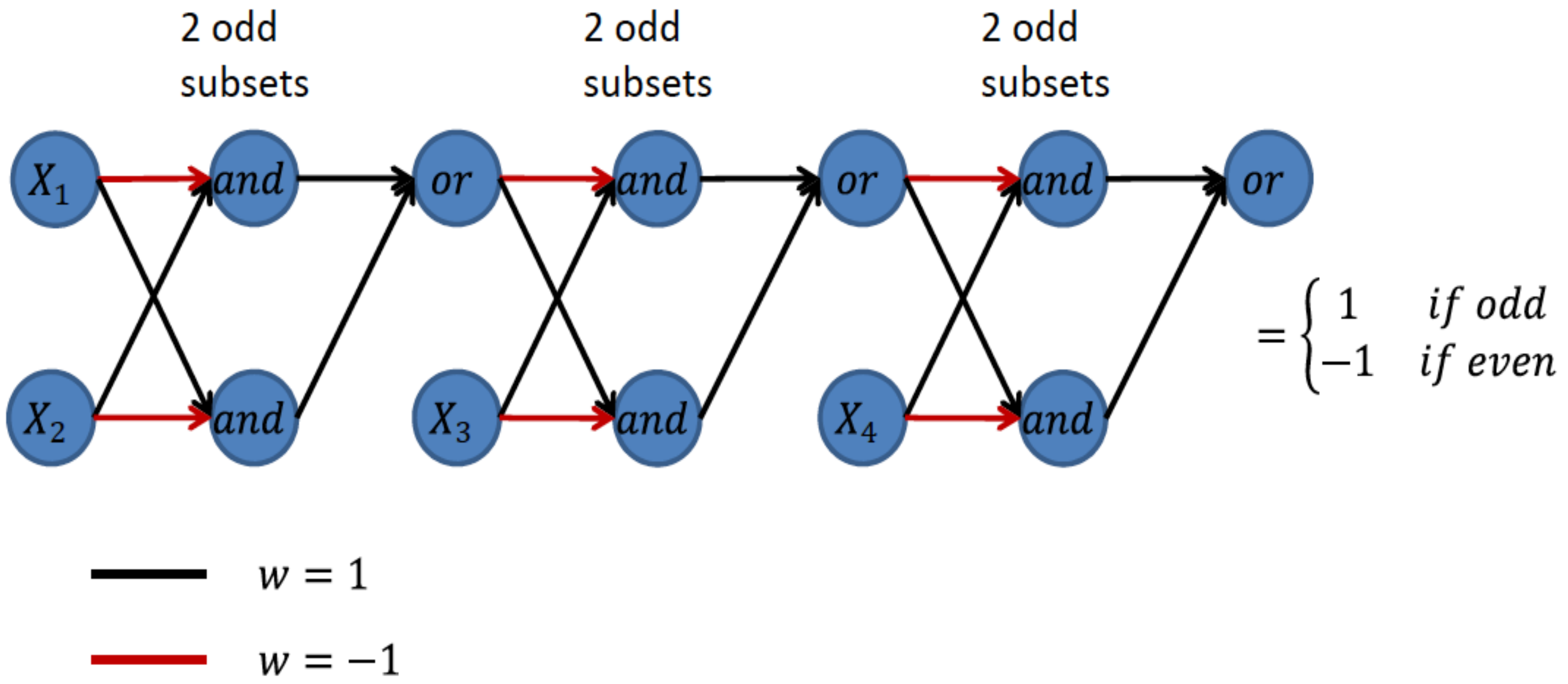
While in theory one only needs a single hidden layer to approximate any continuous function, if you use multiple layers you typically need less units

Parity Function

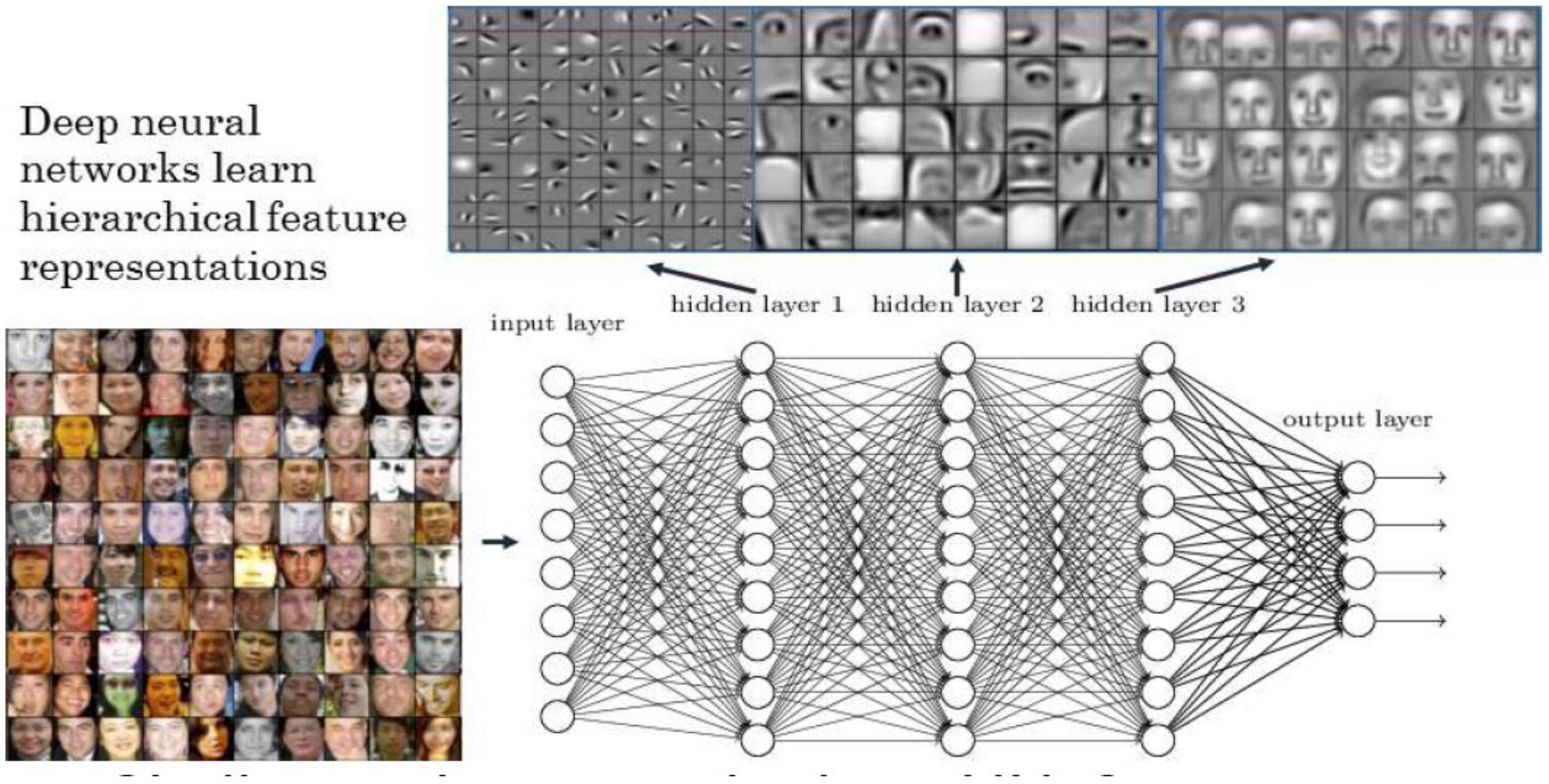


Parity Function

2n-2 hidden layers

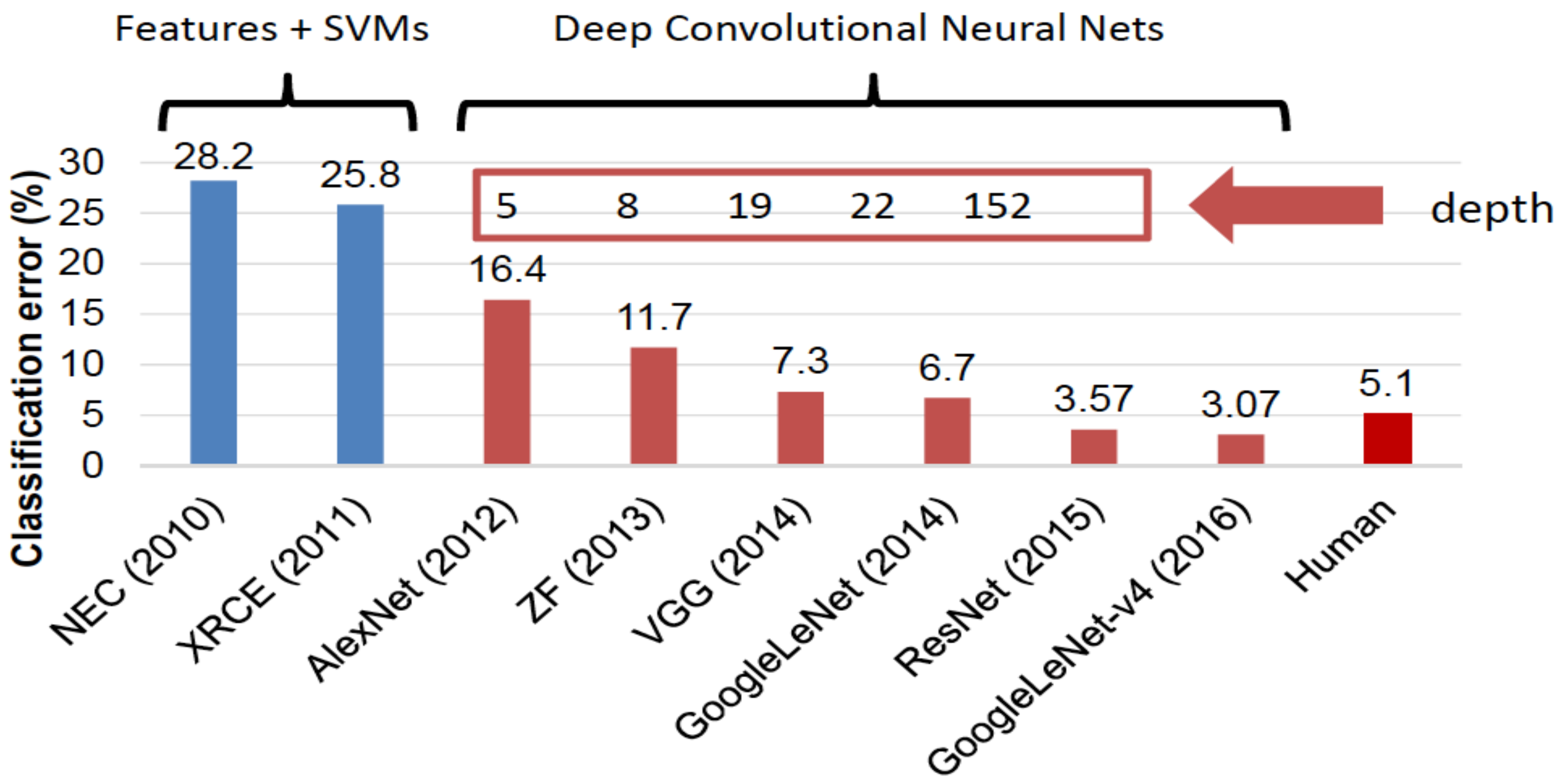


Deep Learning in Practice



How do you train them?

Image Recognition



ImageNet Large Scale Visual Recognition Challenge

When to use ANNs

When we have high dimensional or real-valued inputs, and/or noisy (e.g. sensor data)

Vector outputs needed

Form of target function is unknown (no model)

Not import for humans to be able to *understand* the mapping

Drawbacks of ANNs

Unclear how to interpret weights, especially in many-layered networks.

How deep should the network be? How many neurons are needed?

Tendency to overfit in practice (very poor predictions outside of the range of values it was trained on)