CS 486/686 Introduction to Artificial Intelligence

Hidden Markov Models

Outline

Reasoning under uncertainty over time

- Hidden Markov Models
- Dynamic Bayes Nets

Introduction

So far we have assumed

- The world does not change
- Static probability distribution

But the world does evolve over time

- How can we use probabilistic inference for weather predictions, stock market predictions, patient monitoring, robot localization,...

Dynamic Inference

To reason over time we need to consider the following:

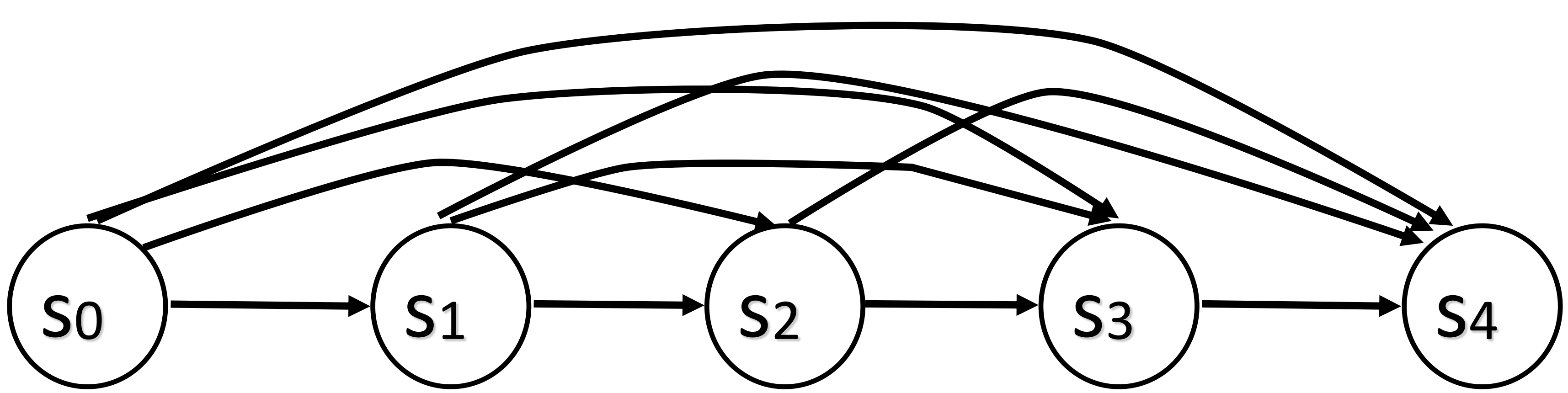
- Allow the world to evolve
- Set of states (all possible worlds)
- Set of time-slices (snapshots of the world)
- Different probability distributions over states at different time-slices
- Dynamic encoding of how distributions change over time

Stochastic Process

Set of states: **S**

Stochastic dynamics: $P(s_t | s_{t-1}, \ldots, s_0)$

Can be viewed as a Bayes Net with one random variable per time-slice



Stochastic Process

Problems:

Infinitely many variables

Infinitely large CPTs

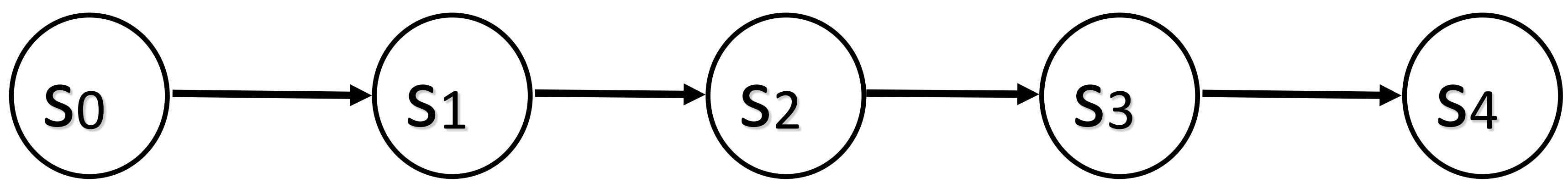
Solutions:

Stationary process: Dynamics do not change over time

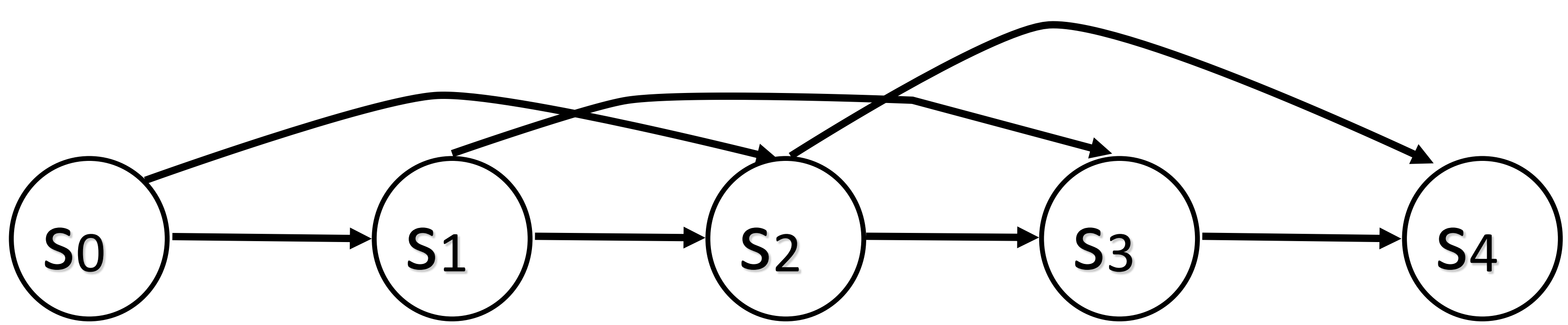
Markov assumption: Current state depends only on a finite history of past states

k-Order Markov Process

- Assumption: last k states are sufficient
- First-order Markov process
  - $P(s_t | s_{t-1}, \ldots, s_0) = P(s_t | s_{t-1})$

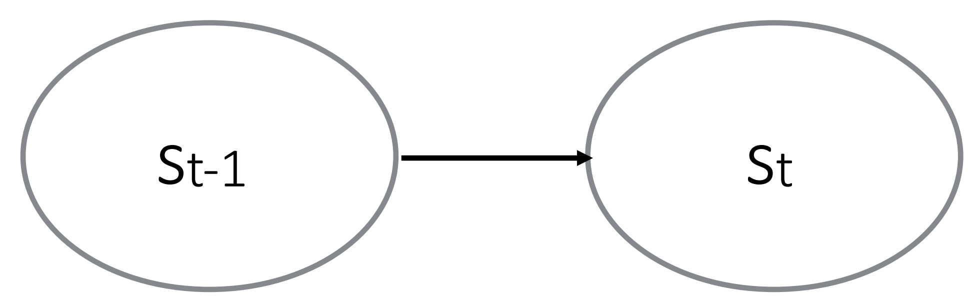


- Second-order Markov process
  - $P(s_t | s_{t-1}, \ldots, s_0) = P(s_t | s_{t-1}, s_{t-2})$



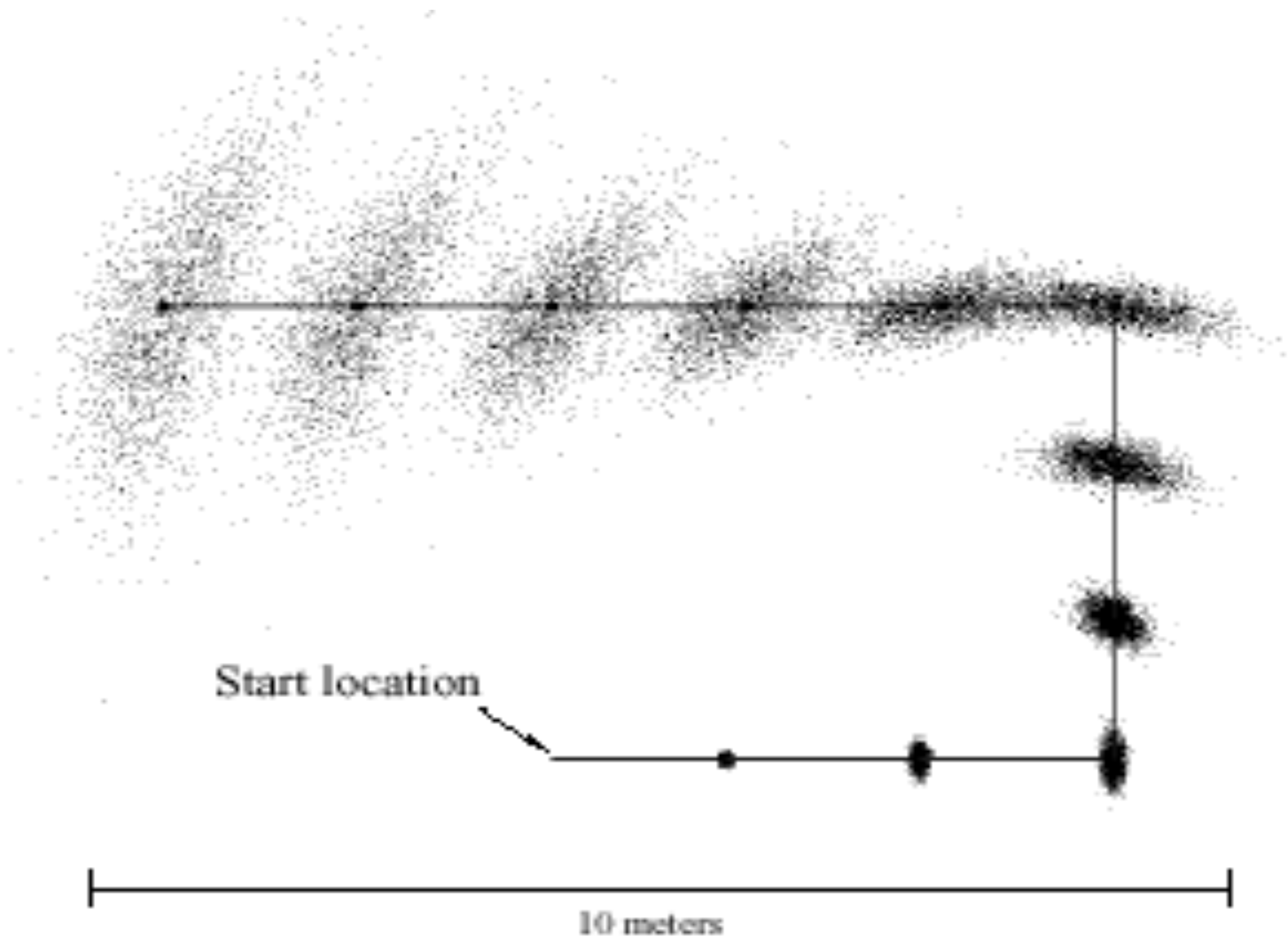
k-Order Markov Process

- Advantages
  - Can specify the entire process using finitely many time slices
- Example: Two slices sufficient for a first-order Markov process
  - Graph:
  - Dynamics: $P(s_t | s_{t-1})$
  - Prior: $P(s_0)$



Example: Robot Localization

Example of a first-order Markov process



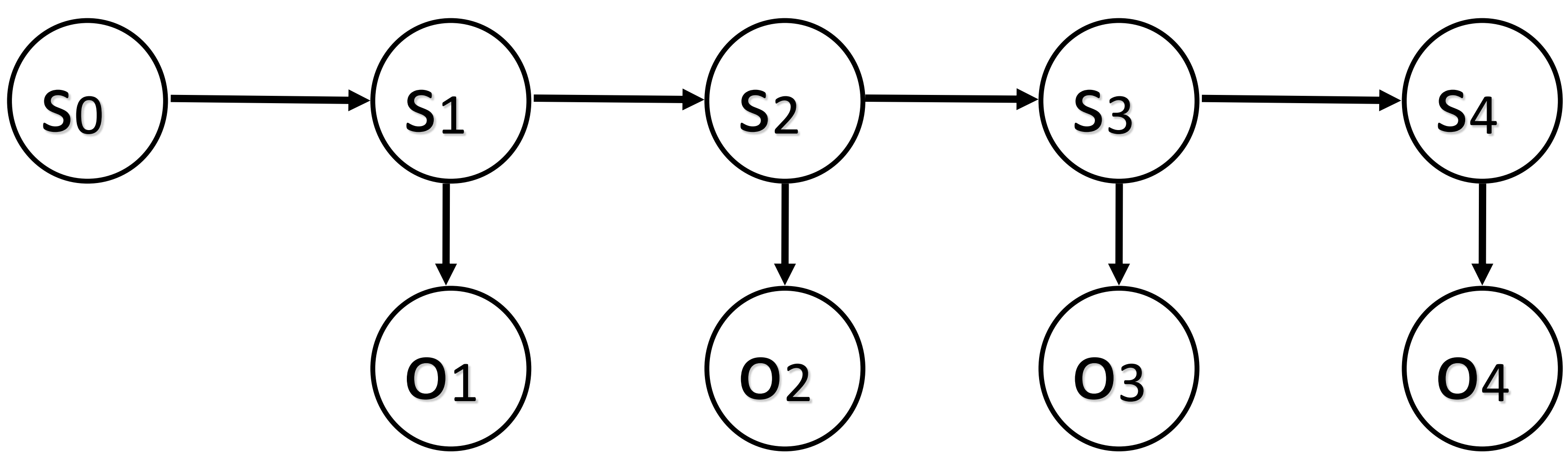
Problem: uncertainty increases over time

Hidden Markov Models

- In the previous example, the robot could use sensors to reduce location uncertainty
- In general:
  - States not directly observable (uncertainty captured by a distribution)
  - Uncertain dynamics increase state uncertainty
  - Observations: made via sensors can reduce state uncertainty
- **Solution**: Hidden Markov Model

First Order Hidden Markov Model (HMM)

- Set of states: S
- Set of observations: O
- Transition model: $P(s_t|s_{t-1})$
- **Observation model**: $P(o_t|s_t)$
- Prior: $P(s_0)$



Example: Robot Localization

- Hidden Markov Model
  - S: (x,y) coordinates of the robot on the map
  - O: distances to surrounding obstacles (measured by laser range fingers or sonar)
  - $P(s_t|s_{t-1})$: movement of the robot with uncertainty
  - $P(o_t|s_t)$: uncertainty in the measurements provided by the sensors
- **Localization** corresponds to the query:
  - $P(s_t|o_t,...,o_1)$

Inference

- There are four common tasks
  - **Monitoring**: $P(s_t | o_t,...o_1)$
  - **Prediction**: $P(s_{t+k} | o_t,...,o_1)$
  - **Hindsight**: $P(s_k | o_t,...,o_1)$
  - **Most likely explanation**: $\text{argmax}_{s_t,...,s_1} P(s_t,...,s_1 | o_t,...,o_1)$

- What algorithms should we use?
  - First 3 can be done with variable elimination and the 4th is a variant of variable elimination

Monitoring

We are interested in the distribution over current states given observations: $P(s_t|o_t,...,o_1)$

- Examples: patient monitoring, robot localization

Prediction

We are interested in distributions over future states given observations: $P(s_{t+k} | o_t, ..., o_1)$

- Examples: weather prediction, stock market prediction

Hindsight

Interested in the distribution over a past state given observations

- Example: crime scene investigation

Most Likely Explanation

We are interested in the most likely sequence of states given the observations: $\text{argmax}_{s0,\ldots st}$ $P(s_0,\ldots,s_t|o_t,\ldots,o_1)$

- Example: speech recognition

Viterbi algorithm:

Complexity of Temporal Inference

Hidden Markov Models are Bayes Nets with a ***polytree structure***!

Variable elimination is

- Linear with respect to number of time slices
- Linear with respect to largest CPT ($P(s_t|s_{t-1})$ or $P(o_t|s_t)$)

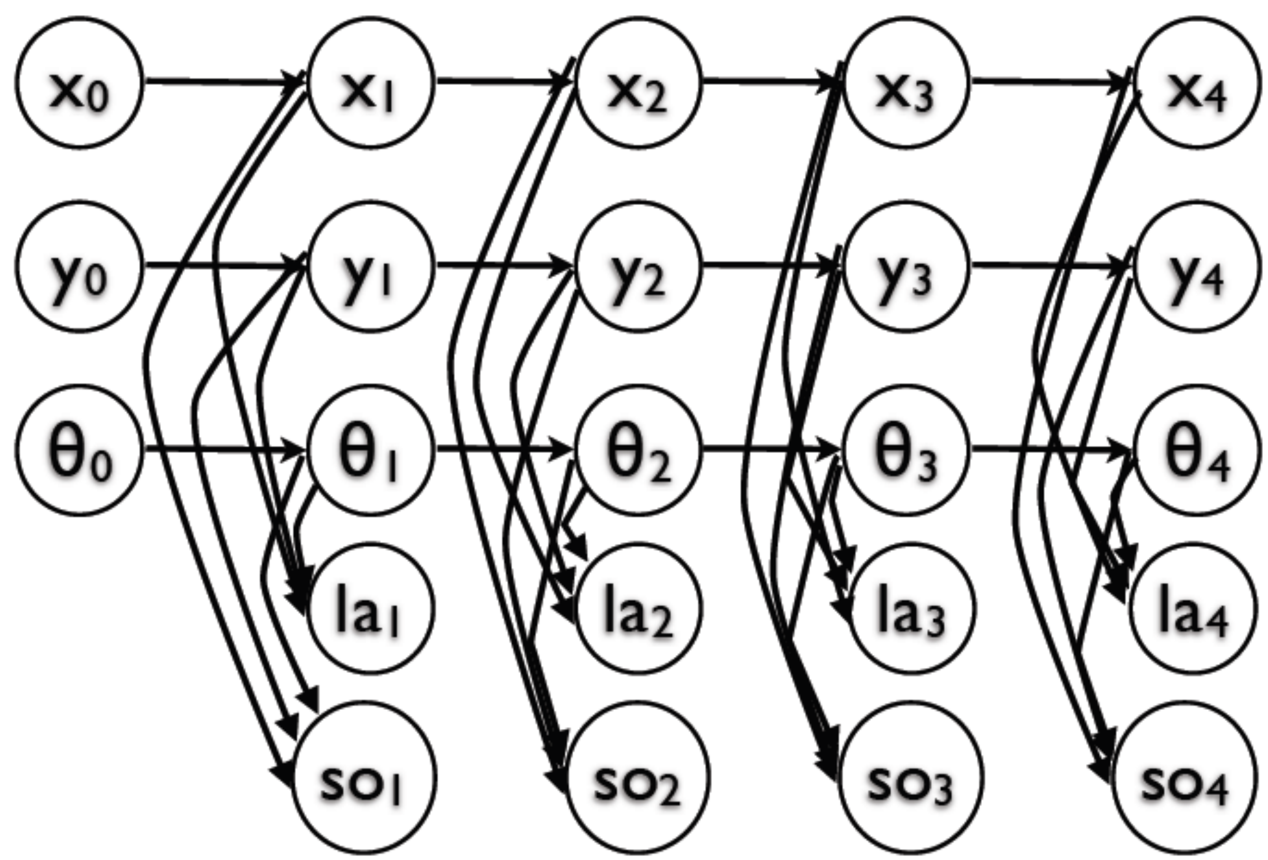
Dynamic Bayes Nets

What if the number of states or observations are exponential?

- Dynamic Bayes Nets
  - **Idea**: Encode states and observations with several random variables
  - **Advantage**: Exploit conditional independence and save time and space
  - **Note**: HMMs are just DBNs with one state variable and one observation variable

Example: Robot Localization

- **States**: (x,y) coordinates and heading θ
- **Observations**: laser and sonar readings, la and so



DBN Complexity

Conditional independence allows us to **represent** the transition and observation models very compactly!

- Time and space complexity of inference: conditional independence rarely helps
  - Inference tends to be exponential in the number of state variables
  - Intuition: All state variables eventually get correlated
  - No better than with HMMs

Non-Stationary Processes

What if the process is not stationary?

- **Solution:** Add new state components until dynamics are stationary
- **Example:** Robot navigation based on (x,y,θ) is nonstationary when velocity varies
  - **Solution:** Add velocity to state description (x,y,v,θ)
  - If velocity varies, then add acceleration,...

Non-Markovian Processes

What if the process is not Markovian?

- **Solution**: Add new state components until the dynamics are Markovian
- **Example**: Robot navigation based on (x,y,θ) is non-Markovian when influenced by battery level
  - **Solution**: Add battery level to state description (x,y,θ,b)

Markovian Stationary Processes

Problem: Adding components to the state description to force a process to be Markovian and stationary may **significantly** increase computational complexity

Solution: Try to find the smallest description that is self-sufficient (i.e. Markovian and stationary)

Summary

- Stochastic Process
  - Stationary
  - Markov assumption
- Hidden Markov Process
  - Prediction
  - Monitoring
  - Hindsight
  - Most likely explanation
- Dynamic Bayes Nets
- What to do if the stationary or Markov assumptions do not hold