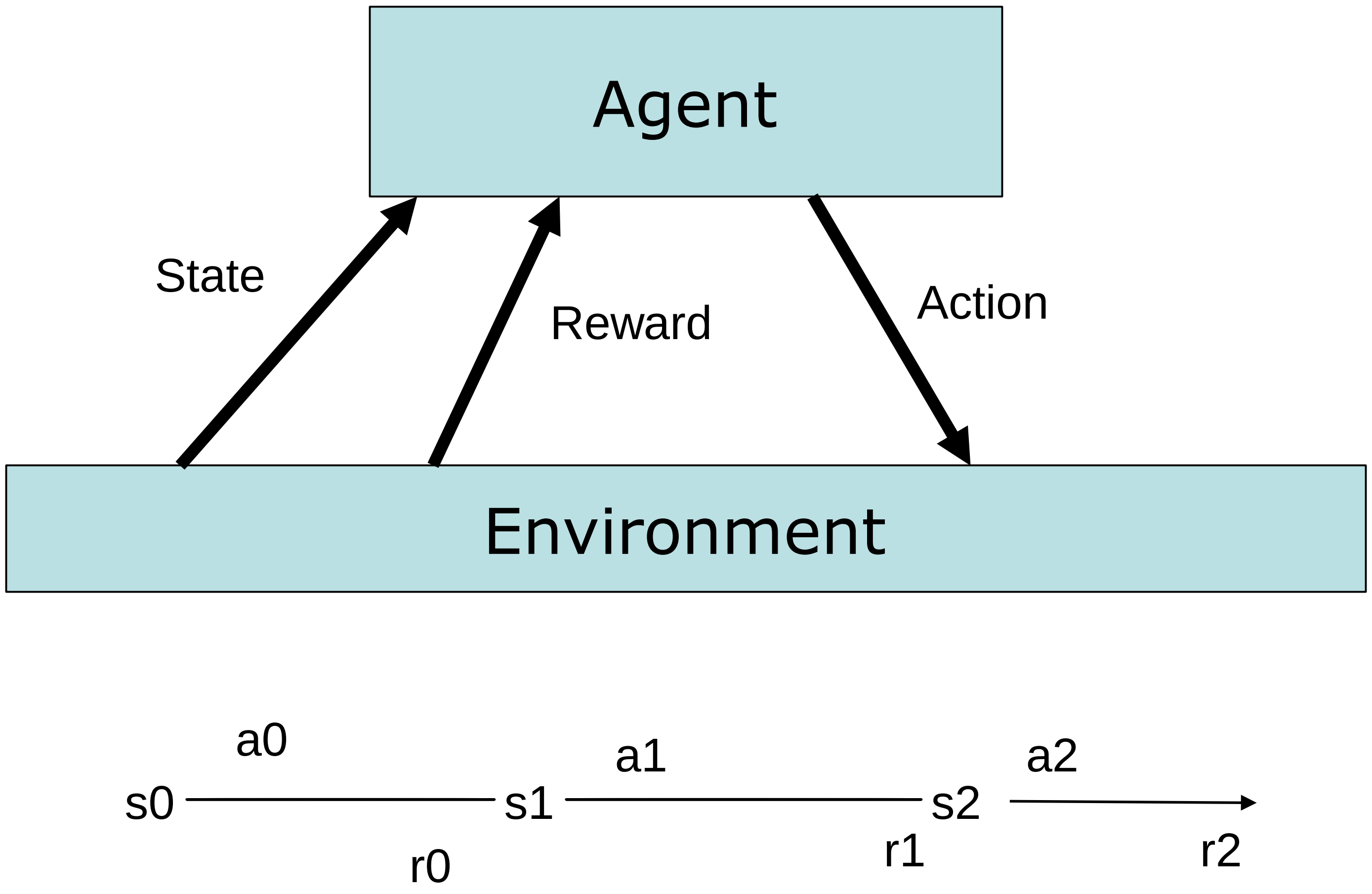
Introduction to Artificial Intelligence

CS 486/686

University of Waterloo

Introduction



Goal: Learn to choose actions that maximize $r_0+\gamma\, r_1+\gamma^2 r_2+\ldots$, where $0\cdot<\gamma<1$

Reinforcement Learning Characteristics

Delayed Reward

- Credit assignment problem

Exploration and exploitation

Possibility that a state is only partially observable

Life-long learning

RL Model

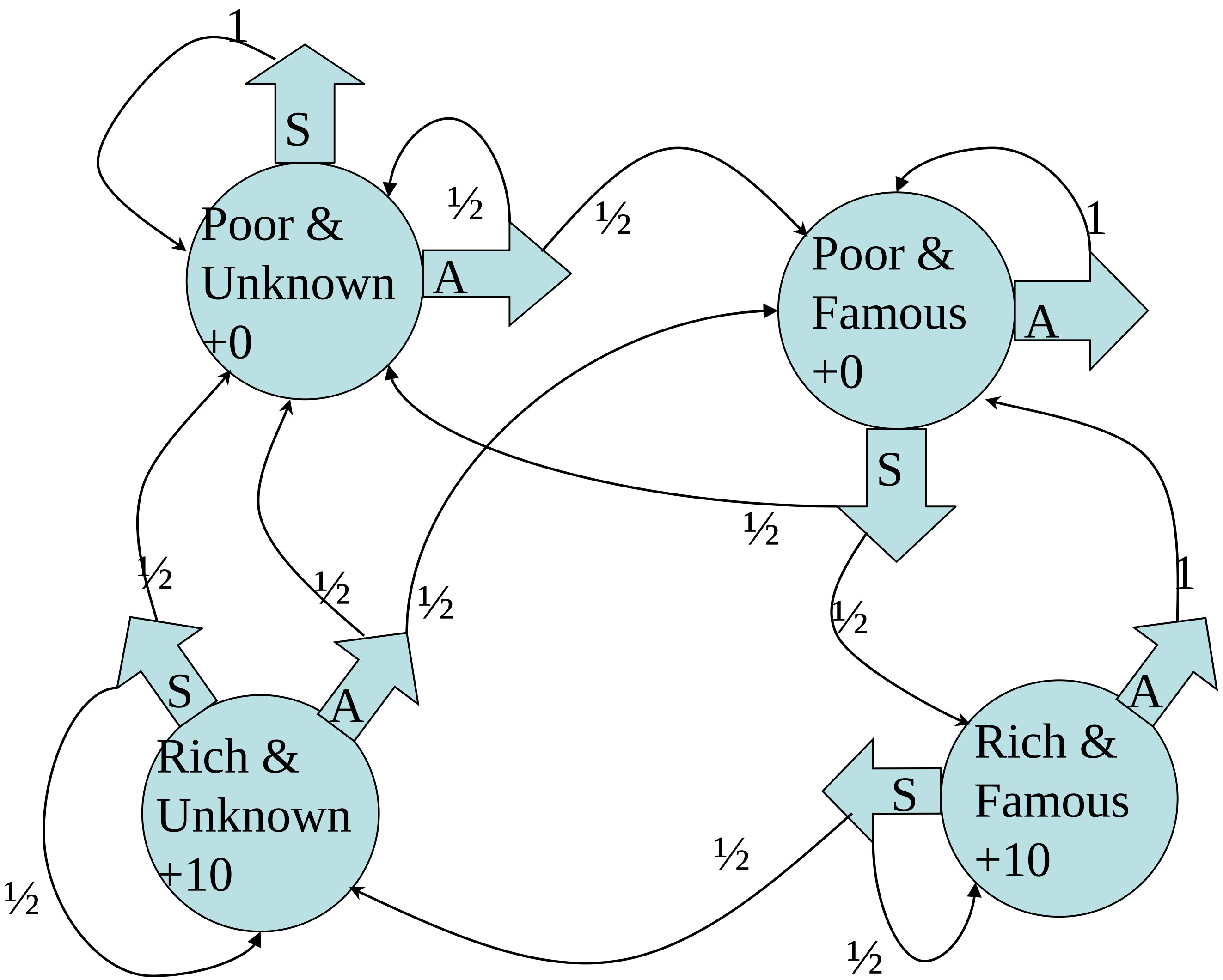
Set of States, S

Set of Actions, A

Set of reward signals, R

- Rewards might be delayed

Markov Decision Process



MDPs and RL

If we were given the MDP then we could compute the optimal policy

$$\pi^*(S_i) = \arg\max_a \left[R_i + \gamma \sum_{j=1}^{n} P(S_j | S_i, a) V^*(S_j) \right]$$

$$V^*(S_i) = R_i + \gamma \sum_{j=1}^{n} P(S_j | S_i, \pi^*(S_i)) V^*(S_j)$$

In RL we are not given the model (rewards/transition probabilities)

Prediction problem: learn V*'s or V^π

Control problem: learn π*

All RL methods are driven by values

- Recall the discounted sum of future rewards:

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \gamma^3 R_{t+4} + \ldots$$

- Value of a state, given a policy π

$$V^{\pi}(s) = \mathbb{E}\{G_t | S_t = s A_{t:\infty} \sim \pi\}$$

- Value of a state-action pair, given a policy π

$$Q^{\pi}(s, a) = \mathbb{E}\{G_t | S_t = s, A_t = a, A_{t+1:\infty} \sim \pi\}$$

All RL methods are driven by values

Optimal value of a state

$$V^*(s) \max_{\pi} V^{\pi}(s)$$

Optimal value of a state-action pair

$$Q^*(s,a) = \max_{\pi} Q^{\pi}(s,a)$$

Optimal policy: π is an optimal policy if and only if

$$\pi^*(s,a) > 0 \text{ only where } Q^*(s,a) = \max_{b} Q^*(s,b) \forall s \in S$$

That is, π* is optimal iff it is greedy with respect to Q*

4 Value Functions

	State Values	**Action Values**
Prediction Problem	V^{π}	Q^{π}
Control Problem	V^{*}	Q^{*}

These are theoretical objects, and are distinct from their estimates V^{t} and Q^{t}

Bellman Optimality Equations

$$\begin{aligned}
V^*(s) &= \max_a Q^{\pi^*}(s,a) \\
&= \max_a \mathbb{E}[R_{t+1} + \gamma V^*(s_{t+1}|S_t = s, A_t = a] \\
&= \max_a \sum_{s',r'} p(s',r|s,a)[r + \gamma V^*(s')]
\end{aligned}$$

$$\begin{aligned}
Q^*(s,a) &= \mathbb{E}[R_{t+1} + \gamma \max_{a'} Q^*(S_{t+1},a')|S_t = s, A_t = a] \\
&= \sum_{s',r} p(s',r|s,a)[r + \gamma \max_{a'} Q^*(s',a')]
\end{aligned}$$

Another way of distinguishing RL methods

- **Model-Based**
  - Learn the model of the environment (i.e. when done, we know the underlying MDP)

- **Model-Free**
  - Never explicitly learn the model (i.e. we never track probability with which we transition between states)

RL and Prediction

- Let’s consider a simple problem

3	r	r	r	+1
2	u		u	-1
1	u	l	l	l
	1	2	3	4

$\gamma = 1$

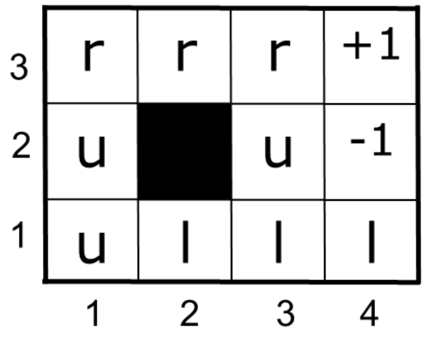
r_i = -0.04 for non-terminal states

We do not know the transition probabilities

$(1,1) \rightarrow (1,2) \rightarrow (1,3) \rightarrow (1,2) \rightarrow (1,3) \rightarrow (2,3) \rightarrow (3,3) \rightarrow (4,3)_{+1}$
$(1,1) \rightarrow (1,2) \rightarrow (1,3) \rightarrow (2,3) \rightarrow (3,3) \rightarrow (3,2) \rightarrow (3,3) \rightarrow (4,3)_{+1}$
$(1,1) \rightarrow (2,1) \rightarrow (3,1) \rightarrow (3,2) \rightarrow (4,2)_{-1}$

What is the value, $V^{\pi}(s)$ of being in state s?

RL and Prediction: Value Estimation



$\gamma = 1$

r_i = -0.04 for non-terminal states

$(1,1) \rightarrow (1,2) \rightarrow (1,3) \rightarrow (1,2) \rightarrow (1,3) \rightarrow (2,3) \rightarrow (3,3) \rightarrow (4,3)_{+1}$

$(1,1) \rightarrow (1,2) \rightarrow (1,3) \rightarrow (2,3) \rightarrow (3,3) \rightarrow (3,2) \rightarrow (3,3) \rightarrow (4,3)_{+1}$

$(1,1) \rightarrow (2,1) \rightarrow (3,1) \rightarrow (3,2) \rightarrow (4,2)_{-1}$

What is the value, $V^*(s)$ of being in state s?

$$V^{\pi}(S) = E\left[\sum_{t=0}^{\infty} \gamma^t R(S_t)\right]$$

Asynchronous Dynamic Programming

$\gamma = 1$

3	**r**	r	r	+1
2	u		u	-1
1	u	l	l	l
	1	2	3	4

r_i = -0.04 for non-terminal states

$$V^{\pi}(s_i) = r(s_i) + \gamma \sum_j P^{\pi}_{ij} V^{\pi}(s_j)$$

(1,1)→(1,2)→ **(1,3)** → (1,2)→**(1,3)**→ (2,3)→(3,3)→$(4,3)_{+1}$
(1,1)→(1,2)→**(1,3)**→(2,3)→(3,3)→(3,2)→(3,3)→$(4,3)_{+1}$
(1,1)→(2,1)→(3,1)→(3,2)→$(4,2)_{-1}$

$$\left.\begin{array}{l} P_{(1,3)(2,3)}{}^{r}=2/3 \\ P_{(1,3)(1,2)}{}^{r}=1/3 \end{array}\right\}$$ Use this information in the Bellman equation

Prediction: Temporal Difference (TD)

- TD is considered to be a bootstrapping and sampling method

- Bootsrapping: update involves an estimate of the value function
  - TD and dynamic programming boostrap
  - Direct utility estimation (a variant of a Monte Carlo method) did not

- Sampling: update does not involve an expected value
  - TD and direct utility estimation samples
  - Dynamic programming does not sample

Prediction: The Simplest TD Method TD(0)

- The simplest temporal-difference method TD(0):

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[\underbrace{R_{t+1} + \gamma V(S_{t+1})}_{\textbf{target}\text{: an estimate of the return}} - V(S_t)\Big]$$

Tabular TD(0) for estimating v_π

Input: the policy π to be evaluated
Initialize $V(s)$ arbitrarily (e.g., $V(s) = 0$, for all $s \in \mathcal{S}^+$)
Repeat (for each episode):
    Initialize S
    Repeat (for each step of episode):
        $A \leftarrow$ action given by π for S
        Take action A, observe R, S'
        $V(S) \leftarrow V(S) + \alpha\big[R + \gamma V(S') - V(S)\big]$
        $S \leftarrow S'$
    until S is terminal

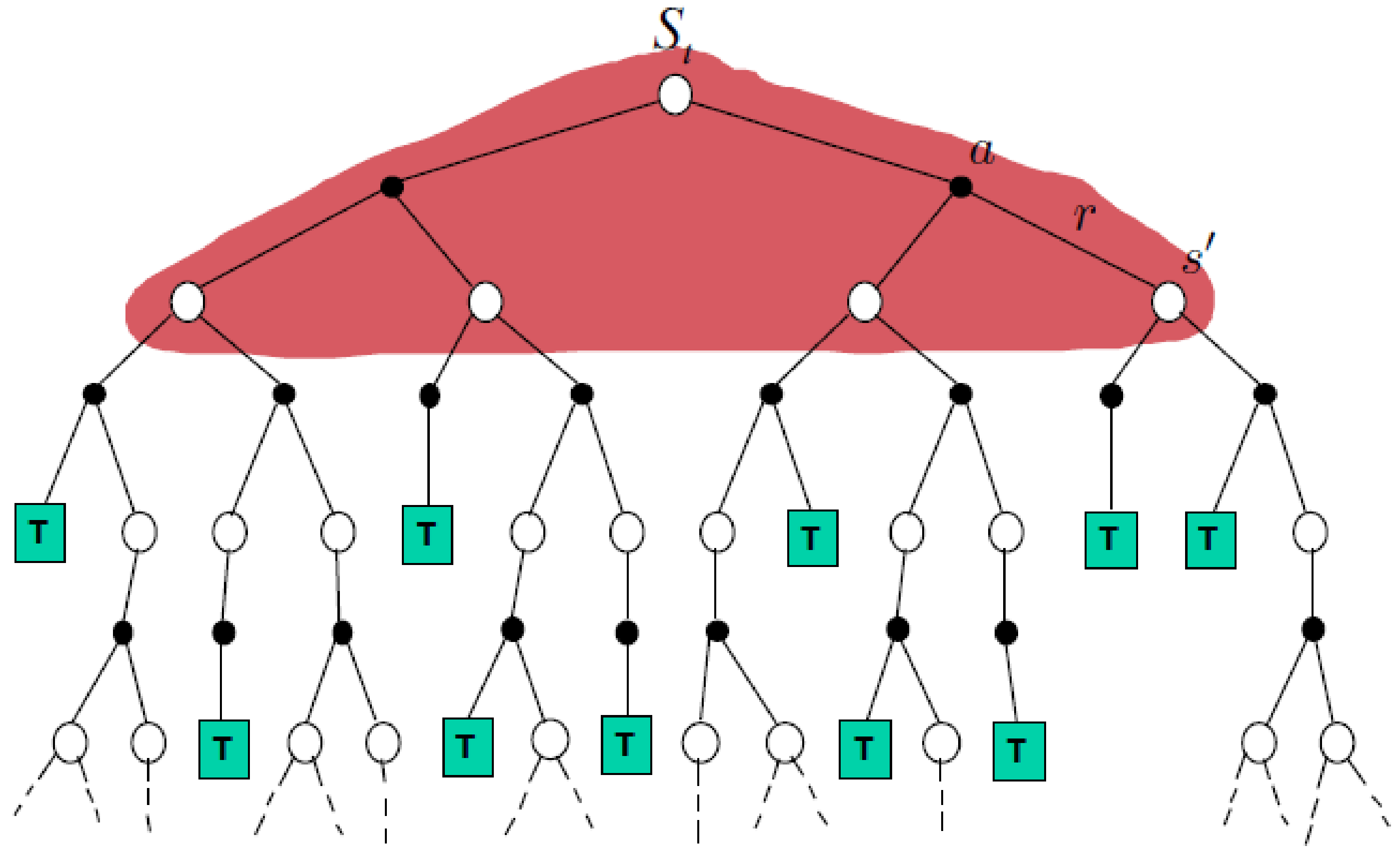
Agent program

Environment program

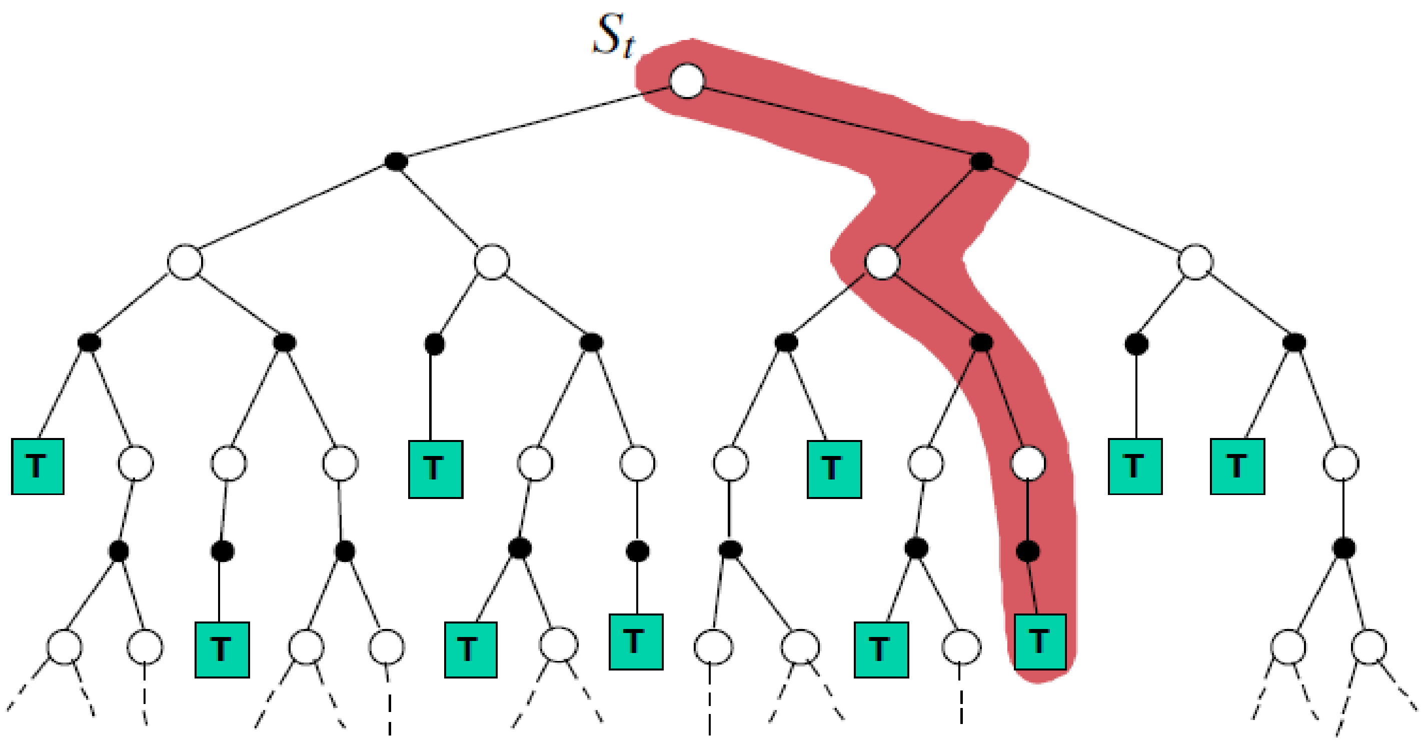
Experiment program

Dynamic Programming

$$V(S_t) \leftarrow E_\pi\left[R_{t+1} + \gamma V(S_{t+1})\right] = \sum_a \pi(a|S_t) \sum_{s',r} p(s',r|S_t,a)[r + \gamma V(s')]$$

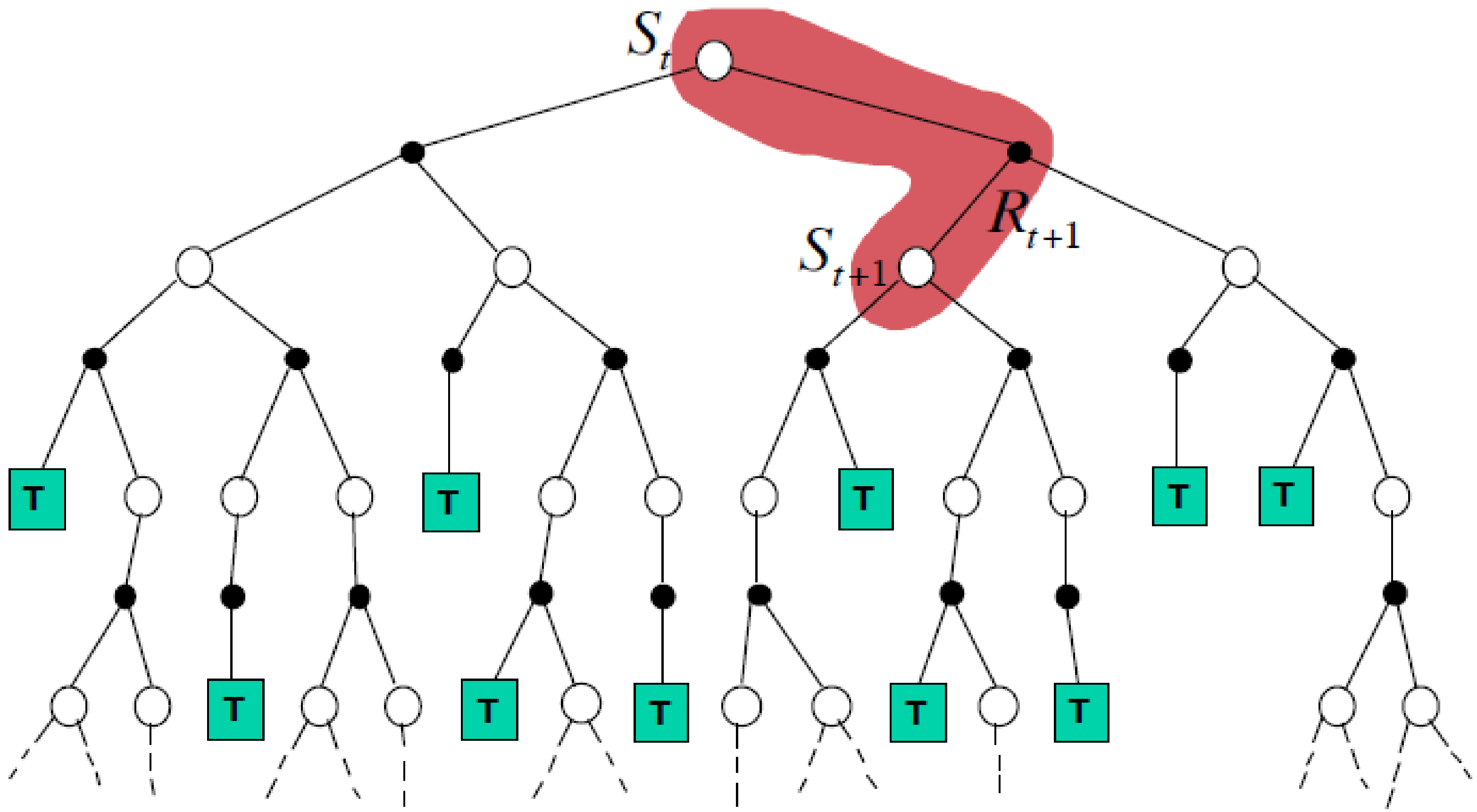


Direct Utility Estimation (and other MC variants)



TD(0)

$$V(S_t) \leftarrow V(S_t) + \alpha\left[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\right]$$



Example: Driving Home (Sutton and Barto)

Driving home:

- Each day you drive home
- Your goal is to try and predict how long your commute will take at particular stages
- When you leave your office you note the time, day, and other relevant information

Policy Evaluation or prediction task

Example: Driving Home

State	*Elapsed Time (minutes)*	*Predicted Time to Go*	*Predicted Total Time*
leaving office, friday at 6	0	30	30
reach car, raining	5	35	40
exiting highway	20	15	35
2ndary road, behind truck	30	10	40
entering home street	40	3	43
arrive home	43	0	43

Example: Driving Home

Rewards = 1 per step

Discount = 1

Gt = time to go from state St

V(St)= expected time to get home from S_t

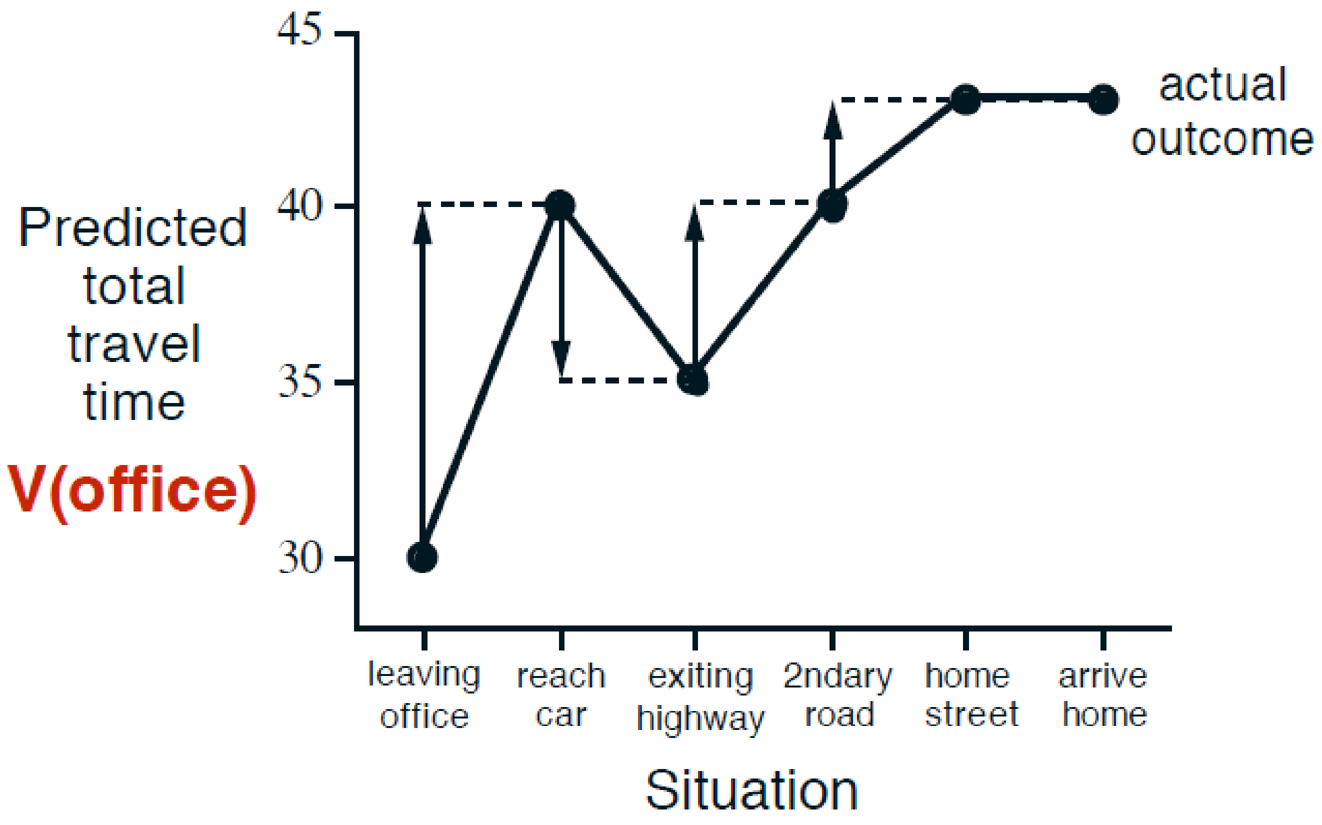
Goal: update the prediction of total time leaving from office, while driving home

Driving Home

State	*Elapsed Time (minutes)*	R	V(s) *Predicted Time to Go*	V(office) *Predicted Total Time*
leaving office, friday at 6	0	5	30	30
reach car, raining	5	15	35	40
exiting highway	20	10	15	35
2ndary road, behind truck	30	10	10	40
entering home street	40	3	3	43
arrive home	43		0	43

- Task: update the value function as we go, based on observed elapsed time—Reward column

Changes Recommended by TD(0) (alpha=1)



TD(λ)

- **Idea**: You can update from the whole training sequence not just a single transition

$$V^{\pi}(s_i) \rightarrow V^{\pi}(s_i) + \alpha \sum_{m=i}^{\infty} \lambda^{m-i} [r(s_m) + \gamma V^{\pi}(s_{m+1}) - V^{\pi}(s_m)]$$