Reasoning Under Uncertainty Over Time

CS 486/686: Introduction to Artificial Intelligence

Outline

Reasoning under uncertainty over time

- Hidden Markov Models
- Dynamic Bayes Nets

Introduction

So far we have assumed

The world does not change

Static probability distribution

But the world does evolve over time

How can we use probabilistic inference for weather predictions, stock market predictions, patient monitoring, robot localization,...

Dynamic Inference

To reason over time we need to consider the following:

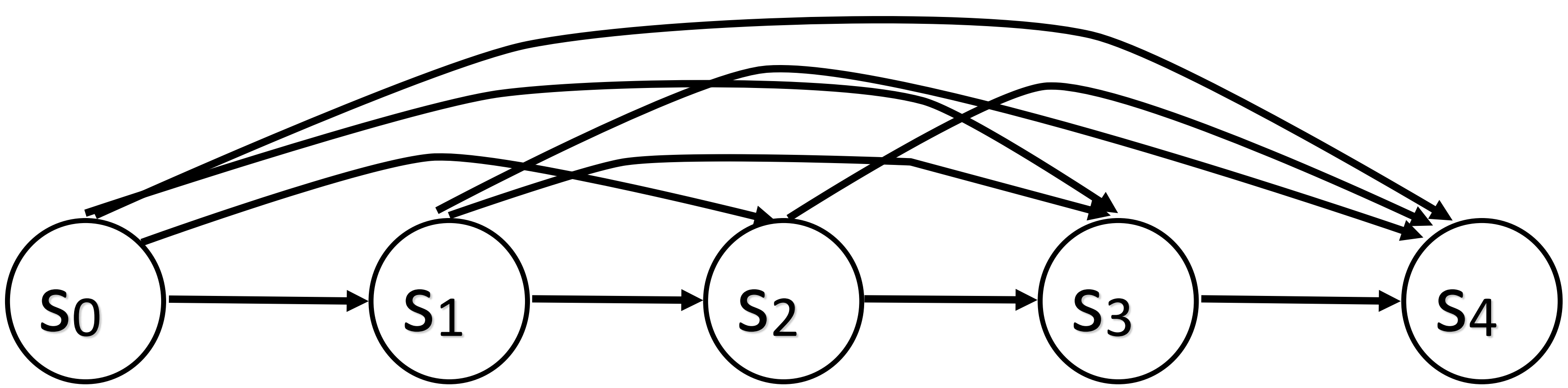
- Allow the world to evolve
- Set of states (all possible worlds)
- Set of time-slices (snapshots of the world)
- Different probability distributions over states at different time-slices
- Dynamic encoding of how distributions change over time

Stochastic Process

Set of states: **S**

Stochastic dynamics: $P(s_t | s_{t-1}, \ldots, s_0)$

Can be viewed as a Bayes Net with one random variable per time-slice



Stochastic Process

Problems:

Infinitely many variables

Infinitely large CPTs

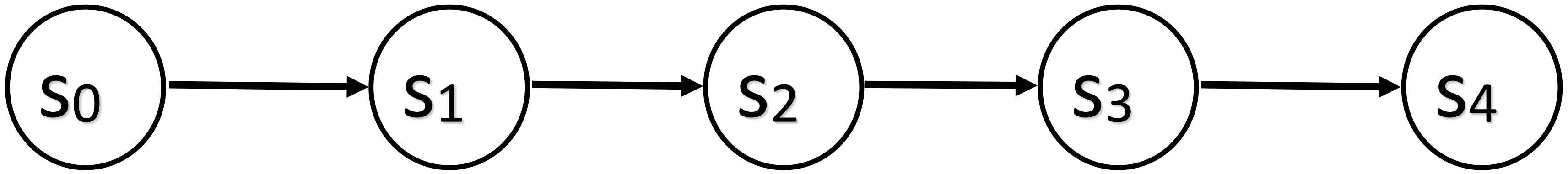
Solutions:

Stationary process: Dynamics do not change over time

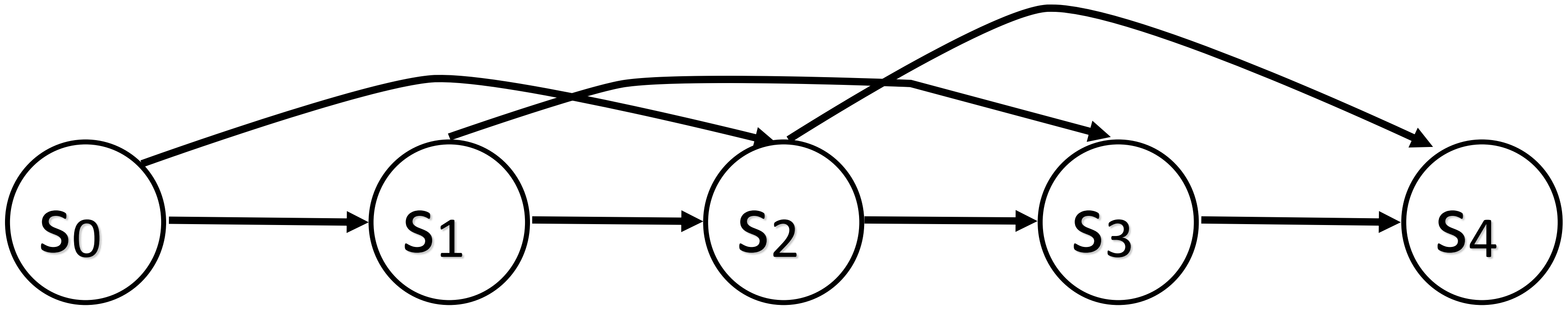
Markov assumption: Current state depends only on a finite history of past states

k-Order Markov Process

- Assumption: last k states are sufficient
- First-order Markov process
  - $P(s_t | s_{t-1}, \ldots, s_0) = P(s_t | s_{t-1})$

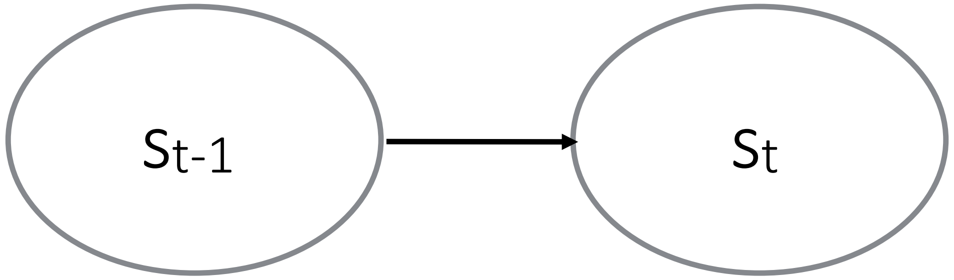


- Second-order Markov process
  - $P(s_t | s_{t-1}, \ldots, s_0) = P(s_t | s_{t-1}, s_{t-2})$



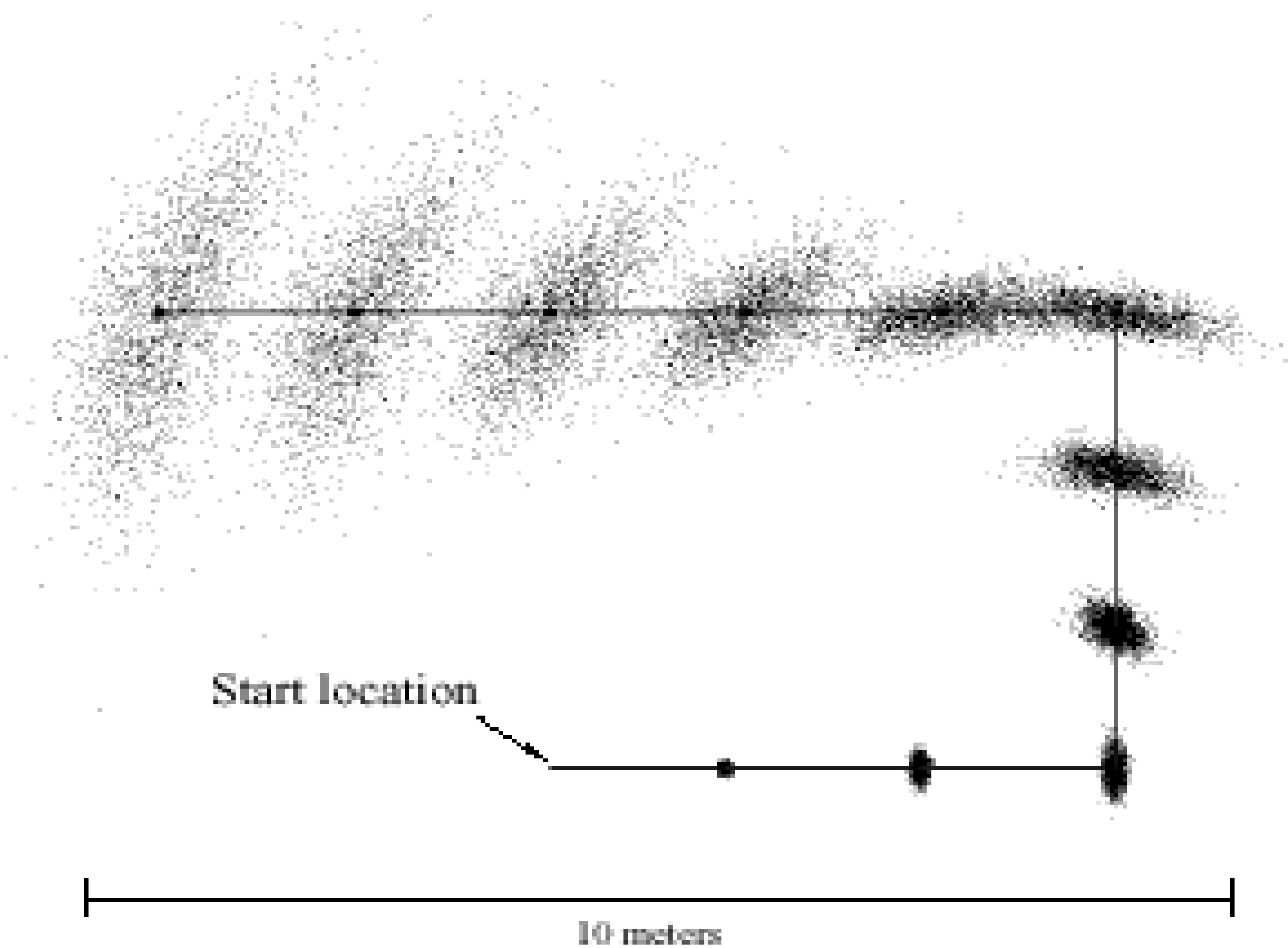
k-Order Markov Process

- Advantages
  - Can specify the entire process using finitely many time slices
- Example: Two slices sufficient for a first-order Markov process
  - Graph:
  - Dynamics: $P(s_t | s_{t-1})$
  - Prior: $P(s_0)$



Example: Robot Localization

- Example of a first-order Markov process



Problem: uncertainty increases over time

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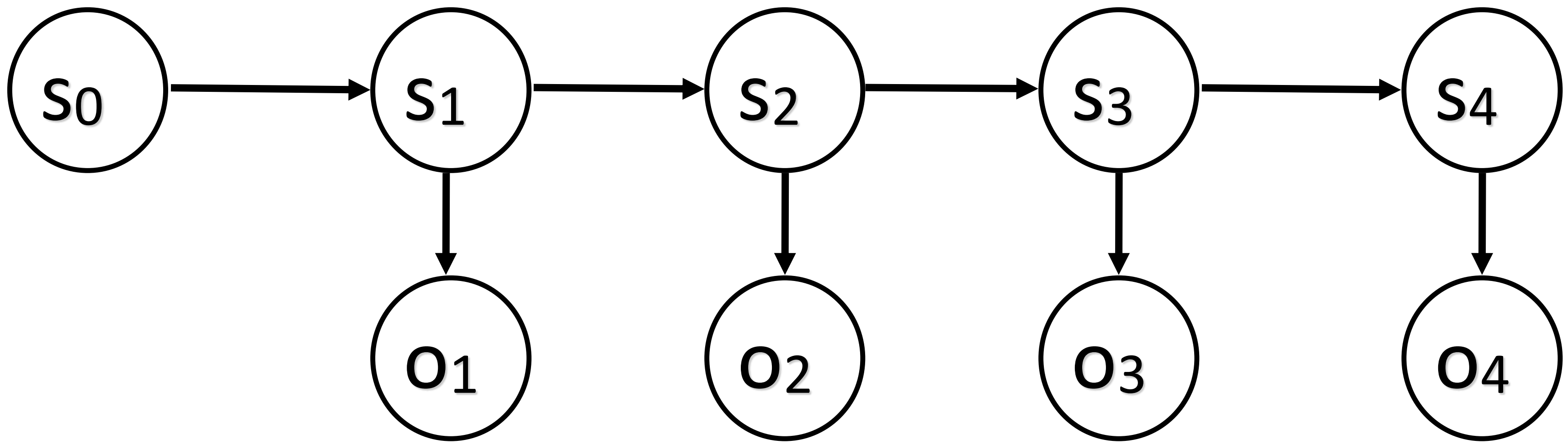
Hidden Markov Models

- In the previous example, the robot could use sensors to reduce location uncertainty
- In general:
  - States not directly observable (uncertainty captured by a distribution)
  - Uncertain dynamics increase state uncertainty
  - Observations: made via sensors can reduce state uncertainty
- **Solution**: Hidden Markov Model

First Order Hidden Markov Model (HMM)

- Set of states: S
- Set of observations: O
- Transition model: $P(s_t|s_{t-1})$
- **Observation model**: $P(o_t|s_t)$
- Prior: $P(s_0)$



Example: Robot Localization

- Hidden Markov Model
  - S: (x,y) coordinates of the robot on the map
  - O: distances to surrounding obstacles (measured by laser range fingers or sonar)
  - $P(s_t|s_{t-1})$: movement of the robot with uncertainty
  - $P(o_t|s_t)$: uncertainty in the measurements provided by the sensors
- **Localization** corresponds to the query:
  - $P(s_t|o_t,...,o_1)$

Inference

- There are four common tasks
  - **Monitoring**: $P(s_t | o_t,...o_1)$
  - **Prediction**: $P(s_{t+k} | o_t,...,o_1)$
  - **Hindsight**: $P(s_k | o_t,...,o_1)$
  - **Most likely explanation**: $\text{argmax}_{s_t,...,s_1} P(s_t,...,s_1 | o_t,...,o_1)$
- What algorithms should we use?
  - First 3 can be done with variable elimination and the 4th is a variant of variable elimination

Monitoring

We are interested in the distribution over current states given observations: $P(s_t | o_t,...,o_1)$

- Examples: patient monitoring, robot localization

Prediction

We are interested in distributions over future states given observations: $P(s_{t+k}|o_t,...,o_1)$

- Examples: weather prediction, stock market prediction

Hindsight

Interested in the distribution over a past state given observations

- Example: crime scene investigation

Most Likely Explanation

We are interested in the most likely sequence of states given the observations: $\text{argmax}_{s0,\ldots st}\ P(s_0,\ldots,s_t | o_t,\ldots,o_1)$

- Example: speech recognition

Viterbi algorithm:

Complexity of Temporal Inference

Hidden Markov Models are Bayes Nets with a ***polytree structure***!

Variable elimination is

- Linear with respect to number of time slices
- Linear with respect to largest CPT ($P(s_t|s_{t-1})$ or $P(o_t|s_t)$)

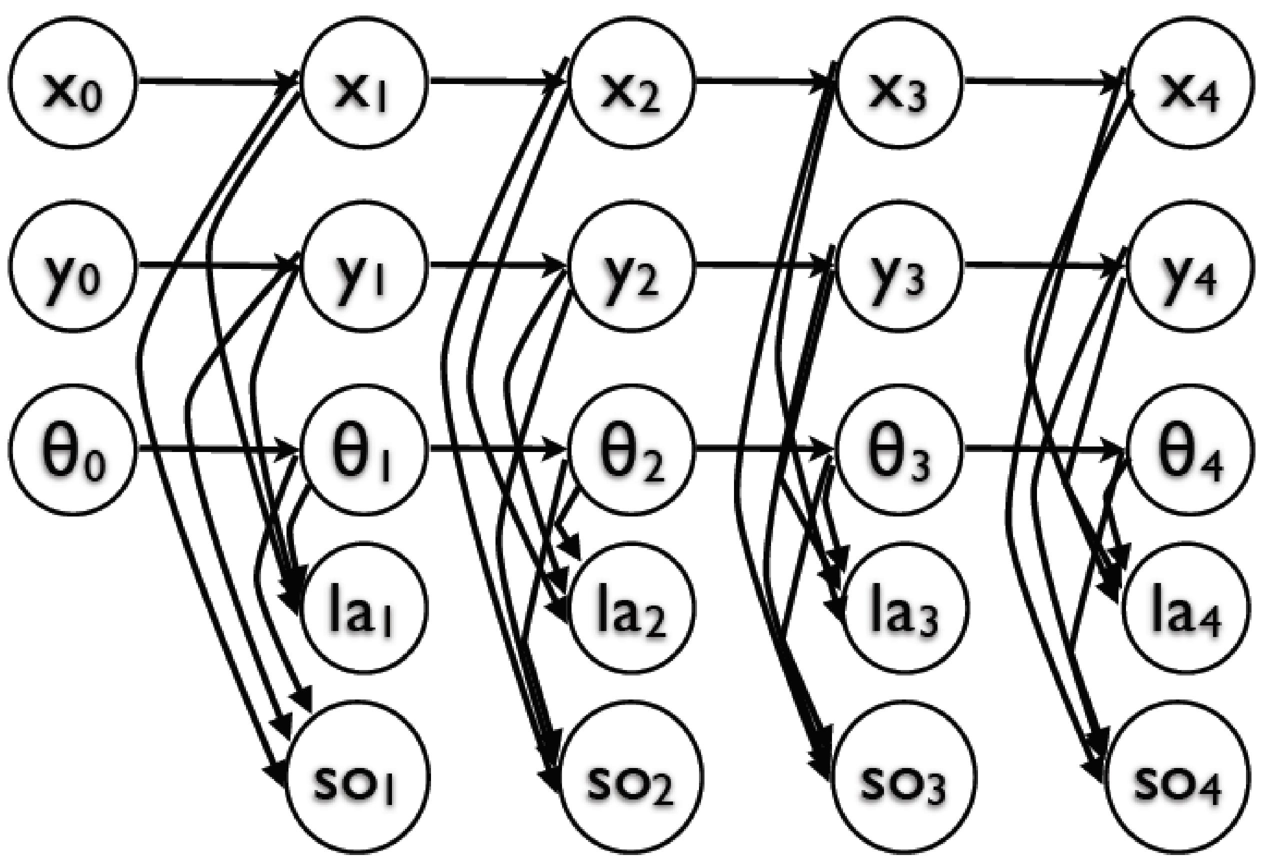
Dynamic Bayes Nets

What if the number of states or observations are exponential?

- Dynamic Bayes Nets
  - **Idea**: Encode states and observations with several random variables
  - **Advantage**: Exploit conditional independence and save time and space
  - **Note**: HMMs are just DBNs with one state variable and one observation variable

Example: Robot Localization

- **States**: (x,y) coordinates and heading θ
- **Observations**: laser and sonar readings, la and so



DBN Complexity

Conditional independence allows us to **represent** the transition and observation models very compactly!

- Time and space complexity of inference: conditional independence rarely helps
  - Inference tends to be exponential in the number of state variables
  - Intuition: All state variables eventually get correlated
  - No better than with HMMs

Non-Stationary Processes

What if the process is not stationary?

- **Solution:** Add new state components until dynamics are stationary
- **Example:** Robot navigation based on (x,y,θ) is nonstationary when velocity varies
  - **Solution:** Add velocity to state description (x,y,v,θ)
  - If velocity varies, then add acceleration,...

Non-Markovian Processes

What if the process is not Markovian?

- **Solution**: Add new state components until the dynamics are Markovian
- **Example**: Robot navigation based on (x,y,θ) is non-Markovian when influenced by battery level
  - **Solution**: Add battery level to state description (x,y,θ,b)

Markovian Stationary Processes

Problem: Adding components to the state description to force a process to be Markovian and stationary may **significantly** increase computational complexity

Solution: Try to find the smallest description that is self-sufficient (i.e. Markovian and stationary)

Summary

- Stochastic Process
  - Stationary
  - Markov assumption
- Hidden Markov Process
  - Prediction
  - Monitoring
  - Hindsight
  - Most likely explanation
- Dynamic Bayes Nets
- What to do if the stationary or Markov assumptions do not hold