Adversarial Search

CS 486/686: Introduction to Artificial Intelligence

Introduction

So far we have only been concerned with a single agent

Today, we introduce an adversary!

Outline

Games

Minimax search

Alpha-beta pruning

Evaluation functions

Coping with chance

Games

Games are the oldest, most well-studied domain in AI

Why?

- They are fun
- Easy to represent, rules are clear
- State spaces can be very large
  - In chess, the search tree has ~10^{154} nodes
- Like the “real world” in that decisions have to be made and time is important
- Easy to determine when a program is doing well

Types of Games

Perfect vs Imperfect Information

Perfect information: You can see the entire state of the game

Imperfect information:

Deterministic vs Stochastic

Deterministic: change in state is fully controlled by the players

Stochastic: change in state is partially determined by chance

Game Search Challenge

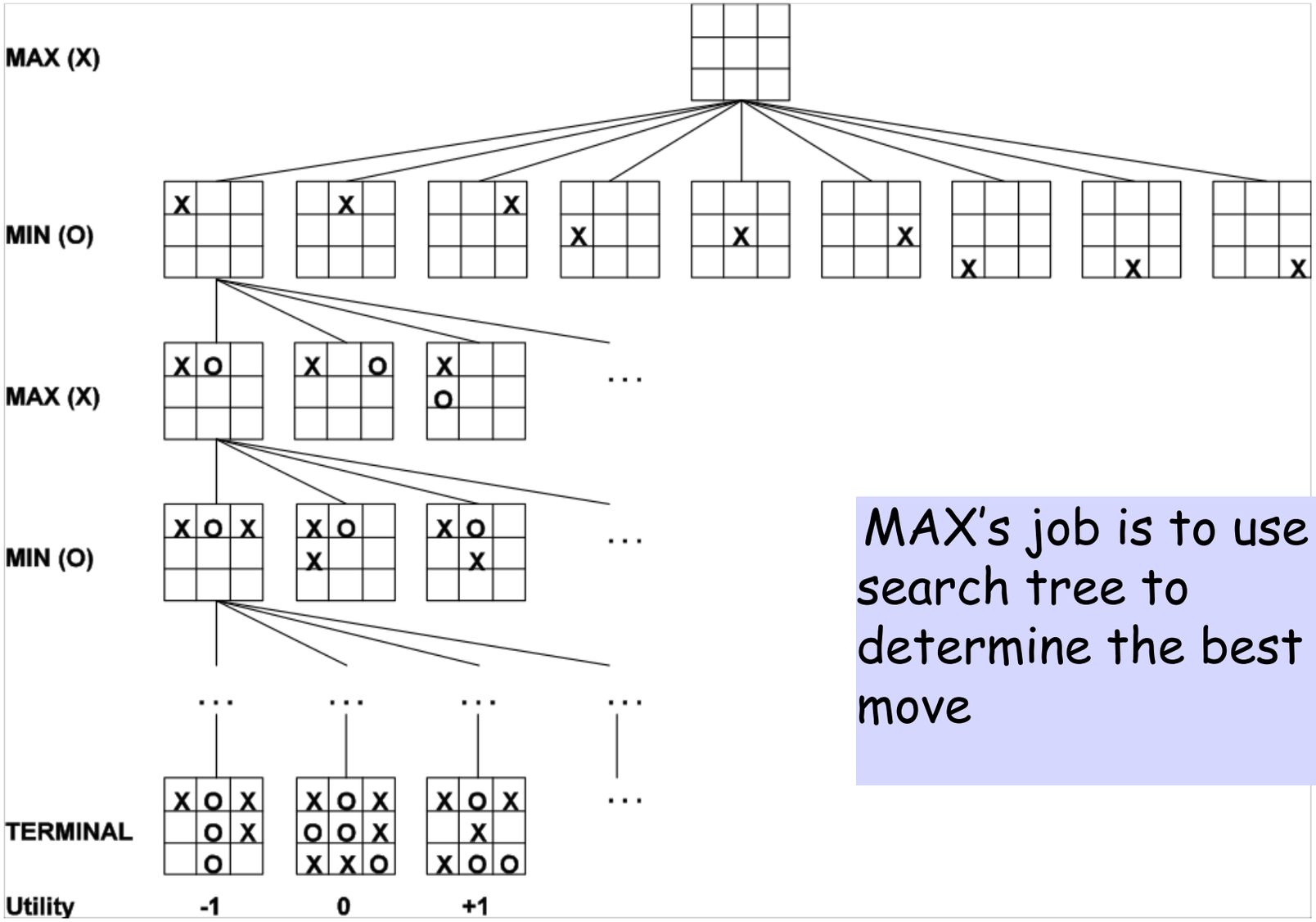
What makes game search challenging?

- There is an opponent
- The opponent is malicious
  - it wants to win (by making you lose)
  - We need to take this into account when choosing moves

Notation:

- **MAX** player wants to maximize its utility
- **MIN** player wants to minimize its utility

Example



MAX's job is to use the search tree to determine the best move

Optimal Strategies

In standard search

Optimal solution is sequence of moves leading to a goal state

Strategy (from MAX's perspective)

Specify a move for the initial state

Specify a move for all possible states arising from MIN's response

Then all possible responses to all of MIN's responses to MAX's previous moves

...

Optimal Strategies

Goal: Find optimal strategy

What do we mean by optimal?

Strategy that leads to outcomes at least as good as any other strategy, *given that MIN is playing optimally*

Equilibrium (game theory)

Today we focus mainly on **zero-sum games of perfect information**

Easy games according to game theory

Game Theory Detour

Game theory is a formal way of reasoning about interactions between multiple agents

To define a game we need the following components:

- The players, N
- Their possible strategies (or actions), S_i
- Their utility functions: $u_i(s_1,\ldots,s_n)$

		Agent 2: One	Agent 2: Two
Agent 1	One	2,-2	-3,3
Agent 1	Two	-3,3	4,-4

Game Theory Detour

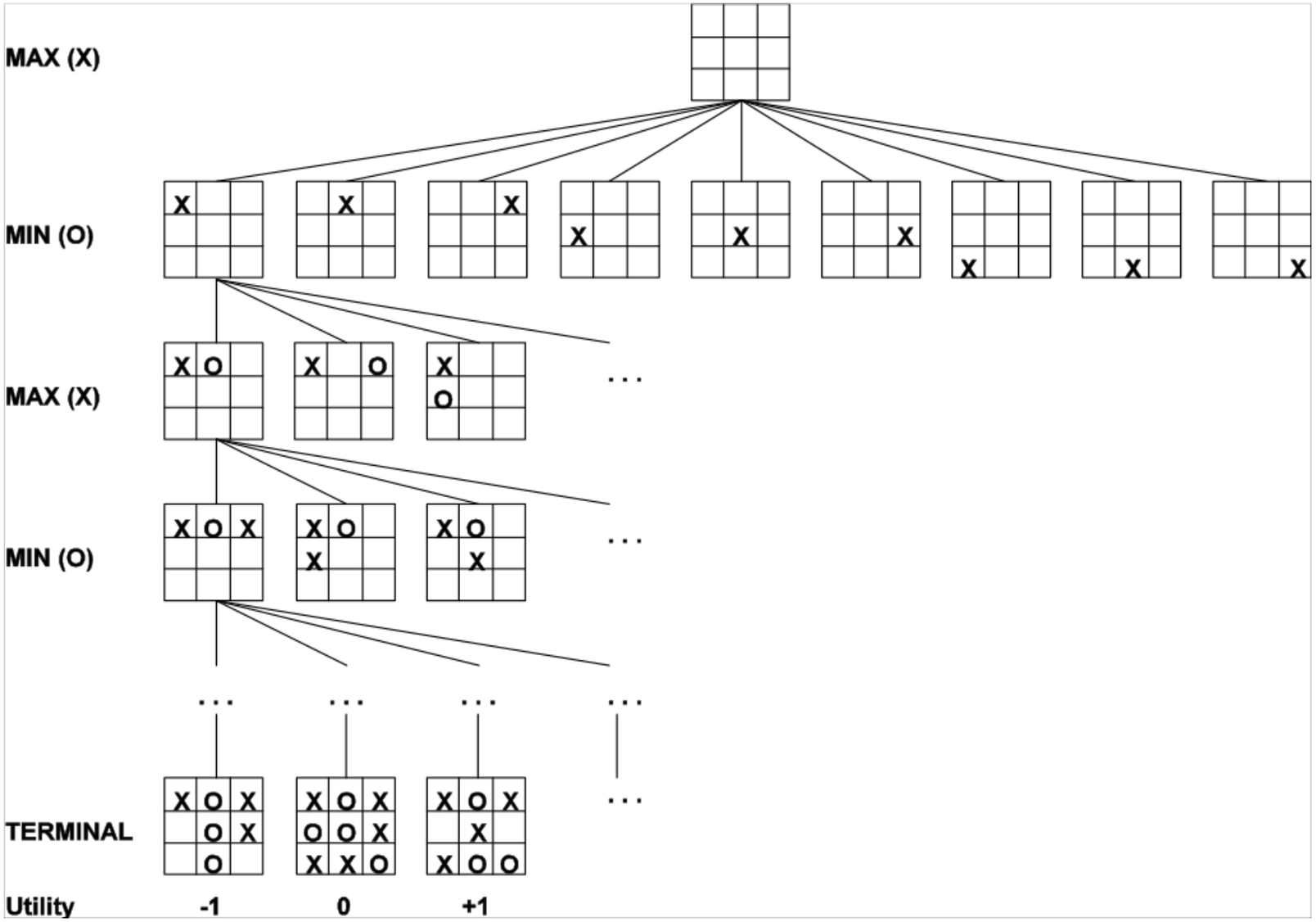
(Nash) Equilibrium: Mutual best-response

$s^*=(s_1^*,\ldots,s_n^*)$ if a NE if for all i in N

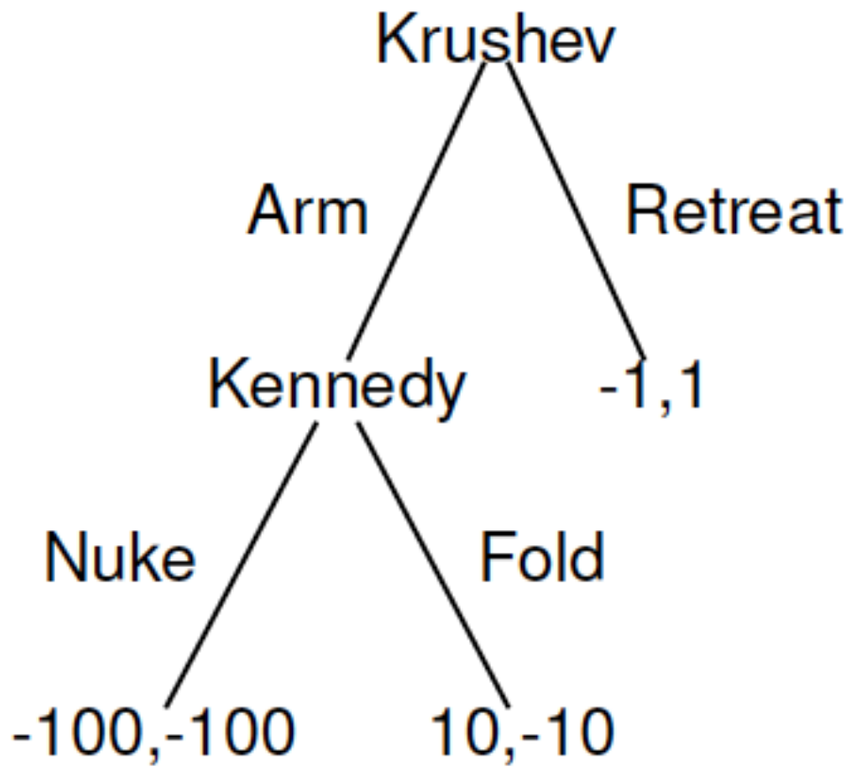
$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i', s_{-i}^*) \forall s_i'$$

For (2-player) zero sum games you can compute NE using linear programming **BUT….**

Extensive Form Games



Subgame Perfect Equilibria



Subgame Perfect Equilibria

s* must be a Nash equilibrium in all subgames

What are the SPE?

Existence of SPE

Theorem (Kuhn): Every finite extensive form game has an SPE.

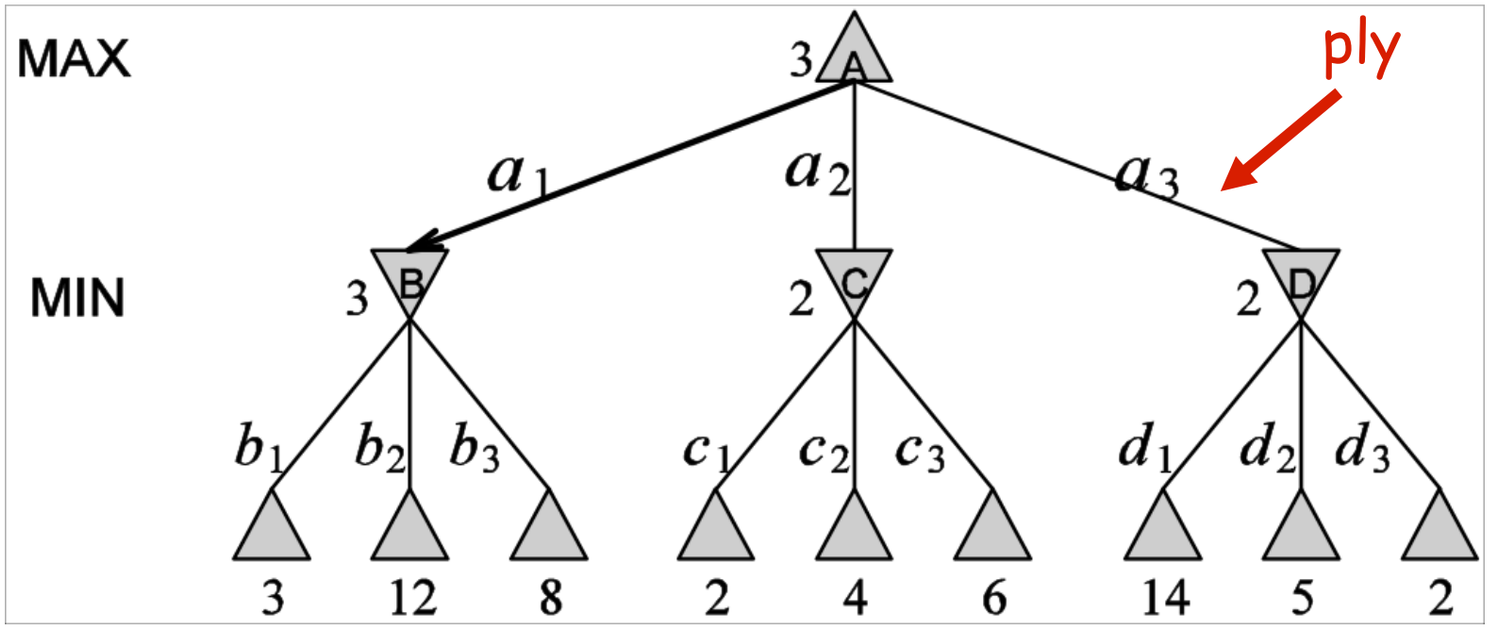
Compute the SPE using **backward induction**

- Identify equilibria in the bottom most subtrees
- Work upwards

Minimax Value

MINIMAX-VALUE(n) =

$$\begin{cases} \text{Utility}(n) & \text{if } n \text{ is a terminal state} \\ \max_{s \text{ in Succ}(n)} \text{MINIMAX-VALUE}(s) & \text{if } n \text{ is a MAX node} \\ \min_{s \text{ in Succ}(n)} \text{MINIMAX-VALUE}(s) & \text{if } n \text{ is a MIN node} \end{cases}$$



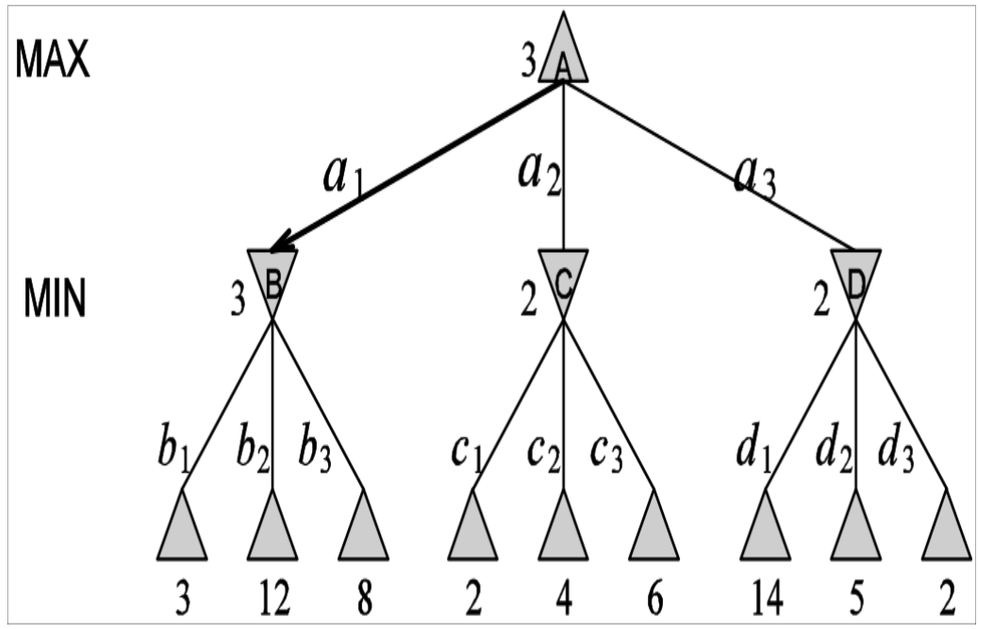
Properties of Minimax

Complete:

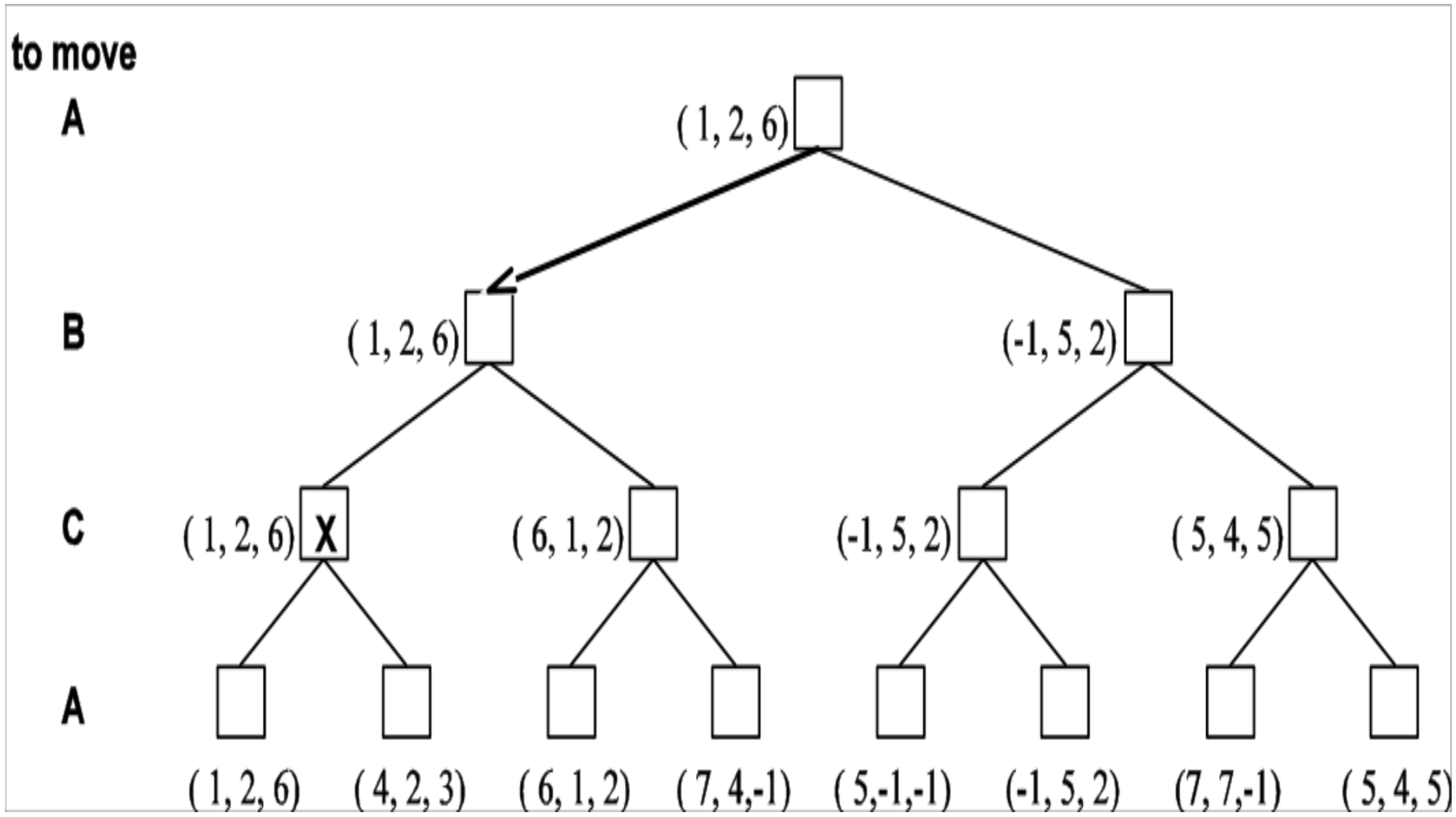
Time complexity:

Space complexity:

Optimal:



Minimax and Multi-Player Games



Question

Can we now write a program that will play chess reasonably well?

Question

Can we now write a program that will play chess reasonably well?

For chess b~35 and m~100

Alpha-Beta Pruning

If we are smart (and lucky) we can do **pruning**

Eliminate large parts of the tree from consideration

Alpha-beta pruning applied to a minimax tree

Alpha-Beta Pruding

Alpha:

Value of best (highest value) choice we have found so far on path for MAX

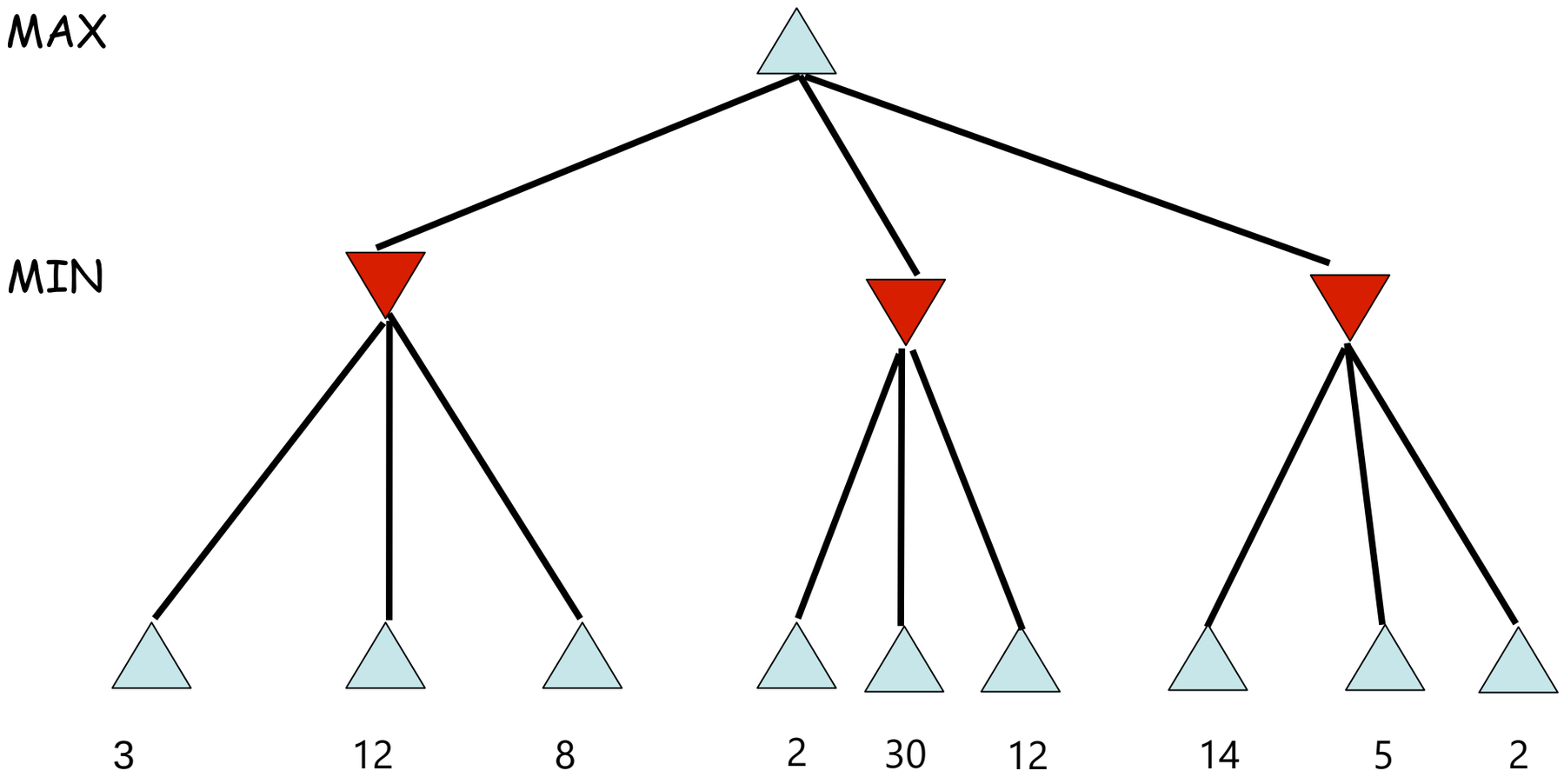
Beta:

Value of best (lowest value) choice we have found so far on path for MIN

Update alpha and beta as search continues

Prune as soon as value of current node is known to be worse than current alpha or beta values for MAX or MIN

Example



Properties of Alpha-Beta

- Can pruning result in a different outcome than minimax search?
- How much can be pruned when searching?

Real-Time Decisions

Alpha-Beta can be a huge improvement over minimax

Still not good enough

Need to search to terminal states for at least part of search space

Need to make decisions quickly

Solution

Heuristic evaluation function + cutoff tests

Evaluation Functions

Apply an evaluation function to a state

- If terminal state, function returns actual utility
- If non-terminal, function returns estimate of the expected utility

Function must be fast to compute

Evaluation Functions

How do we get evaluation functions?

- Expert knowledge
- Learned from experience

Look for features of states

- Weighted linear function $\mathrm{Eval}(s)=\sum_i w_i f_i(s)$

Cutting Off Search

Do we have to search to terminal states?

No! Cut search early and apply evaluation function

When?

Arbitrarily (but deeper is better)

Quiescent states

States that are “stable”

Singular extensions

Searching deeper when you have a move that is “clearly better”

Can be used to avoid the **horizon effect**

Cutting Off Search

How deep?

Novice player

- 5-ply (minimax)

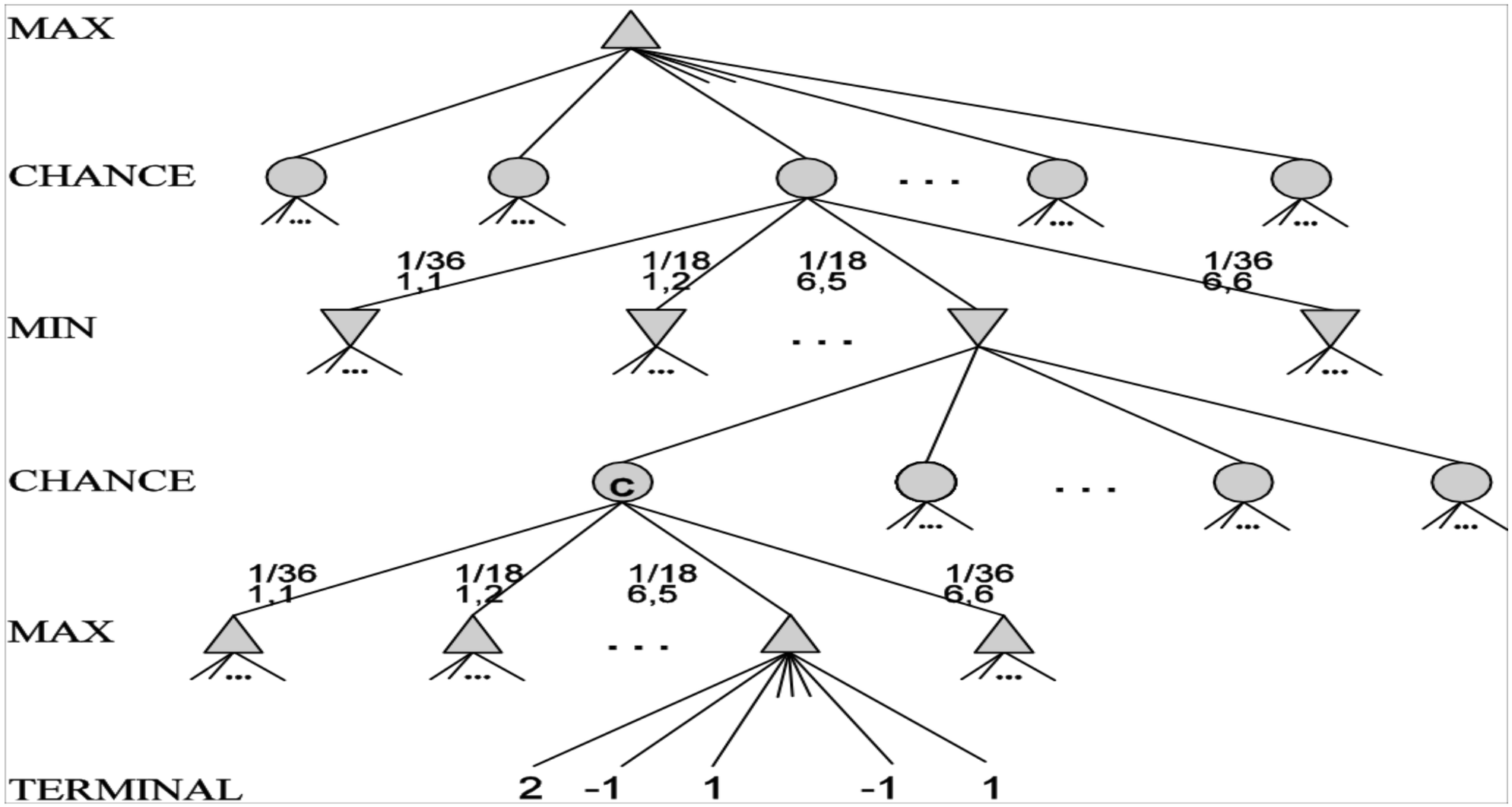
Master player

- 10-ply (alpha-beta)

Grandmaster

- 14-ply + fantastic evaluation function, opening and endgame databases,...

Stochastic Games



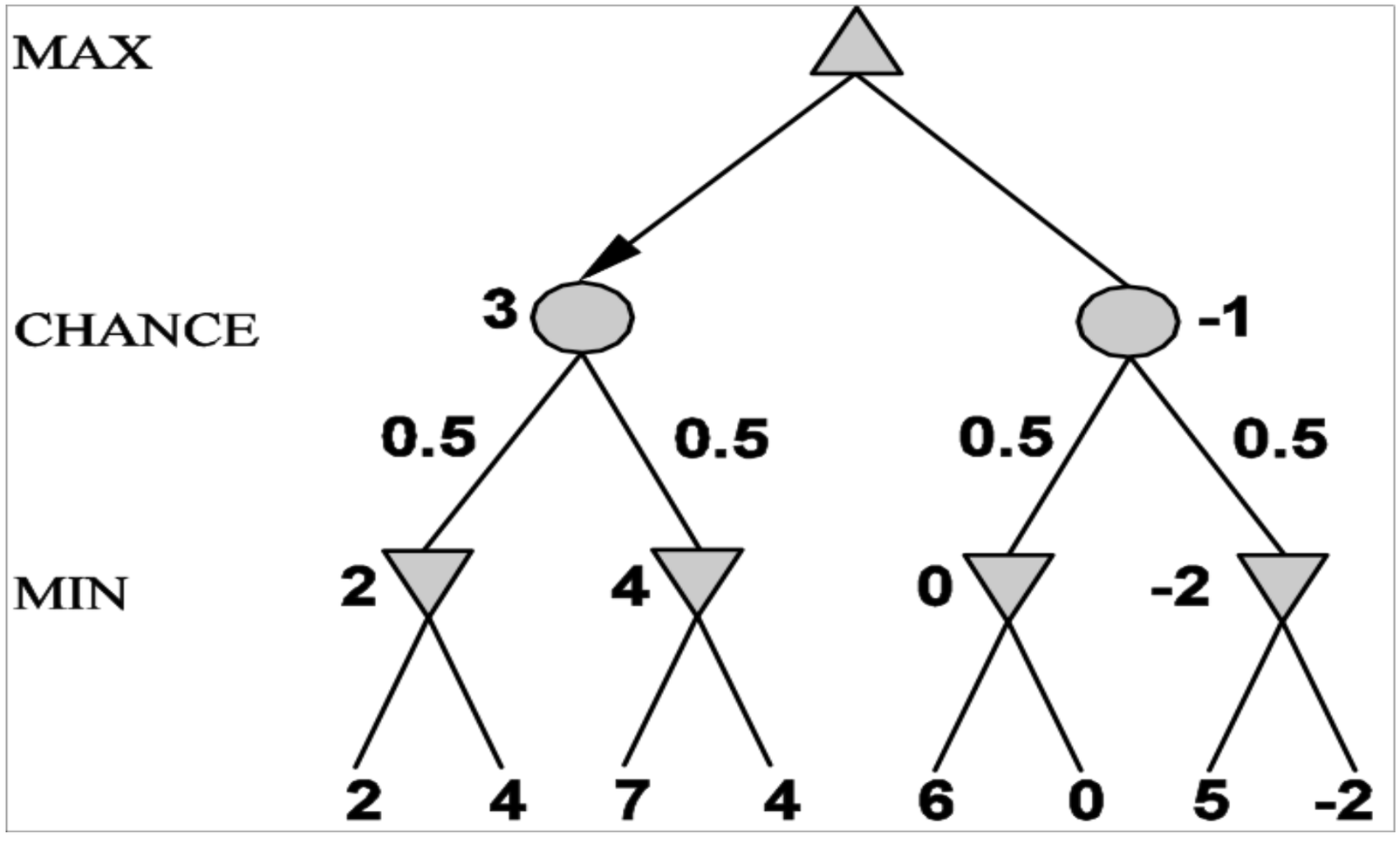
Stochastic Games

Need to consider **best/worst cases + probability** they will occur

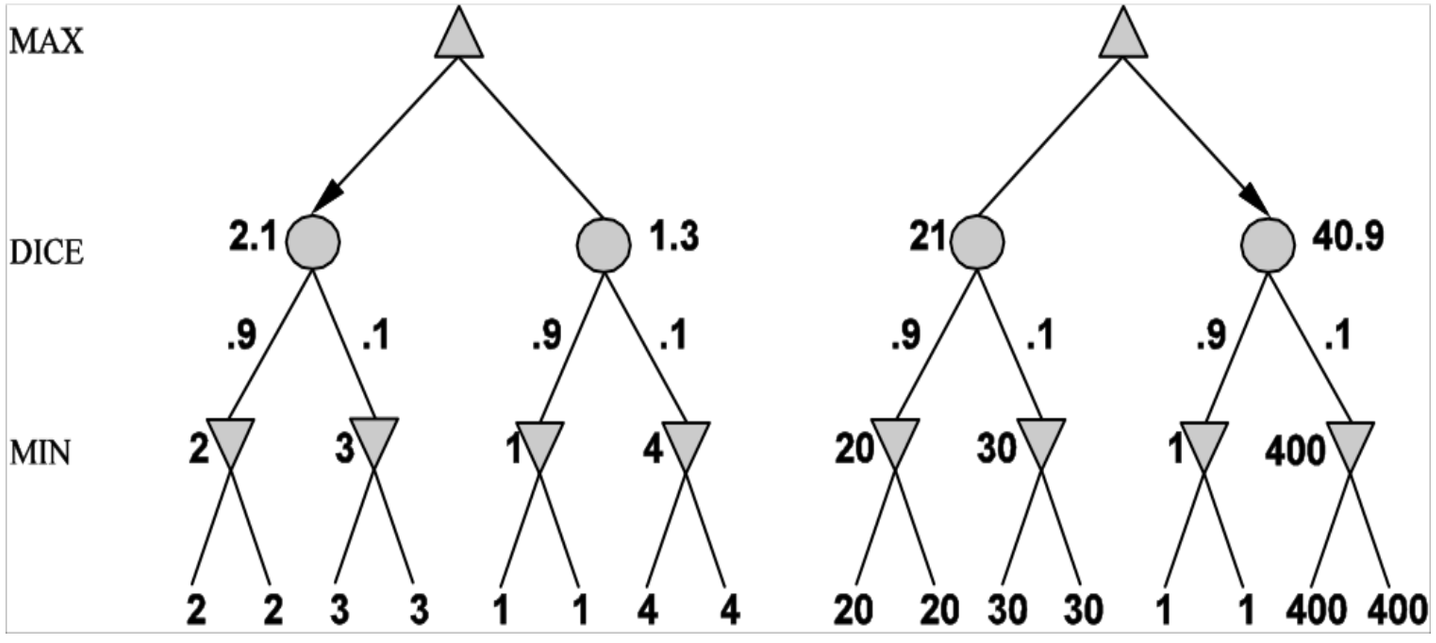
Recall: Expected value of a random variable x
$E[x]=\sum_{x \text{ in } X} P(x)x$

Expectiminimax: minimax but at chance nodes compute the expected value

Expectiminimax



Expectiminimax



WARNING: exact values do matter! Order-preserving transformations of the evaluation function can change the choice of moves. Must have **positive linear transformations** only

What about Go?

b=250, d=150

What about Go?

Monte-Carlo Tree Search (MCTS)

- Build search tree according to outcomes of simulated plays

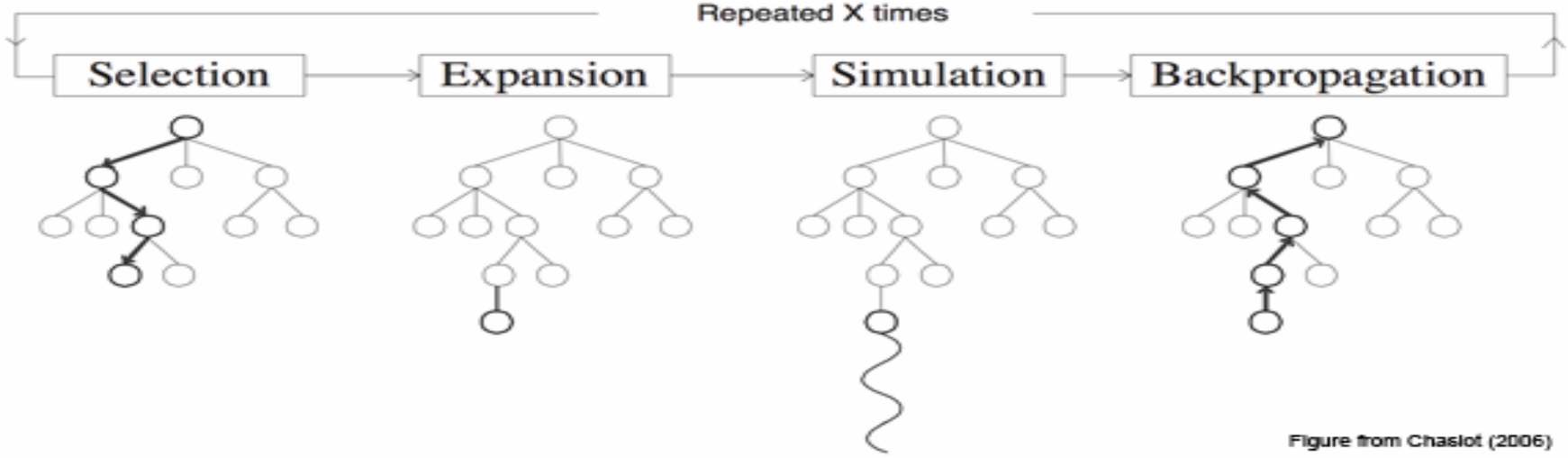


Figure from Chaslot (2006)

Upper Confidence Bounds for Trees (UCT): "Minimax search" using UCB

$$v_i + C\sqrt{\frac{\ln N}{n_i}}$$

MCTS

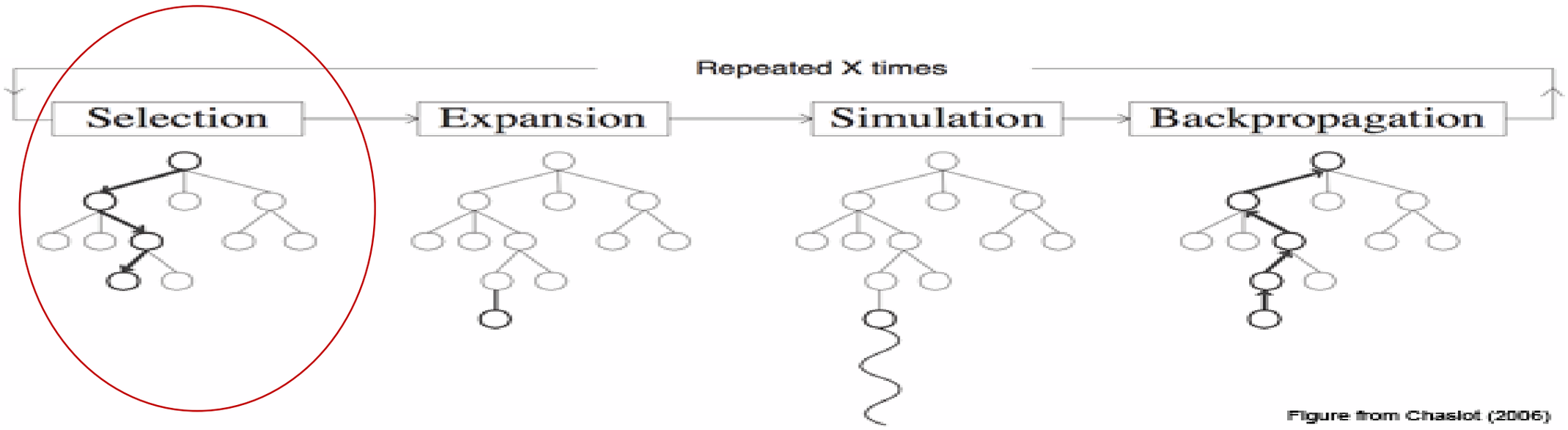


Figure from Chaslot (2006)

Selection:

Starting at the root, traverse tree following a policy using Upper Confidence Bounds (UCB) until you run out of statistics needed

i.e. treat the problem as a Multi-Armed Bandit

Upper Confidence Bounds for Trees (UCT): "Minimax search" using UCB

$$v_i + C\sqrt{\frac{\ln N}{n_i}}$$

MCTS

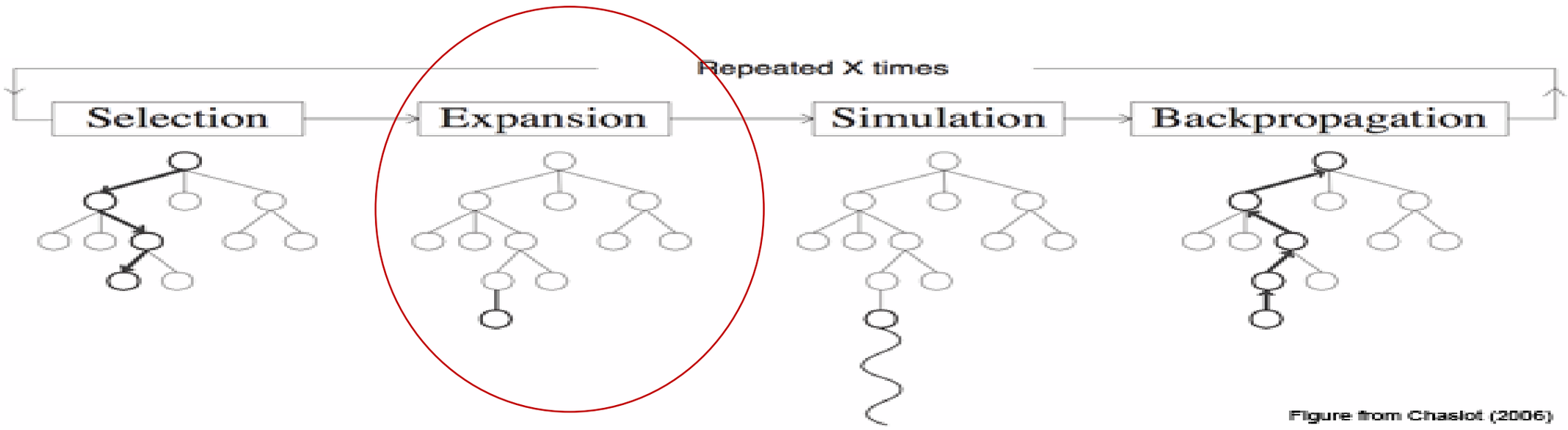


Figure from Chaslot (2006)

Expansion:

Once you reach a node on which you have no statistics, select a random child node

MCTS

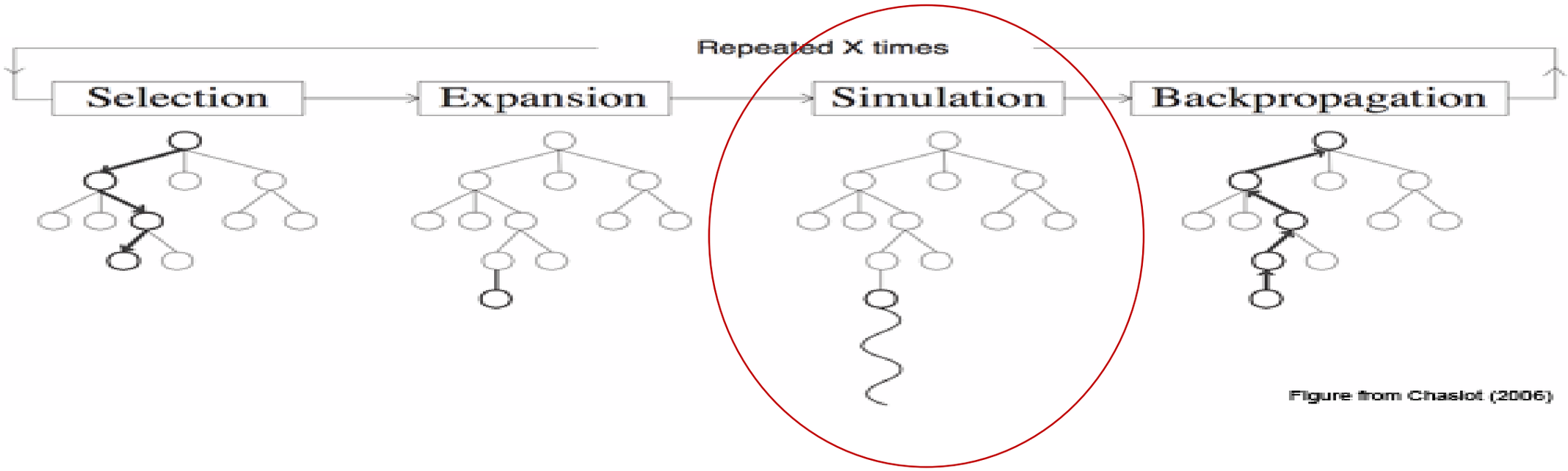
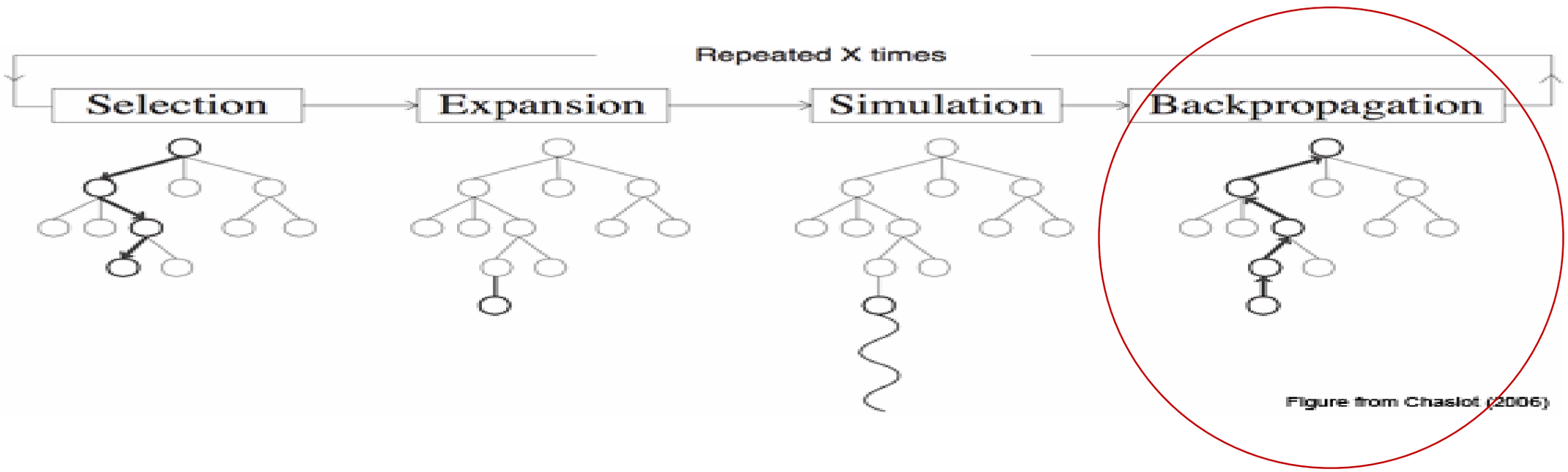


Figure from Chaslot (2006)

Simulation:

Using a rollout policy (possibly a random policy or something lightweight) play out to the end of the game.

MCTS



Backpropagation:

Using the outcome of the rollout in the simulation phase (i.e. was it a win or a loss), update the statistics for all nodes from the child you expanded back up to the root.

Summary

Games pose lots of fascinating challenges for AI researchers

Minimax search allows us to play optimally against an optimal opponent

Alpha-beta pruning allows us to reduce the search space

MCTS is needed for large games

A good evaluation function is key to doing well

Games are fun!