Constraints and Local Search

CS 486/686: Introduction to Artificial Intelligence

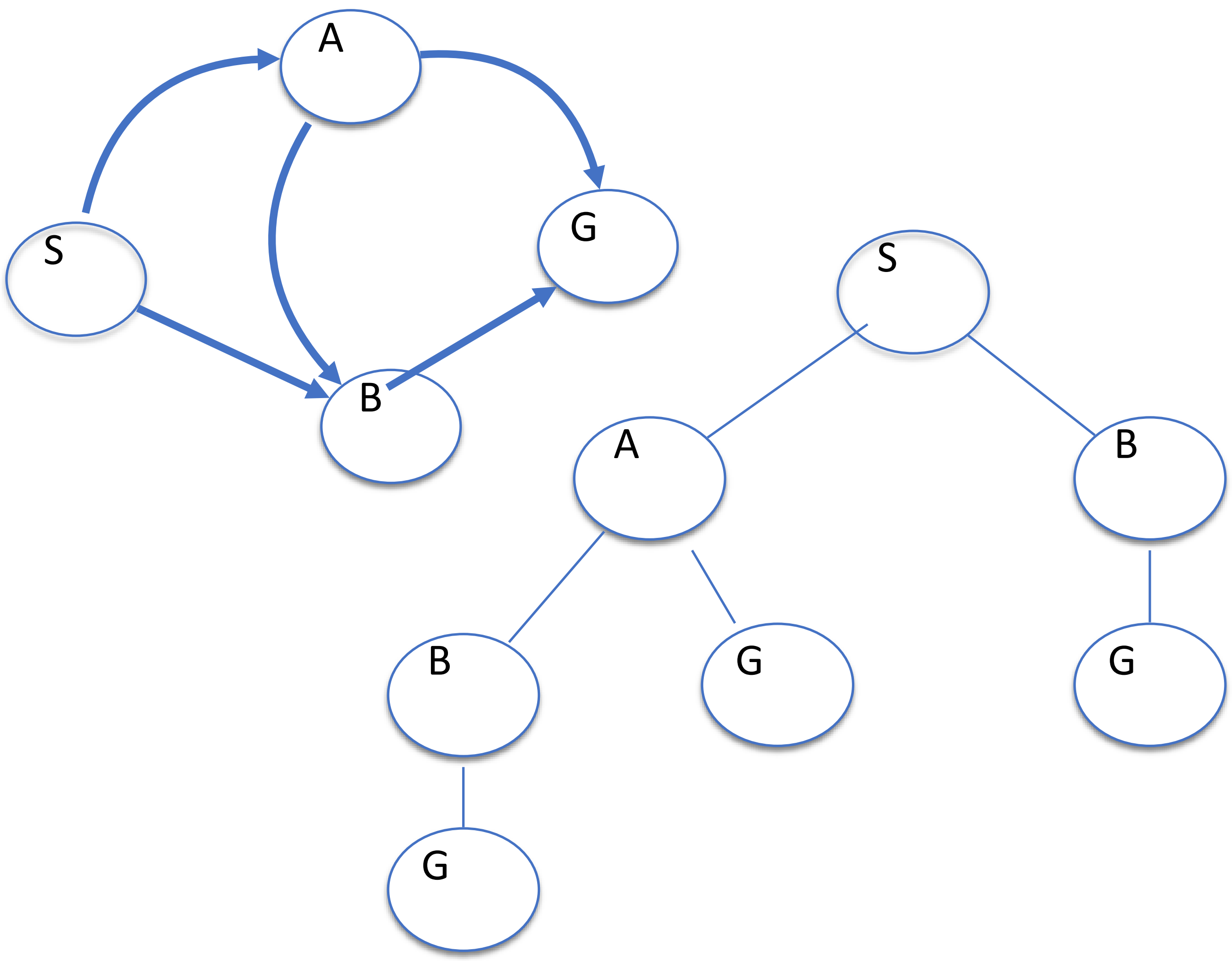
Overview

- Uninformed Search
  - Very general: assumes no knowledge about the problem
  - BFS, DFS, IDS
- Informed Search
  - Heuristics
  - A* search and variations
- **Search and Optimization**
  - What are the problem features?
  - Iterative improvement: hill climbing, simulated annealing
  - Genetic algorithms

Introduction

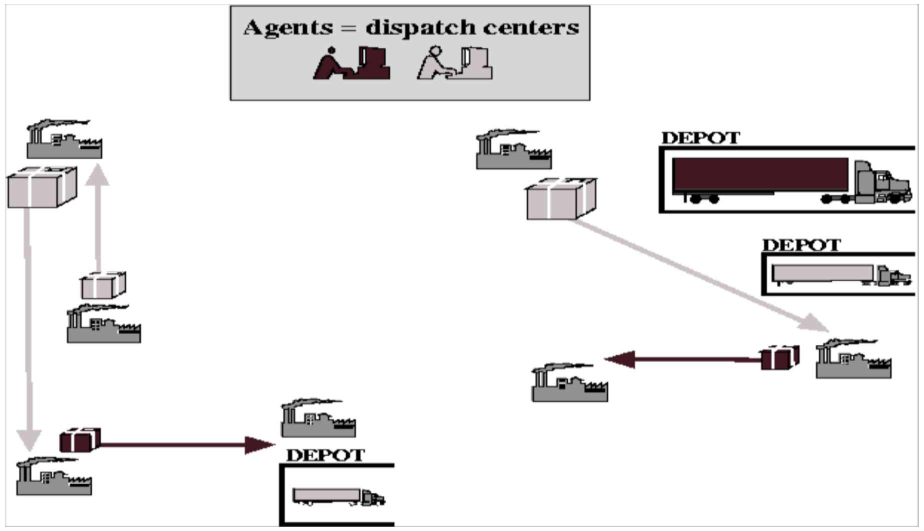
Both uninformed and informed search systematically explore the search space

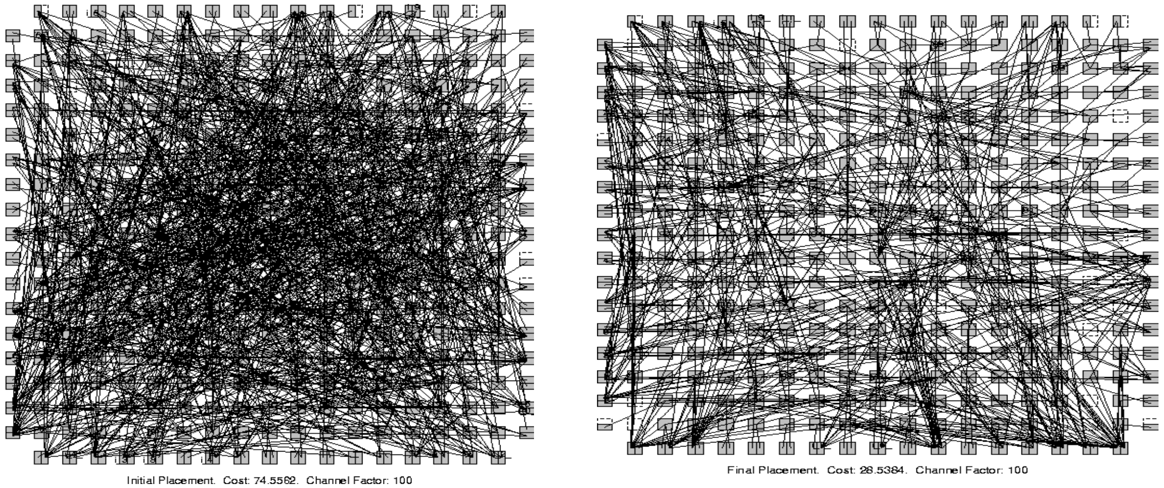
- Keep 1 or more paths in memory
- **Solution is a path to the goal**



For many problems the path is unimportant

Examples





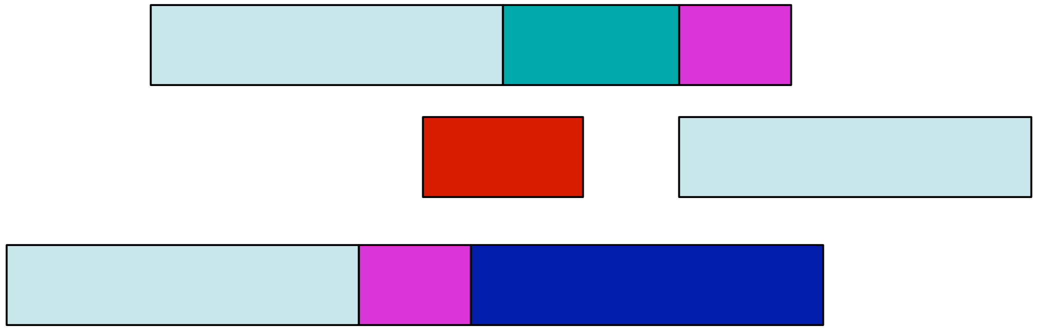
AV ~B V C

~A V C V D

B V D V ~E

~C V ~D V ~E

...



Informal Characterization

Combinatorial structure being optimized

Constraints have to be satisfied

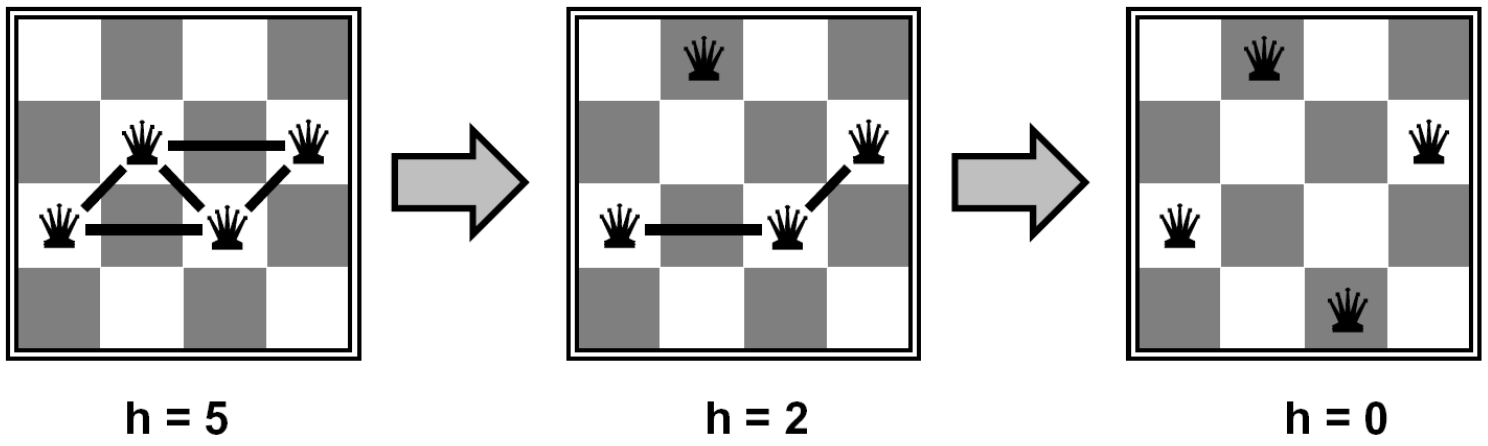
There is a cost function

- We want to find a **good** solution

Search all possible states is infeasible

- Often easy to find **some solution** to the problem
- Often provably **hard** (NP-complete) to find the **best** solution

Typical Example: 4 Queens



Start with a "complete" state

Operators reassign variables

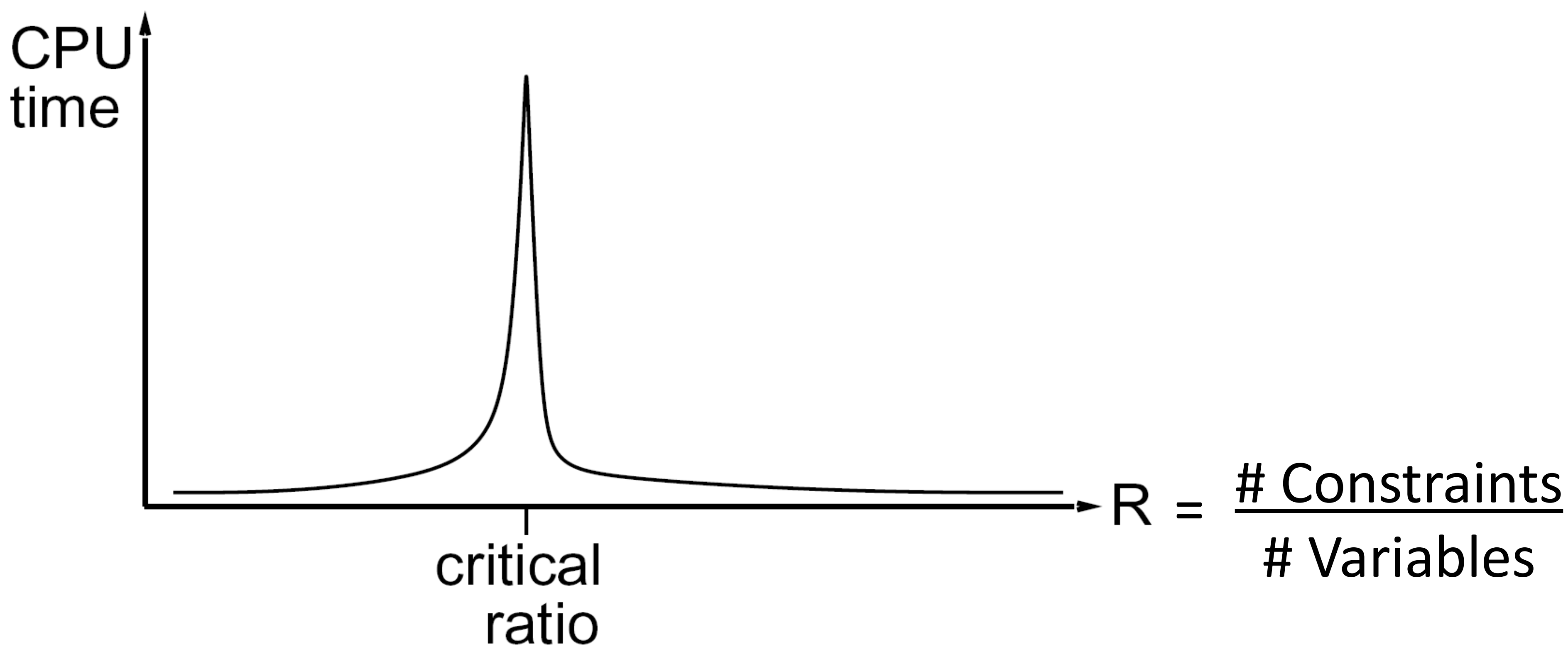
- Choose variable at random
- Choose value using **min-conflicts** heuristic

Continue until solved

Performance for N-Queens

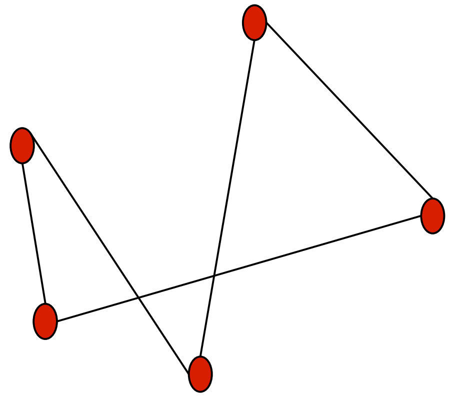
Given a random initial state, can solve n-queens in almost constant time for arbitrary n *with high probability* (n=10,000,000)!

This seems to hold for almost any randomly generated CSP except for a small set!



Typical Example: TSP

Goal is to minimize the length of the route



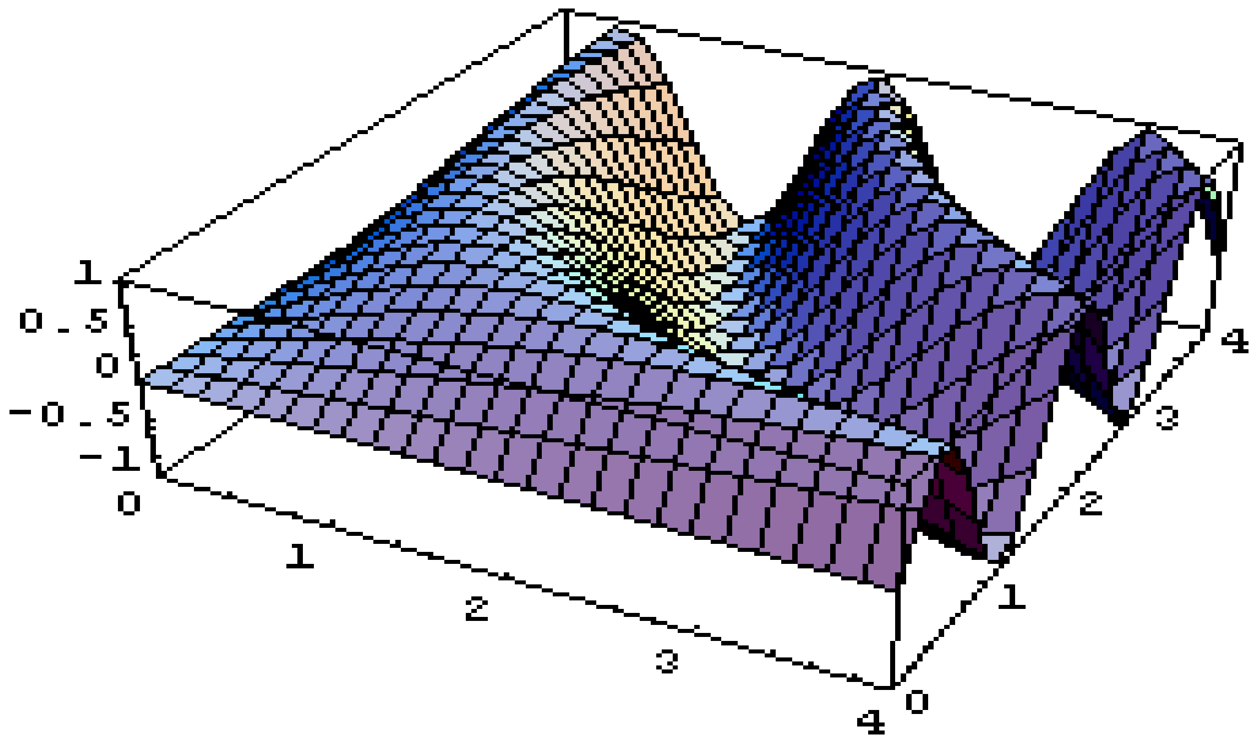
Constructive method: Start from scratch and build up a solution (using A* etc)

Iterative improvement method: Start with solution (may be suboptimal or broken) and improve it

Iterative Improvement Methods

Idea: Imagine all possible solutions laid out on a landscape

Goal: find the highest (or lowest) point



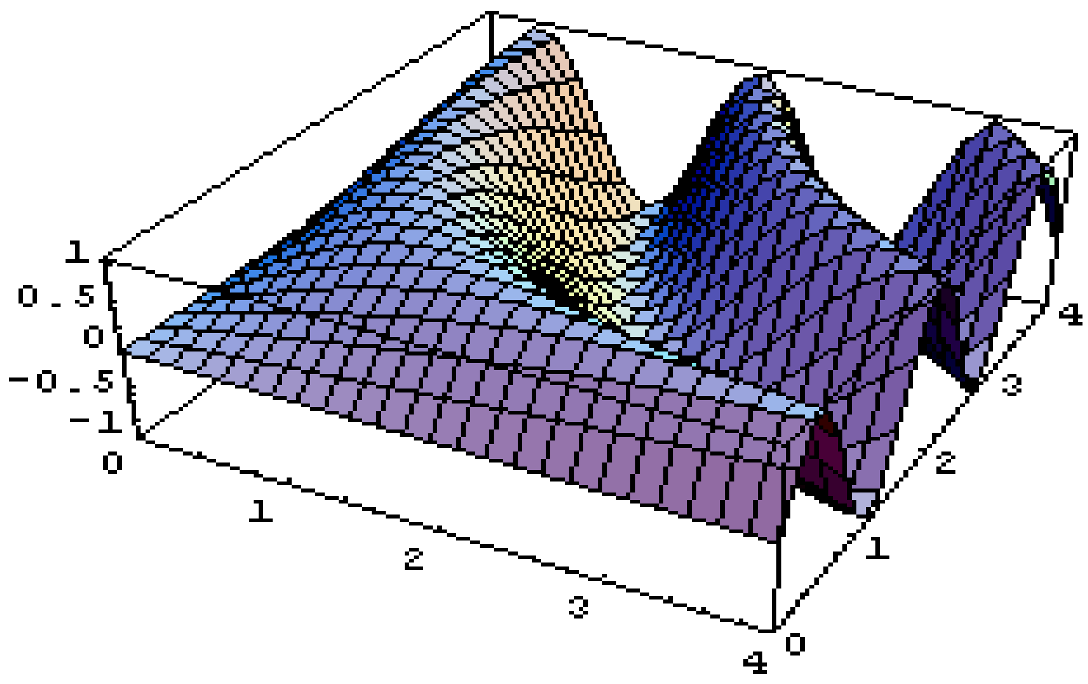
Iterative Improvement Methods

Start at some random point (potential solution)

Generate all possible points to move to

If the set is not empty, choose a point and move to it

If you are stuck (set is empty), then restart



Hill Climbing (Gradient Descent)

Main idea: Always take a step in the direction that improves the current solution value the most

Note: Variation of best-first search

Application: Very popular for learning algorithms

"…like trying to find the top of Mt Everest in a thick fog while suffering from amnesia", Russell and Norvig

Hill Climbing (Discrete Version)

1. Start with some initial configuration S, with value V(S)
2. Generate Moveset(S)= $\{S_1,\ldots,S_n\}$
3. S_{max}=argmax_{S_i} $V(S_i)$
4. If $V(S_{max})<V(S)$ return S (local optimium)
5. Let $S \leftarrow S_{max}$ Go to 2

Judging Hill Climbing

Good news

Easy to program!

Requires no memory of where we have been!

Judging Hill Climbing

Good news

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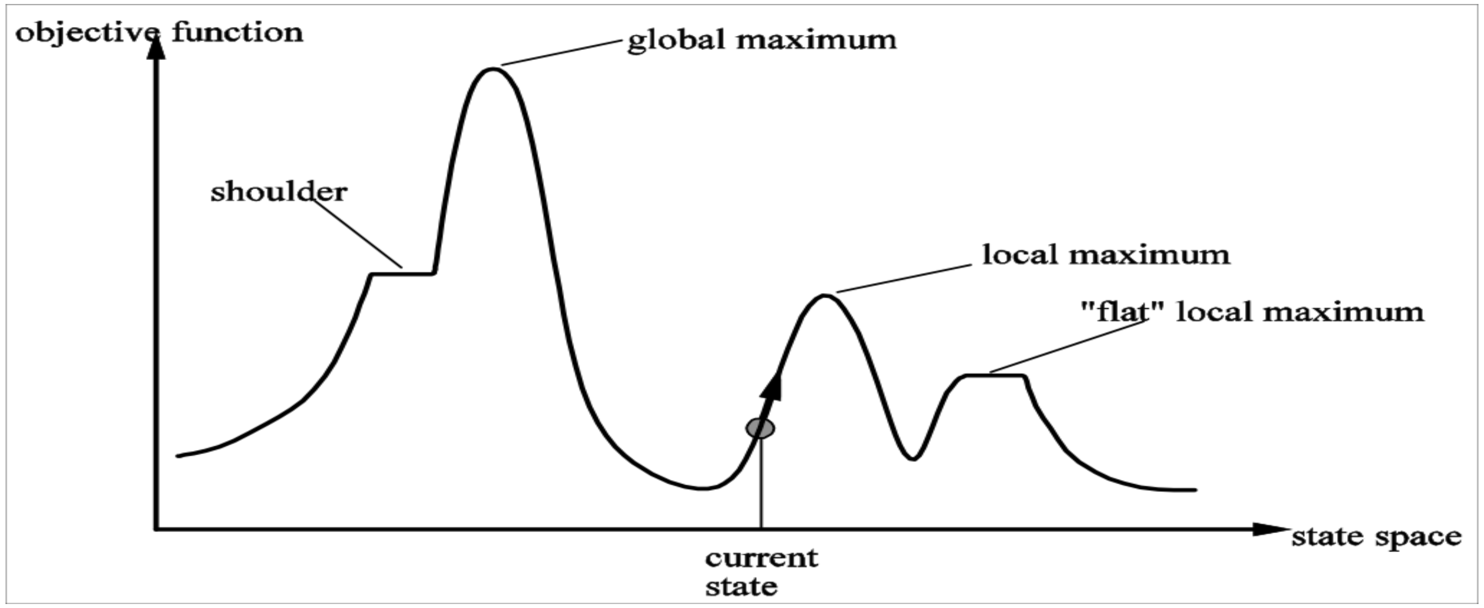
Bad news

Not necessarily complete

Not optimal

It can get stuck in local optima/plateaus



Improving Hill Climbing

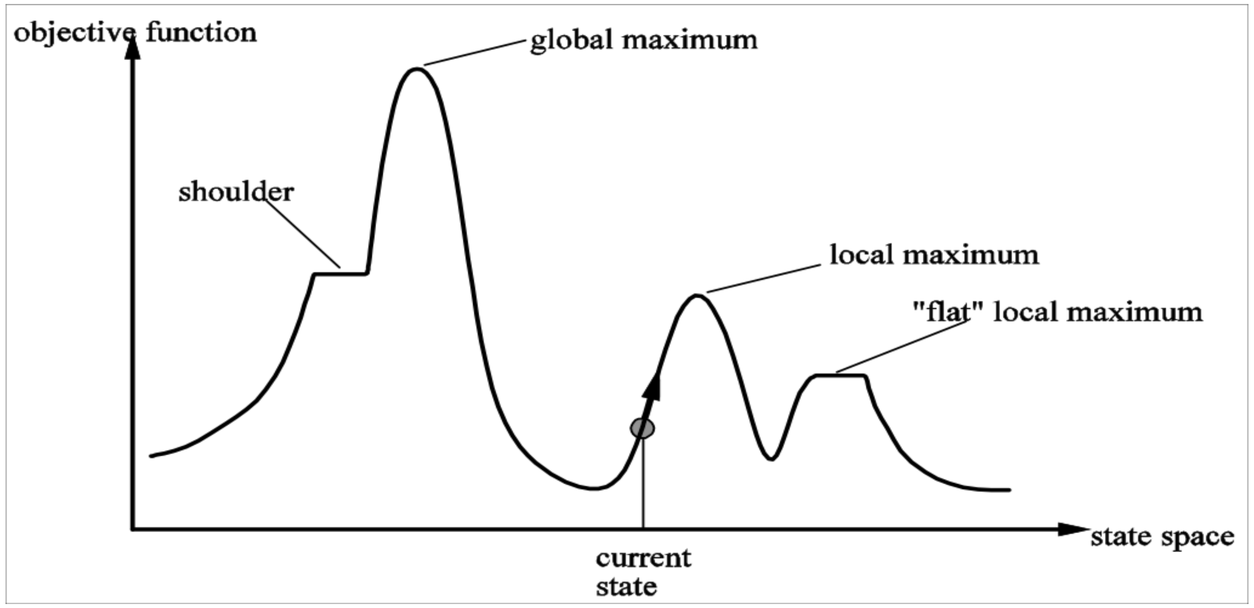
Plateaus

- Allow for sideways moves
  - But be careful since might move sideways forever

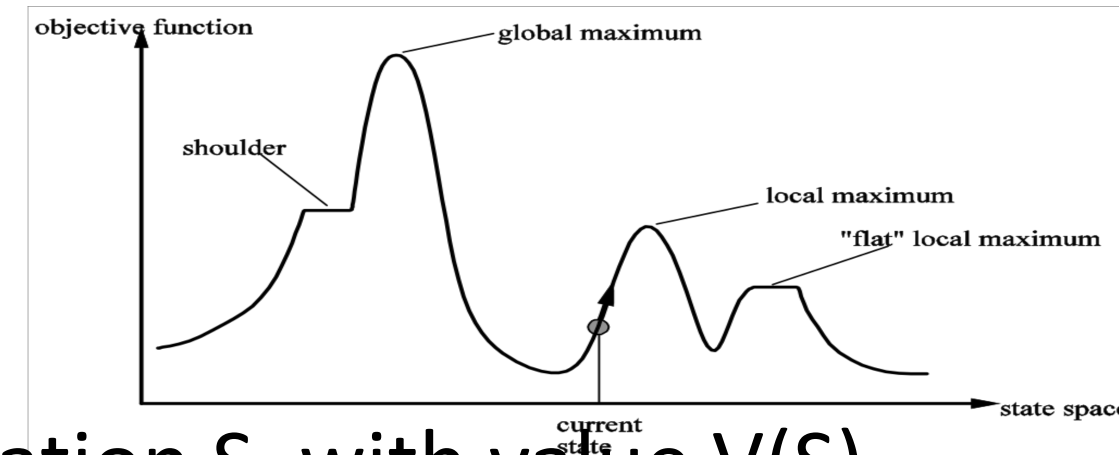
Local Maxima

- **Random restarts:**

If at first you do not succeed, try, try again!



Simulated Annealing



Escape local maxima by allowing "downhill moves"

1. Start with some initial configuration S, with value V(S)
2. Generate Moveset(S)= $\{S_1,\ldots,S_n\}$
3. Choose S_i **randomly** from Moveset(S)
4. Define $\Delta V=V(S_i)-V(S)$
5. If $\Delta V>0$ then $S \leftarrow S_i$

   else **with probability p, $S \leftarrow S_i$**
6. Go to 2

What About p?

Main Issue: How should we choose the probability of making a "bad" move?

Ideas:

p=0.1 (or some fixed value)?

Decrease p with time?

Make p a function of $|V-V_i|$?

…

Selecting Moves in Simulated Annealing

- If new value V_i is **better** than old value V then **definitely** move to new solution
- If new value V_i is **worse** than old value V then move to new solution with *some probability*

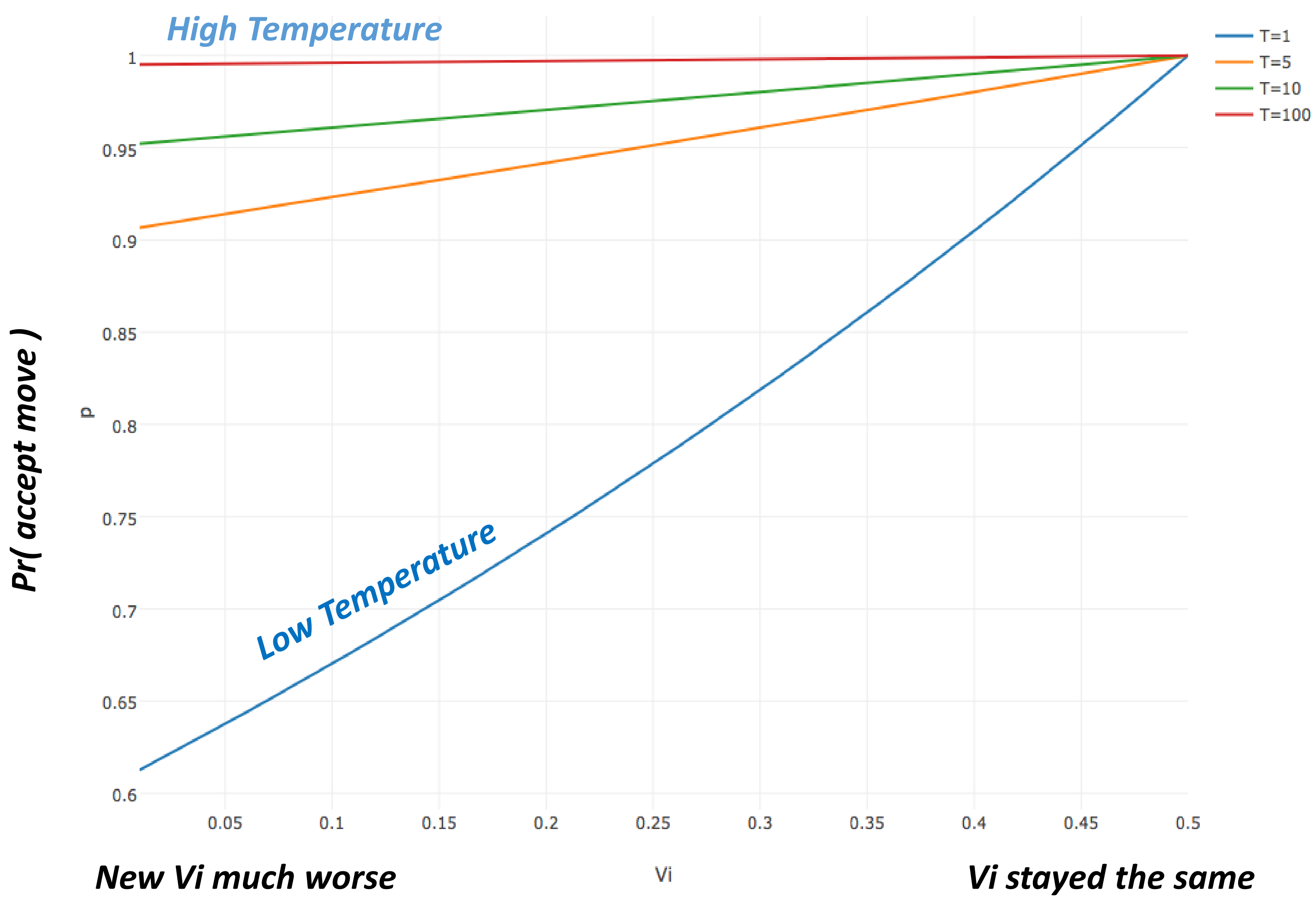
Boltzmann Distribution

$$e^{\frac{\Delta V}{T}}$$

Selecting Moves in Simulated Annealing

Boltzmann Distribution: T>0 is a parameter called temperature. It starts high and decreases over time towards 0. If T is close to 0 then the prob. of making a bad move is almost 0.

$$e^{\frac{\Delta V}{T}}$$



Properties to Simulated Annealing

When T is high:

Exploratory phase: even bad moves have a chance of being picked (~ random walk)

When T is low:

Exploitation phase: “bad” moves have low probability of being chosen (randomized hill climbing)

If T is decreased slowly enough then simulated annealing is (theoretically) guaranteed to reach optimal solution

Genetic Algorithms

- Populations are encoded into a representation which allows certain operations to occur
- An encoded candidate solution is an **individual**
- Each individual has a **fitness**
  - Numerical value associated with its quality of solution
- A **population** is a set of individuals
- Populations change over **generations** by applying operators to them
  - Operations: selection, mutation, crossover

Typical Genetic Algorithm

- Initialize: Population P←N random individuals
- Evaluate: For each x in P, compute fitness(x)
- Loop
  - For i=1 to N
    - **Select** 2 parents each with probability proportional to fitness scores
    - **Crossover** the 2 parents to produce a new bitstring (child)
    - With some small probability **mutate** child
    - Add child to population
  - Until some child is fit enough or you get bored
- Return best child in the population according to fitness function

Selection

- Fitness proportionate selection:

$$P(i) = \frac{\mathrm{fitness}(i)}{\sum_j \mathrm{fitness}(j)}$$

  - Can lead to overcrowding
- Tournament selection
  - Pick i, j at random with uniform probability
  - With probability p select fitter one
- Rank selection
  - Sort all by fitness
  - Probability of selection is proportional to rank
- Softmax (Boltzmann) selection:

$$P(i) = \frac{e^{\mathrm{fitness}(i)/T}}{\sum_j e^{\mathrm{fitness}(j)/T}}$$

Crossover

- Combine parts of individuals to create new ones
- For each pair, choose a random crossover point
  - Cut the individuals there and swap the pieces

101|0101 011|1110

Cross over

011|0101 101|1110

Implementation: use a crossover mask m

Given two parents a and b the offspring are

(a^m)V(b^~m) and (a^~m)V (b^m)

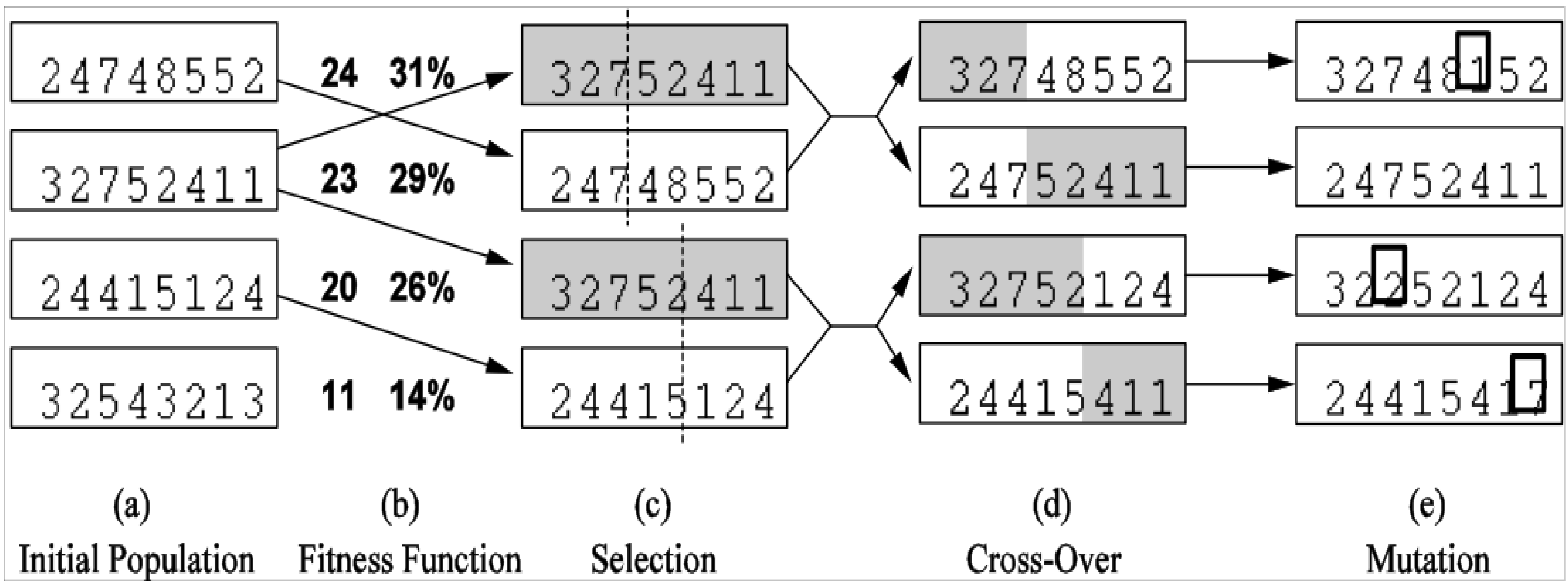
Mutation

- Mutation generates new features that are not present in original population
- Typically means flipping a bit in the string

100111 mutates to 100101

- Can allow mutation in all individuals or just in new offspring

Example


24748552
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24415124
32748552
24752411
32752124
24415411
32748152
24752411
32252124
24415417
(a)
Initial Population
(b)
Fitness Function
(c)
Selection
(d)
Cross-Over
(e)
Mutation

Summary

Useful for optimization problems

Often the **second-best way** to solve a problem

- If you can, use A* or linear programming or ...
- But there are cool applications: Scheduling umpires for US Open, solving jigsaw puzzles,…

Need to think about how to escape from local optima

- Random restarts
- Allowing for bad moves
- ...